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Equations of Motion of Robotic System With Piezo-Modified Viscoelastic and Magnetorheological Elements of Fractional Order

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In this communication, the Rodriguez method is proposed for modeling dynamics of the robotic system, where Lagrange's equations of second kind of rigid bodies system in covariant form are used. Discrete hybrid elements with so called piezo-modified Kelvin-Voight (PKV) and magnetorheological (MRD) type of viscoelastic models with fractional order derivatives are introduced in to the system of motion equations by means of generalized forces. The results obtained are illustrated by numerical examples.

1 Introduction

Coupling of a rigid multibody system with a spring, damper or actuator elements can be introduced in to the system of Lagrange's equations of second kind by using the generalized forces of the elements [1, 2]. Many papers have been published on the subject of fractional calculus and its applications in modelling of viscoelastic materials. By experimental validation it has been concluded that such a models are much better representation of real material properties with less parameters used than in models with integer order derivatives. Analysis and applications of fractional calculus in robotic systems are well presented in [3]. Numerical approximation methods for fractional order derivatives are given in [4]. In this paper, we considered motion equations and generalized forces of the robotic system (see Fig. 1) in which for a two different bodies in the system PKV or MRD elements are connected and described with fractional order derivative models. In a numerical example simple three rigid bodies robotic system is given and generalized forces are determined.

2 Formulation of the problem

Equations of motion of a robotic system are

$$\sum_{\alpha=1}^{n} a_{\alpha\gamma}(q)\cdot\ddot{q}^{\alpha}+\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\Gamma_{\alpha\beta,\gamma}(q)\cdot\dot{q}^{\alpha}\dot{q}^{\beta}=Q_{\gamma},\qquad \gamma=1,...,n, \tag{1}$$

where q and $\dot{q}$ are generalized coordinates and velocities, respectively, $a_{\alpha\gamma}$ are elements of the basic metric tensor, $\Gamma_{\alpha\beta,\gamma}$ are Christoffel symbols of the first kind and Q_γ are generalized forces. In (1) we can introduce a discrete magnetorheological damping element (MRD) or a discrete piezo-modified viscoelastic element (PKV) by using the generalized forces of these elements. The mathematical models of the MRD element with neglected hysteresis part and the PKV element are given respectively as

$$F_{mr}=E_c\cdot x+E_m D^{\lambda}(x),\qquad E_m=\mu_0+u\cdot\mu, \tag{2}$$

$$F_{pz}=(E_{pz}+E_c)\cdot x+E_d D^{\lambda}(x)-\alpha_{pz}E_{pz}U_{pz}, \tag{3}$$

where x is dilatation of the element, F_{pz} and F_{mr} are forces of the PKV and MRD elements, rescpectively, E_{pz}, E_d, E_m and E_c are constant coefficients, α_{pz} is piezoelectric constant and U_{pz} is piezo voltage. In (3) u is the voltage applied to the electromagnet of the MRD element, μ_0 is the initial constant of the MR fluid and μ is the constant which vary linearly with the applied voltage u. The operator D^λ is fractional order derivative operator of Riemann-Liouville definition. In both relations (2) and (3) dilatation of the elements $x(q^i(t),q^{i+1}(t),...,q^k(t))$ is equal to the difference of the instant length l of the elements and constant initial length l_0 and it is the composite function depending on the generalized coordinates which are time dependent functions. By using the princilple of virtual work we obtained generalized forces of the MRD and PKV elements, respectively as

$$Q_{\beta}^{w}=-\left[E_c\cdot x\cdot\frac{\vec{l}}{l}\frac{\partial(\vec{l})}{\partial q^{\beta}}+E_m\cdot\frac{\vec{l}}{l}\frac{\partial(\vec{l})}{\partial q^{\beta}}\cdot D^{\lambda}(x)\right],\ \vec{l}=\overrightarrow{D_iD_j}, \tag{4}$$

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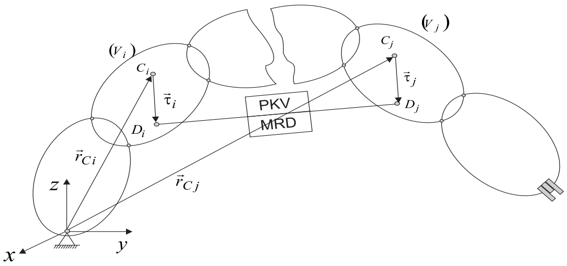

Fig. 1: Multibody robotic system with PKV or MRD element.

Table 1: Values of parameters used in numerical examples.

MRD	Value	PKV	Value
l_0	70 cm	l_0	70 cm
u	1 V	U_{pz}	60 V
μ_0	50 Ns/cm	E_d	50 Ns/cm
μ	10 Ns/cm · V	E_c	20 Ns/cm
E_c	25 N/cm	E_{pz}	30 N/cm
-	-	α_{pz}	10^{-3} cm/V

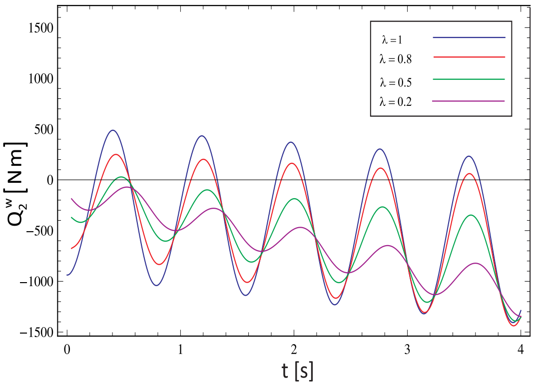

Fig. 2: Generalized force Q_2^w of the MRD element for different fractional order parameters.

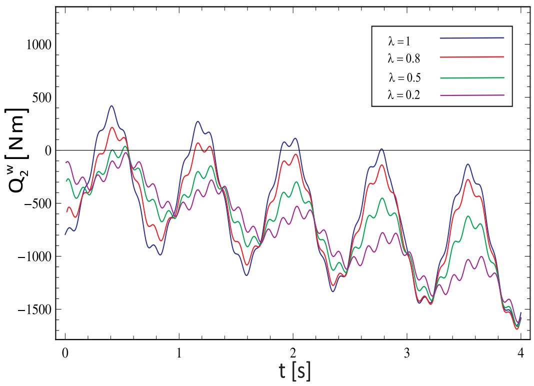

Fig. 3: Generalized force Q_2^w of the PKV element for different fractional order parameters.

$$Q_\beta^w = -\left[(E_c + E_{pz}) \cdot x \cdot \frac{\vec{l}}{l}\frac{\partial(\vec{l})}{\partial q^\beta} + E_d \cdot \frac{\vec{l}}{l}\frac{\partial(\vec{l})}{\partial q^\beta} \cdot D^\lambda(x) - \alpha_{pz} E_{pz} U_{pz} \cdot \frac{\vec{l}}{l}\frac{\partial(\vec{l})}{\partial q^\beta}\right]. \quad (5)$$

3 Numerical examples

We used the simple three rigid body system with the PKV or MRD element connected to the first and the third body in the system. To determine generalized forces, first we solved inverse kinematics positioning problem for sinusoidal trajectory of the gripper and then we obtained generalized coordinates. In our case x is the composite function and to find the fractional order derivative it is not correct to apply the classical chain rule. Therefore, we used the numerical approximation scheme proposed by Atanacković [4]. In Figs. 2 and 3 are presented only the values of Q_2^w for the MRD and PKV element, respectively. In Fig. 2 one can see values of Q_2^w of the MRD element for different values of fractional order parameters. It can be seen that changing the fractional order of derivative one can obtain values in the region which is unavailable by using the integer order derivative models. Also, it is possible to investigate the influence of system parameters on generalized forces by changing the voltage applied to the electromagnet. For the fractional order PKV element and applied sinusoidal signal on the piezo ceramics with the frequency of 8 Hz we can see that the value of Q_2^w oscillates with the frequency of applied signal (Fig. 3) and its behavior can be interpreted as some external force acting on the system. Similar as in case of MRD element it is possible to investigate the influence on the generalized forces by changing the system parameters. For the future investigation the interesting cases are: to introduce the hysteresis part in to the MRD model, to investigate damping of parasite vibrations in the system and influence on motion equations and positioning accuracy of the gripper.

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