Warping Torsion of Non-uniform Thin-walled Open section at Cantilever Beams

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The problem of torsion of the I-section beam is considered in this work, with emphasis on the variable cross section of the beam. Linear variation of height is concerned as most practical one for the design of cantilever beams. The solution for adopted cases of beams is obtained numerically, according to the given ordinary differential equation which deals with pure torsion along with warping torsion. The models are based on practical tailore-made beams. The comparison of results is done with uniform cantilever beam models which can be used as one view for stress check of cantilever beams subjected to bending with torsion.

Keywords: *I-beam, open cross section, variable section, warping,bending, torsion*

1. INTRODUCTION

The usage of variable cross section elements in structural systems is present since the beginning of the industrialization. In early stages, it was strongly connected with limits of production technology for steel beams [1], which forced engineers to tailor beams with existing ones. Nowadays, the variation in beam cross section is mostly related to the optimization due to reduction of the weight as the goal.

The variation of the cross section can be in one or two directions (height, width or both) with linear or parabolic mode [2]. However, the most practical variation is linear variation of beam height which will be also addressed in this paper. This positive effect for beam carrying capacity due to bending is obvious and authors will not underline this furthermore.

The basis for cross section variation will be carried out at I-section beam which is standard structural element with wide application in material handling machines. Although this section has allocation which is appropriate for bending over section major axis, it belongs to the class of "open" thin-walled sections which are highly sensitive to torsional effects. The one view to the optimization of uniform I-section beam due to torsional effects, considering minimum mass (minimum section area) as a goal, is given in [3,4].

The torsion of I-section beams can occure with loads such as inertia in horizontal direction or wind in outdoor structural systems. Faced with such loads, the designer needs to evaluate the magnitudes of the torsional effects and to consider the resistance of the member under the combined bending and torsion. Tipical design problem which complies with previous notes is design of outdoor foot mounted jib crane, Fig.1.

In some circumstances, torsional effects (with significant torsional moment) leads to redesign the beam into "closed" structural hollow section.

The aim of this paper is to describe the problem of torsional effects on variable I-section cantilever beam subjected to operational torsional moments. Warping torsion on variable open section is probably unfamilliar to most structural engineers, due to scarce literature in that field of structural analysis.

Figure 1. Foot-mounted jib crane

Also, this paper gives some practical aspects for the designers of cantilever girders such as long span jib crane where this postulation needs to be performed due to safety check.

2. WARPING TORSION

Only brief overview is given here since the torsional performance of open structural section distinguishes St Venant torsional effects (pure torsion) and warping torsional effects. The interaction between these two types depends on parameters of the cross section, loads and element length. The classical formulation for open thin-walled sections subjected to torsion was developed by Vlasov [5].

The problem arises with complexity with non-uniform sections as it will be addressed here.

At any point in the span, the torsion is defined with the expression

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$$M = M_t + M_w \tag{1}$$

where M_t is St Venant torsional moment and M_w is warping moment.

These two values can be determined with formulation of deflected shape of the beam given by the following ordinary differential equation (ODF)

$$\theta'' - \frac{GI_t}{EI_w}\theta = -\frac{M}{EI_w} \tag{2}$$

where:

I_t - torsional constant,
I_w - warping constant,
G - shear modulus,
E - modulus of elasticity and
$\theta=\theta(z)$ - rotation per unit length.

The boundary conditions arise from the fact that the beam is fixed at one end, while it is free at the other, which leads to

$$\theta(0) = 0,\ \ \theta'(L) = 0 \tag{3}$$

where L is beam length.

In the case of uniform cross sections, I_t and I_w have constant values and the solution of the differential equation (2) can be obtained analytically. On the other hand, for variable cross sections considered in this paper, where $I_t = I_t(z)$ and $I_w = I_w(z)$, shooting method [6] for solving this boundary value problem is applied. This method is implemented in the code written in Python programming language, using appropriate modules and functions from SciPy package, [7].

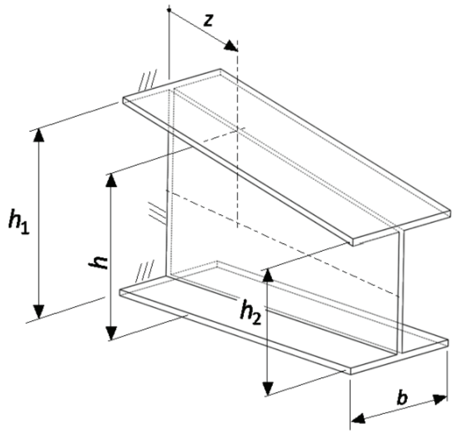

Figure 2. Linear variation of I-beam cross section

The linear variation of height $h=h(z)$ is assumed known with known values of section height at the clamped end, h_1, and section height at the free end, h_2 (Fig. 2). The heights of I-beam cross section are measured between the centerlines of flanges.

The torsional constant is given by

$$I_t = \frac{1}{3}(2bt_f{}^3 + h(z)t_w{}^3) \tag{4}$$

where:

b - beam width,
t_f - flange thickness and
t_w - web thickness.

The warping constant can be determined with the normalized warping function (Fig. 3) as

$$I_w = \frac{1}{24}t_f b^3 h(z)^2 \tag{5}$$

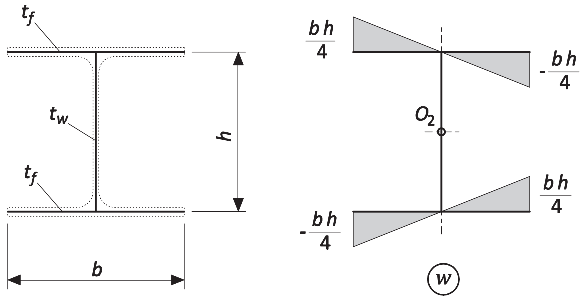

Figure 3. I-beam cross section warping function

The torsional moment will be adopted per each case of beam model. Determination of title problem needs numerical analysis for specific beam models.

3. CANTILEVER BEAM MODELS

The solution of (2) will be obtained for several models of the cantilever beam. With respect to practical aspects of this work, the results will be obtained for cantilever beam models with the length of L=6 m. According to authors, this can be established as starting length where inclusion of linear variation of beam height can give benefits versus increased costs due to manufacturing process of such beam.

The modulus are taken as G=8000 kN/cm^2 and E=21000 kN/cm^2, i.e. for construction steel material.

The torsional moment will be adopted due to horizontal loads of cantilever beams. Regardless if concerned as horizontal inertial effects or wind load on the payload which is carried by bottom flange of the section, it is assumed that torsional moment originates from the horizontal force which is 10 % of vertical capacity of the cantilever beam.

There are calculated 6 cases, with parameters given in Table 1. The parameters are relying on technical characteristics of IPE sections, where data correspond to "tailor" modus of sizes 160, 200, 240, 300, 400, 500, respectively to cases.

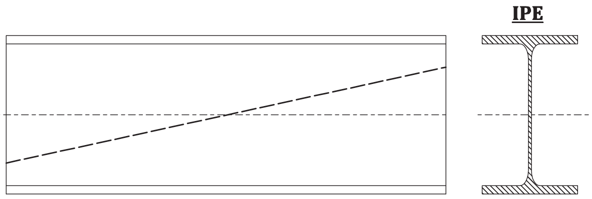

Figure 4. Production modus for variaton of I-beam

Parallel to these cases with variable sections, the solutions are obtained for uniform section beams where the height corresponds to h_1 (which assures same bending resistance at the clamped end of cantilever beam).

Table 1. Beam models

	h_1	h_2	b	t_f	t_w	M
CASE	[cm]	[cm]	[cm]	[cm]	[cm]	[kNcm]
I	24.86	5.66	8.2	0.74	0.5	1
II	31.15	7.15	10	0.85	0.56	2.4
III	37.42	8.62	12	0.98	0.62	4.7
IV	46.93	10.93	15	1.07	0.71	10.2
V	62.65	14.65	18	1.35	0.86	24
VI	78.40	18.40	20	1.6	1.02	44.5

4. NUMERICAL RESULTS AND DISCUSSION

The solution of (2) with boundary conditions (3) is obtained with the in-house software. The results are depicted with following charts, for each case.

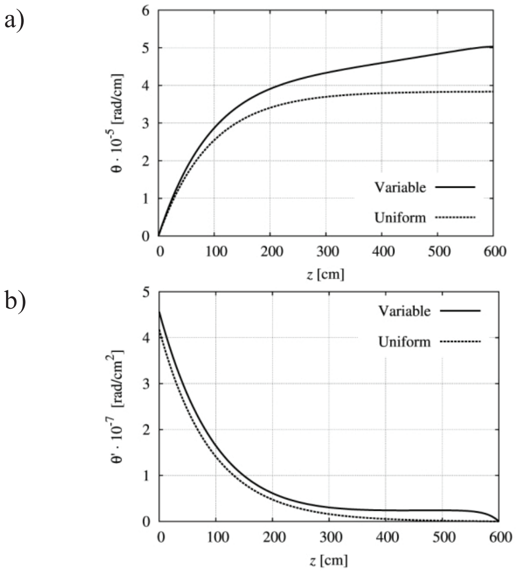

Figure 5. Case I

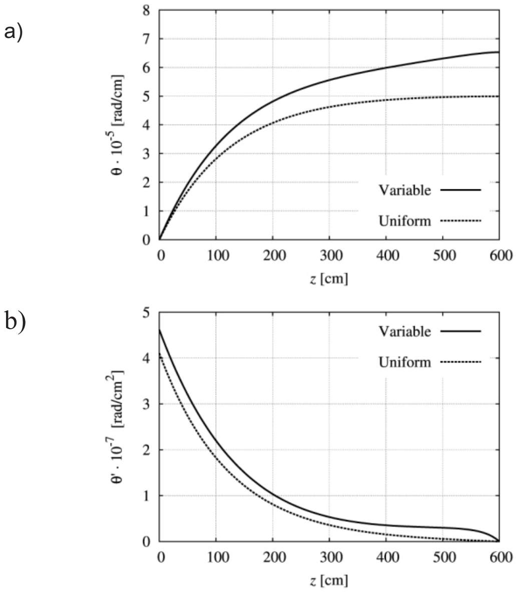

Figure 6. Case II

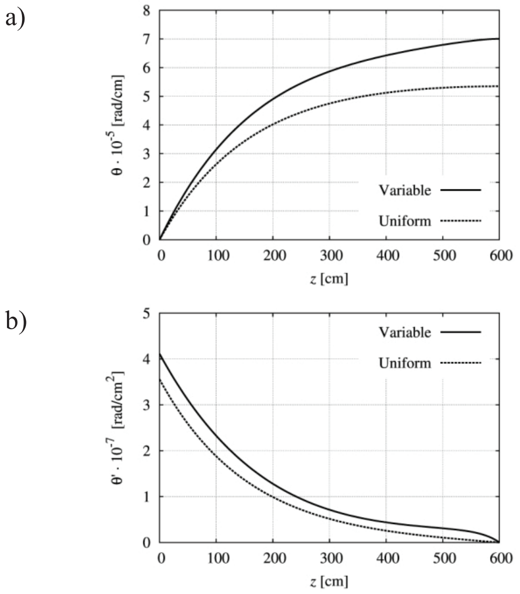

Figure 7. Case III

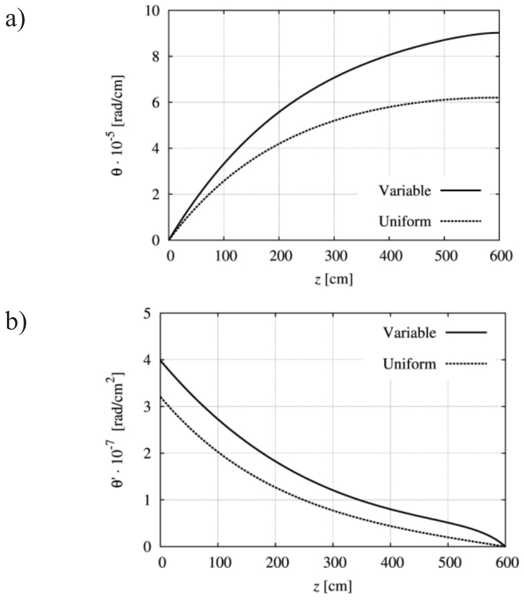

Figure 8. Case IV

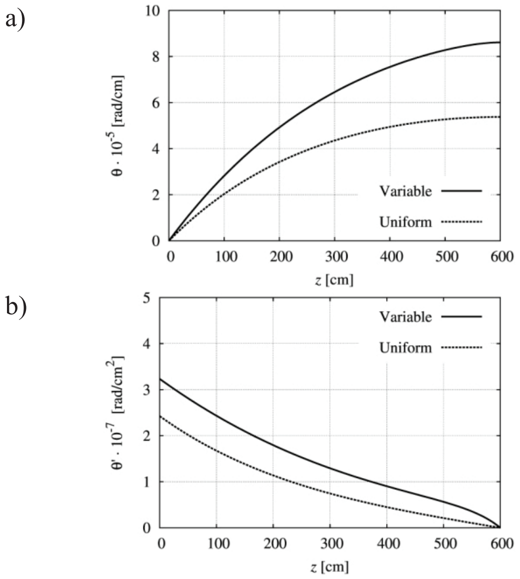

Figure 9. Case V

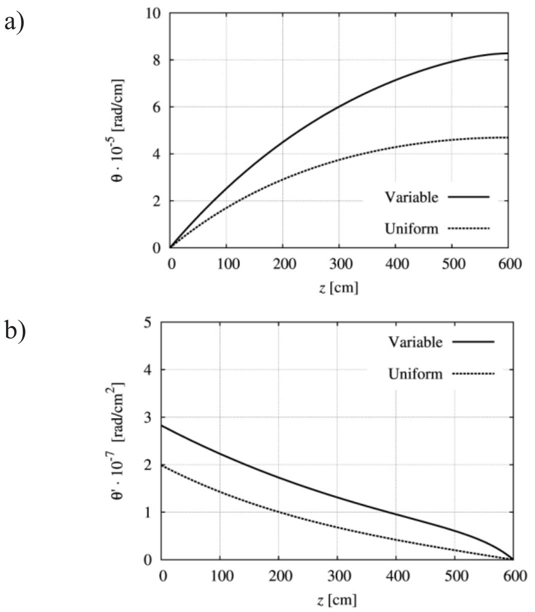

Figure 10. Case VI

There are obtained main torsional characteristics of the deflected beam, i.e. θ(z) and θ'(z), which can provide beam loads and stresses with known expressions [8]. It is obvious that values for θ(z) and θ'(z) are higher for the variable cross section which is expected, for each case.

Only stresses due to torsion will be mentioned here. St Venant shear stresses (τ) are proportional to θ(z), thus maximum values occures at the free end of the cantilever beam. Restraint of warping produces longitudinal stresses and shear stresses. In practice, the warping shear stresses are small enough to be neglected. However, the longitudinal warping stresses (σ_w) are of importance and they are greatest at the flange tips with maximum values for clamped end of cantilever beams. Since this is the place where bending is also checked, their calculation is needed. Table 2 gives the summary of results for uniform and variable sections for each case.

Table 2. Comparation of uniform vs. variable section

	UNIFORM		VARIABLE		
	max(τ)	max(σ_w)	Mass reduction	τ increas.	σ_w increas.
CASE	[kN/cm^2]	[kN/cm^2]	[%]	[%]	[%]
I	0.23	0.44	19.53	33	10.2
II	0.34	0.67	19.50	31	12.2
III	0.44	0.83	19.09	28	15.7
IV	0.61	1.18	19.51	32	24.3
V	0.76	1.43	20.1	31	33.4
VI	0.86	2.32	21.2	35	42.4

One may see that usage of the variable cross section gives average 20% reduction of beam mass. This can save operational energy costs of the crane.

The stresses in beams are higher than for uniform sections, mentioned as expected. The shear stresses are 30 % bigger but their initial values are very small and can be neglected in stress check. However, the longitudinal warping stresses have values which are important for stress check. Their increasment goes up to 42%. The backup for this can be the partial factor for resistance of beam which is smaller when inertial forces or wind forces are implemented in the calculation.

5. CONCLUSION

It is introduced warping torsion for cantilever beams with the variable section. Since numerous factors have the influence on postulated problem, no general remarks can be given at this stage. However, it is recommended to use variable sections as one way of optimization of crane structure which needs detailed calculation methods. Due to the complexity of the problem, the extension would be to postulate simplified assessment of warping effects in order to facilitate the design problem for practical usage.

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