

Dynamic responses of a gantry crane structure due to an accelerating moving mass

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The dynamics of a two-dimensional gantry crane structure subjected to a moving trolley with hoist and payload is examined. Dynamic responses of structure, both in vertical (Y) and horizontal direction (X), are postulated using the combined finite element and analytical method and solved with the direct integration method. Instead of conventional moving force problem, the two-dimensional inertial effects due to the overall mass of trolley, hoist and payload have been considered in this paper. The title problem was solved by calculating the forced vibration responses of the jib crane structure with time-dependent overall mass and subjected to an equivalent moving force. Factors as speed and acceleration of the moving trolley were studied. Numerical results reveal that used approach is useful and can draw conclusions for structural design purposes of gantry cranes and approaches for creation of dynamic models.

Keywords: *gantry crane, moving mass, FEA, dynamic response, direct integration.*

1. INTRODUCTION

High-performance machines (HPM) for material handling and conveying, such as e.g. container cranes, huge gantry cranes, ship unloaders and ship loaders, etc., have found an extremely wide application in almost all areas of industrial activities. However, regardless of the differences, almost all considered machines from this class of equipment are exposed to the effects of a working load whose basic characteristic is the changeability of intensity and/or position relative to the support structure of the machine [1]. The expressed facts point to the exceptional significance of identifying their behavior under dynamic loads, as well as response of the considered machines or their subsystems, as an extremely important stage in their design, particularly having in mind that the improvement of performances is not adequately followed by methods of calculation in many cases. Also, good awareness of dynamic characteristics is necessary to reduce vibrations of the machine and, subsequently, to increase durability of the structure.

Dynamic behavior of HPM depends on relatively numerous factors and an analysis of dynamic behavior demands to solve previously, in a suitable manner, the following two problems [2]: how to create dynamic model of the machinery and how to create model of the external load.

This paper studies the dynamic responses of the large gantry crane structure subjected to the moving mass. It has following contributions: (i) Both the horizontal and the vertical response of a gantry crane structure under moving load are investigated; (ii) It is given comparasion of the two basic approaches in dynamic modelling; (iii) Real cycle of trolley movement is included, with acceleration/deceleration periods in speed pattern.

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2. MOVING LOAD (MASS) PROBLEM IN STRUCTURAL DYNAMICS OF CRANES

Over time, cranes sizes and lifting capacities have increased. While the size, mass and strength of the crane structure have also increased, the stiffness of the crane structure has not been increased proportionally. That means the crane response to the trolley motion has changed and can cause unwanted crane deflections. Increased trolley and hoist speeds are obvious targets for increased productivity [2]. Vibrations are a serious problem in crane systems that are required to perform precise motion in the presence of structural flexibility. In practice it is very difficult and expensive to do experimental research on a real-size high-performance crane or even on a scale-model. For this reason investigations on mathematical models are a necessity, especially during the design stage.

Previous notes are related with moving load problem as special topic in structural dynamics. Irrespective of the many viewpoints and analytical methods proposed to solve the dynamic problems, most research can be grouped into two categories: the moving force problem and the moving mass problem. The additional aproach is moving oscillator model which is only reasonable to be used in some special structures because of its complexity [3]. The basic understanding of the moving force phenomena is given in excellent monograph by Fryba, [4]. In most moving force models the magnitude of the contact force is constant in time which implies that the inertia forces of a moving body are neglected. Evenso, the moving force models are simple to use and yield reasonable structural results in some cases [5]. The moving mass problem implies the existence of an interaction force between the moving mass and the structure during the time the mass travels along the structure, to which the following factors contribute: the inertia of the mass, the centrifugal force, the Coriolis force and the time-varying speed-dependent forces. Hence, the speed of the moving mass, structural flexibility and the ratio of the moving mass and structure mass are important factors that contribute to the creation of the interaction force. Michaltsos et al. [6] have studied the effect of a moving mass and other parameters, such as magnitude and speed of the moving mass, on the dynamic response of a simply supported beam. Modern researches in this field are based on combined finite element and analytical method for the crane dynamics [7]. Moving mass approach is needed in analysis of dynamic behaviour of crane structure, [8], because this gives more accurate results for responses than moving force approach. This paper improves the concept of moving mass approach in dynamics of high-performanse gantry crane.

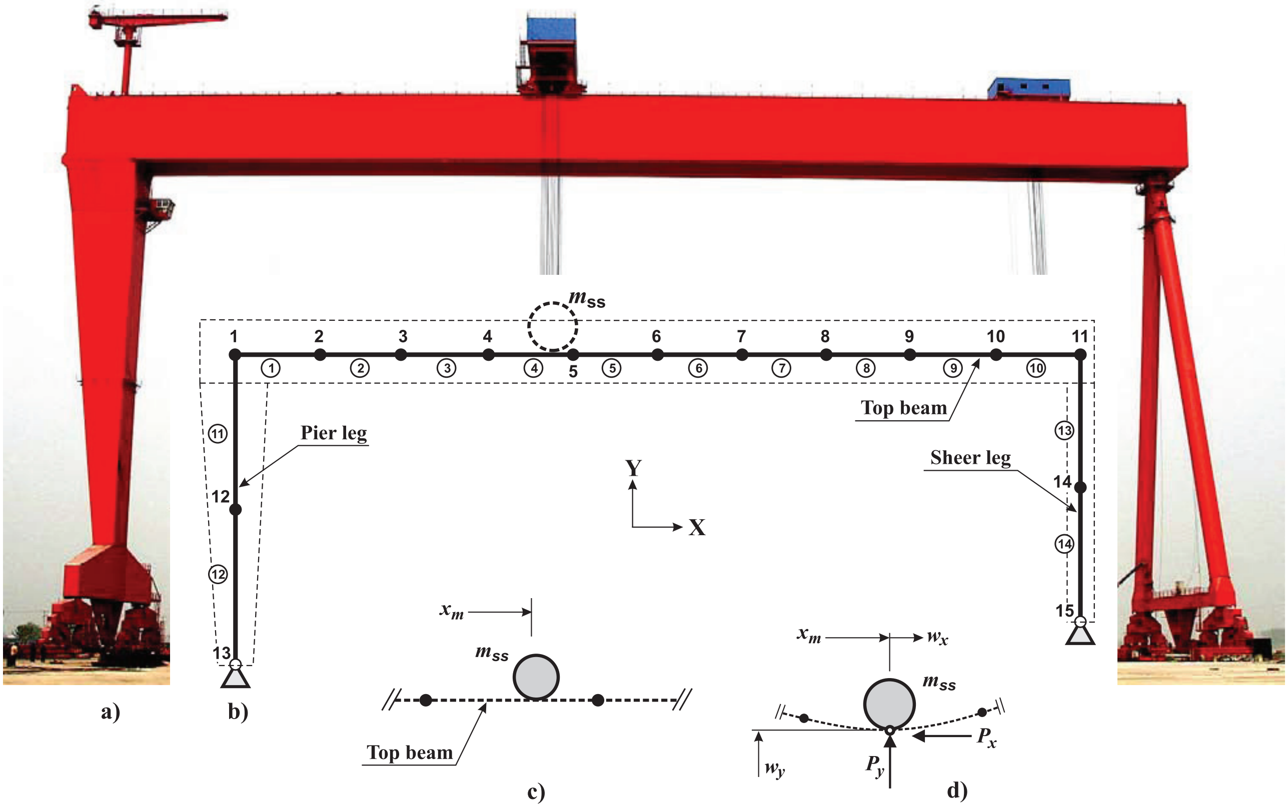


Figure 1. a) Real model of the gantry crane, b) FE model of the framework, c) Moving mass, d) Dynamic interaction

3. MODEL FORMULATION

The general approach in moving load problems at cranes is also used here, thus the system of the gantry crane (Fig.1.a) is divided into two parts: the framework (structure) and the moving system. The framework is a 2D discrete model consisted of top beam with length L, pier leg with height H and sheer leg with height h. The discretization of the framework (Fig.1.b) is done by using FEM, with plane-frame elements, as shown more detailed in [9]. The top beam is divided in 10 identical elements and each leg by 2 elements. Hence, framework has 41 DOF's (with extraction of the restrained displacements from supports) forming the structure displacement vector **U**.

The moving system (Fig.1.c) with total mass m_{ss} is consisted of the mass of trolley and mass of hoist with payload. It is assumed that mass m_{ss} is always in contact with the top beam. The global position of the moving system on the top beam, Fig. 2.a, is assumed known and defined by coordinate $x_m(t)$. Here, the acceleration-deceleration is also included in calculation because of the trolley trapezoidal speed pattern, Fig.3. It is assumed by model of the gantry crane system, that a loading is symmetrically distributed on the top beam rail(s) and furthermore that relationship between the framework and the moving system can be simplified into one moving load $P(t)$, with projections in two-dimensional directions $P_x(t)$ and $P_y(t)$, Fig. 2.a.

4. PROPERTY MATRICES AND EXTERNAL LOAD VECTOR

The equation of motion for a framework (structural system) is represented as follows

$$\mathbf{M}_{st}\ddot{\mathbf{U}} + \mathbf{C}_{st}\dot{\mathbf{U}} + \mathbf{K}_{st}\mathbf{U} = \mathbf{P}(t) \qquad (1)$$

where $\mathbf{M}_{st}$, $\mathbf{C}_{st}$, $\mathbf{K}_{st}$ are the mass, damping and stiffness matrices of the structural system; $\ddot{\mathbf{U}}, \dot{\mathbf{U}}, \mathbf{U}$, are the respective acceleration, velocity and displacement vectors for the system and $\mathbf{P}(t)$ is the external force vector acting upon the structure.

4.1 Structural stiffness and mass matrix

According to the shown FE model of the framework, the stiffness matrix can be obtained by assembling all the element stiffness matrices [10] up to forming the square matrix $\mathbf{K_{st}}$ that corresponds with 41 DOF's of the structure. Similarly, the mass matrix of the framework $\mathbf{M_{st}}$ can be obtained.

4.2 Structural damping matrix

Here, as usually practiced in FE dynamic analysis of cranes, the damping matrix is constructed by using the theory of Rayleigh damping in following form [11]

$$\mathbf{C_{st}} = a\,\mathbf{M_{st}} + b\,\mathbf{K_{st}} \tag{2}$$

with determined stiffness and mass matrix in last subsection, while constants a and b are

$$a = \frac{2\omega_i\omega_j(\xi_i\omega_j - \xi_j\omega_i)}{\omega_j^2 - \omega_i^2},\ b = \frac{2(\xi_j\omega_j - \xi_i\omega_i)}{\omega_j^2 - \omega_i^2} \tag{3}$$

where damping ratios ξ_i, ξ_j, correspond to first two frequencies of the structure ω, thus i=1, j=2.

4.3 Equivalent nodal forces

The *equivalent nodal force vector* for the element s, at which the moving mass is located (Fig. 2), takes the following form

$$\{f^s(t)\} = [f_1^s(t)\quad f_2^s(t)\quad f_3^s(t)\quad f_4^s(t)\quad f_5^s(t)\quad f_6^s(t)] \tag{4}$$

where

$$f_1^s(t) = N_1(x)P_x(t),\ f_4^s(t) = N_4(x)P_x(t),$$
$$f_2^s(t) = -N_2(x)P_y(t),\ f_3^s(t) = -N_3(x)P_y(t),$$
$$f_5^s(t) = -N_5(x)P_y(t),\ f_6^s(t) = -N_6(x)P_y(t)$$

Noting that l is the element length and x is the distance along the element s to the point of the application of the forces (Fig. 2), the relative distance is given by $\xi = x/l$, and the shape functions [9], $N_i = N_i(x) = N_i(\xi)$ (i =1-6), take the following form

$$N_1 = 1-\xi,\ N_4 = \xi,$$
$$N_2 = 1-3\xi^2+2\xi^3,\ N_3 = l(\xi - 2\xi^2 + \xi^3),$$
$$N_5 = 3\xi^2 - 2\xi^3,\ N_6 = l(-\xi^2+\xi^3) \tag{5}$$

In order to model the moving load, one may apply forces and moments which are function of time to all the nodes of the FE model of the structure. The nodes of the sth element are s and s+1. The nodal forces for the nodes of the element s, where the moving load is located at, can be calculated from Eqs. (4,5), while other values are equal to zero.

With known position of the moving system, the element number s can be found as

$$s = IntegerPart[\frac{x_m(t)}{l}]+1 \tag{6}$$

The nodal forces (4) can be calculated in terms of the global position, by

$$\xi = \frac{x_m(t) - (s-1)l}{l} \tag{7}$$

The total time τ needed for moving system to travel from the left end to the right end of the top beam, is now divided into p steps with a time interval Δt. At any time $t = r\Delta t$ the nodal forces can be calculated with (7,6,5,4). Hence, the equivalent force vector due to moving load is determined.

According to [11], the structural external force vector can take the following form

$$\mathbf{P}(t) = \begin{bmatrix} 0..0 & f_1^s & f_2^s & f_3^s & f_4^s & f_5^s & f_6^s & 0..0 \end{bmatrix}^T \tag{8}$$

In matrix form, one may represent the element force vector as

$$\{f^s(t)\} = \mathbf{N_x}^T P_x - \mathbf{N_y}^T P_y \tag{9}$$

where

$$\mathbf{N_x} = [N_1\quad 0\quad 0\quad N_4\quad 0\quad 0] \tag{10}$$
$$\mathbf{N_y} = [0\quad N_2\quad N_3\quad 0\quad N_5\quad N_6] \tag{11}$$

Adjustment with total DOF's give the following matrices

$$\mathbf{N_X} = [0\quad 0\quad \ldots\quad \mathbf{N_x}\quad \ldots\quad 0] \tag{12}$$
$$\mathbf{N_Y} = [0\quad 0\quad \ldots\quad \mathbf{N_y}\quad \ldots\quad 0] \tag{13}$$

These matrices are with non-zero elements which correspond to the DOF's of element s where the moving load is located at. Other elements are zero. The submatrices $\mathbf{N_x}$ and $\mathbf{N_y}$ are calculated by (7,6,5) at each time step. One may see that these matrices engage only 6 values which *move along* in the (12,13), as moving load changes the position on the top beam.

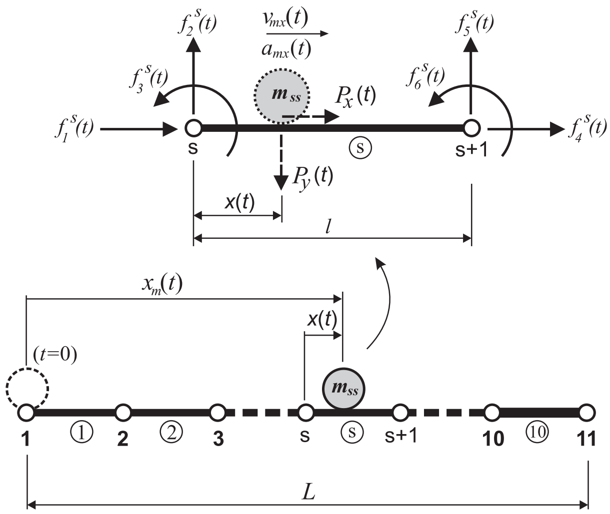


Figure 2. Equivalent nodal forces of the (s)th beam element subjected to the moving mass

It should be pointed that this approach lead to exact finite element modelling of the moving externall load and can be used for any runway.

5. DERIVATION OF DIFFERENTIAL EQUATION OF MOTION FOR THE SYSTEM

The governing equation of motion for a multiple degree of freedom system becomes

$$\mathbf{M_{st}}\ddot{\mathbf{U}}+\mathbf{C_{st}}\dot{\mathbf{U}}+\mathbf{K_{st}}\mathbf{U}=\mathbf{N}_X^T P_x-\mathbf{N}_Y^T P_y \qquad (14)$$

The interaction forces between the structure and the moving load can be obtained from the dynamic analysis of moving system (Fig. 1.d), such as

$$P_x=-(m_{ss})\ddot{x}_m-(m_{ss})\ddot{w}_x \qquad (15)$$

$$P_y=(m_{ss})g+m_{ss}\ddot{w}_y \qquad (16)$$

The axial deformation at any location within the finite element $w_x=w_x(x,t)$ and transversal deformation at any location within the finite element $w_x=w_x(x,t)$, can be presented in matrix form [12]

$$w_x=\mathbf{N_X}\mathbf{U}\,,\; w_y=\mathbf{N_Y}\mathbf{U} \qquad (17)$$

Second derivates of the expression (25) can be presented [12], in following form

$$\ddot{w}_x\approx\mathbf{N_X}\ddot{\mathbf{U}} \qquad (18)$$

$$\ddot{w}_y=\mathbf{N}''_{\mathbf{Y}}\mathbf{U}\,\dot{x}^2+\mathbf{N}'_{\mathbf{Y}}\mathbf{U}\,\ddot{x}+2\mathbf{N}'_{\mathbf{Y}}\dot{\mathbf{U}}\,\dot{x}+\mathbf{N_Y}\ddot{\mathbf{U}} \qquad (19)$$

where the superscripts ('), (") are representing the first and second derivate of expressions (5) with respect to x, while $\dot{x}$ is the velocity and $\ddot{x}$ is the acceleration of the trolley.

With substituting (18,19) into (15,16), and substituting with (17) in (14), overall equation of motion becomes

$$(\mathbf{M}_{st}+\mathbf{M}_1)\ddot{\mathbf{U}}+(\mathbf{C}_{st}+\mathbf{C}_1)\dot{\mathbf{U}}+(\mathbf{K}_{st}+\mathbf{K}_1)\mathbf{U}= \\ =-\mathbf{N}_Y^T m_{ss}g-\mathbf{N}_X^T m_{ss}\ddot{x}_m \qquad (20)$$

where

$$\mathbf{M_1}=m_{ss}\mathbf{N_Y}^T\mathbf{N_Y}+m_{ss}\mathbf{N_X}^T\mathbf{N_X}$$

$$\mathbf{C_1}=2m_{ss}\,\dot{x}\mathbf{N_Y}^T\mathbf{N}'_{\mathbf{Y}}$$

$$\mathbf{K_1}=m_{ss}\dot{x}^2\mathbf{N_Y}^T\mathbf{N}''_{\mathbf{Y}}+m_{ss}\ddot{x}\mathbf{N_Y}^T\mathbf{N}'_{\mathbf{Y}}$$

6. NUMERICAL RESULTS AND DISCUSSION

Dynamic behaviour of the gantry crane subjected to moving mass is obtained by solution of the (20). Original, in-house software is created to solve the title problem with direct integration method based on the Newmark algorithm [13]. The time interval Δt is 0,005 s, unless particularly stated. The gravitational acceleration g is taken to be 9,81 m/s^2. The crane structure is made of steel with density 7850 kg/m^3 and modulus of elasticity 2,1 10^{11} Pa. Initial mathematical model include structural damping with $\xi=\xi_1=\xi_2=0{,}05$ %.

Geometric characteristics of the gantry crane are L=40 m and H=h=15 m. Element properties are: A_n=0,09 m^2, I_n=0,041 m^4 (n=1-10), A_{11}=0,085 m^2, I_{11}=0,036 m^4, A_{12}=0,07 m^2, I_{12}=0,024 m^4 and A_n=0,048 m^2, I_n=0,01 m^4 (n=13-14). The moving system is 60 t. The system moves with 2 velocity patterns, shown in Figure 3. Pattern v_1 corresponds to maximum speeds of nowadays systems of trolleys (www.liebherr.de), while the pattern v_2 is expected in the close future.

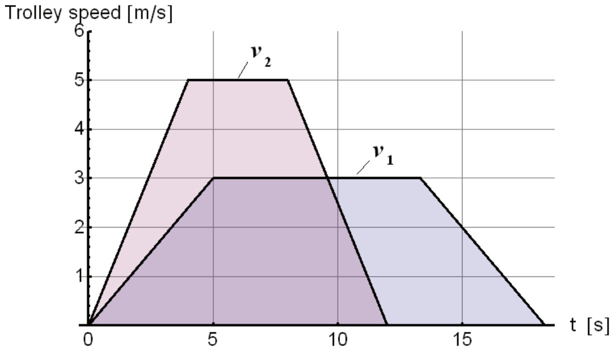


Figure 3. Trolley speed patterns

As expected from physical intuition, the biggest values of vertical displacements are for node 6, i.e. the middle (central) point. Horizontal displacement for all the top beam points are the same, because of axial stiffness of the elements. Figure 4 shows the results for both the speed patterns. The responses are higher for pattern v_2. This influence is very significant for horizontal dispacement U_{X1} where values reach the maximum of 5,41 cm in the deceleration period, while maximum values with pattern v_1 is 4,1 cm, Figure 4.a.

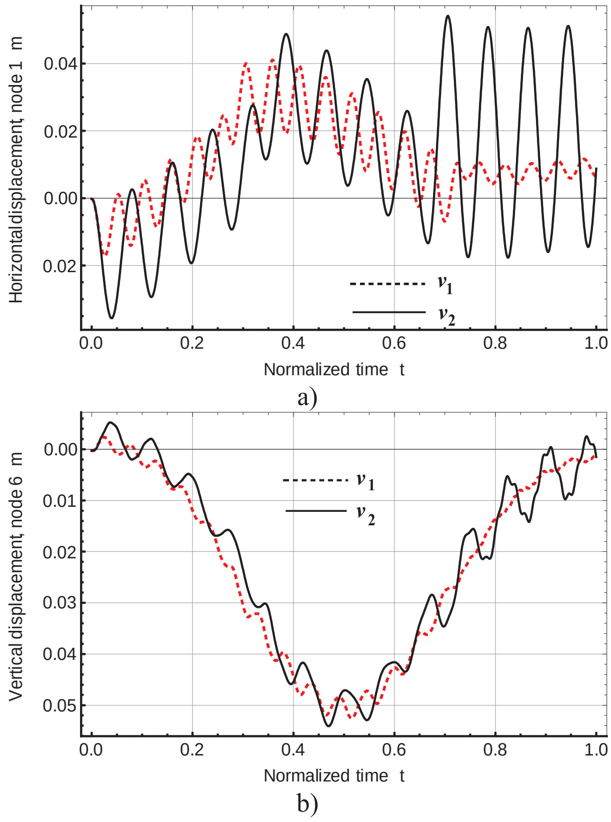


Figure 4. a) Horizontal displacement U_{X1}, b) Vertical displacement U_{Y6}; v_1, v_2

The difference is much smaller for vertical displacement of middle point, Figure 4.b. Maximum values occure when trolley is at midspan, and for v_1 is 5,2 cm and 5,4 cm for pattern v_2. It can be concluded that increase of acceleration, here 1,25 m/s^2, don't have significant influence on vertical displacements of structure, but only for horizontal displacements because horizontal inertia force is proportional to the value of acceleration or decceleration of the moving mass.

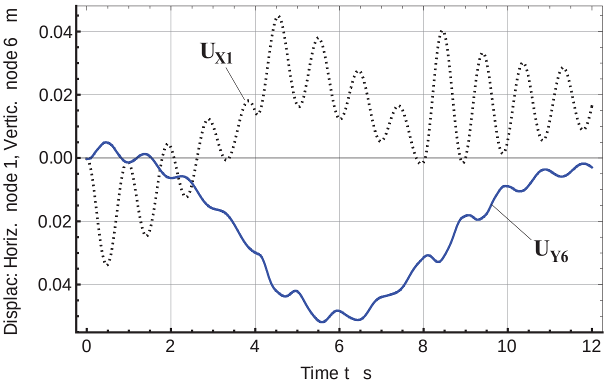
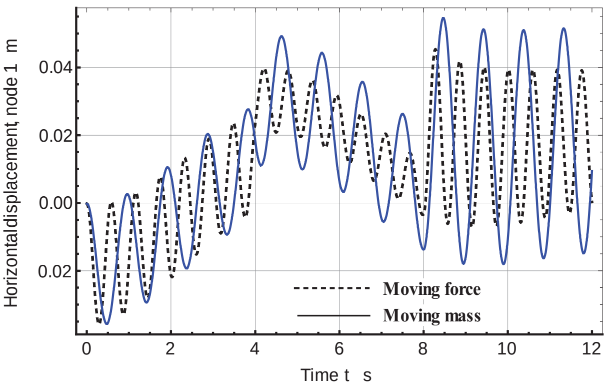
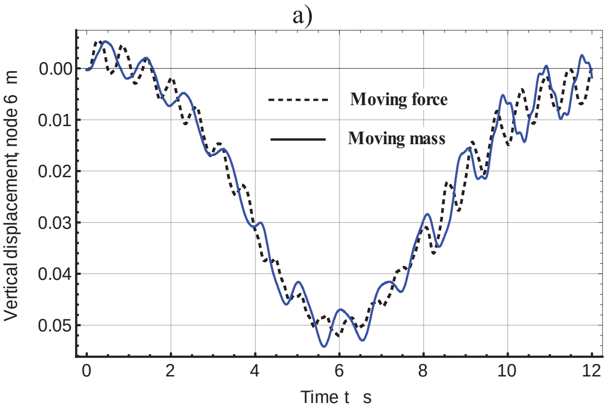


Figure 5. Displacements of node 6; **ξ**=0,06; ***v***$_2$

Here, it would be presented structural damping with ξ=0,06, Fig. 5. The damping has influence for both displacements of the node 6 (horizontal is the same as for node1). One can see the decrease of amplitudes, with comparison with results from Figure 4. This can lead to a conclusion that increase of structural damping can be the way to slightly decrease the influence of higher speed pattern. This is only descriptive because it is very hard to achieve damping in structure even as 6 %. Decrease of amplitude is also noticeable at horizontal displacement. With comparison with Fig. 6, one can see this decrease in constant speed period, before the deceleration. In the acceleration/deceleration periods maximal values are slightly decreased.



a)



b)

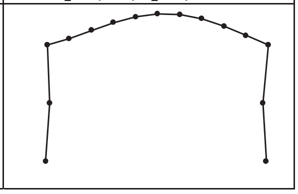
Figure 6. a) Horizontal displacement U_{X1}, b) Vertical displacement U_{Y6}; ***v***$_2$; Moving mass/moving force

The main advantage of presented model is that all the influence of moving mass is included. Figure 6 shows the comparison of these results with results when inertia effect of mass is ignored (moving force approach) for characteristic points of structure. Vertical displacements shows small difference in values, while horizontal displacement has significant errors. In constant speed period, horizontal displacement of node 1 is about 25% higher for moving mass model.

Amplitudes are higher (Figure 6) for both the displacements and oscillation period is bigger when moving mass approach is studied. This is more accurate approach for calculating the frequencies of the model.

For framework only, considering (1), the eigenvalue problem $\det(\mathbf{K_{st}} - \omega_i^2 \mathbf{M_{st}}) = 0$ gives following first and second frequency, with appropriate mode shapes (Table 1).

Table 1. Framework eigenfrequencies

1. mode	2. mode
T_1=0,58s, f_1=1,72 Hz	T_2=0,2 s, f_2=4,93 Hz
	

However, when mass is included one may calculate the frequencies of the whole system at each time step. The results for first 3 frequencies are shown at Figure 7. It is obvious that frequencies are dependent of position of mass is changed on top beam. Only 1. frequency, which is most important, is the same beacuse of character of the 1. mode shape. The second frequency is changing from 2 to 5 Hz, and 3. frequency goes from 9 to 14,5 Hz. These are important notice for gaining the frequency spectrum for the given system.

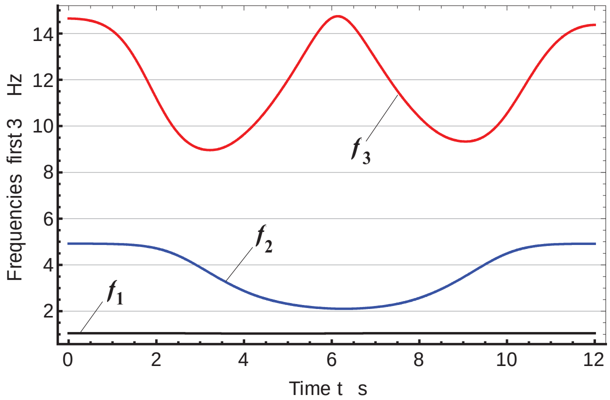


Figure 7. First 3 frequencies with moving mass model

7. CONCLUSION

Two-dimensional inertial effects due to the mass of the moving load are taken into consideration for obtaining the dynamical responses of the flexible gantry crane structure in both the vertical and horizontal direction.

It is shown that speed, acceleration (or deceleration) and structural damping have effect on the dynamic responses of the structure. The responses of the structure in the horizontal direction have significant influence due to the increase of the moving speed and acceleration of the moving load. This needs to be studied because the most researchers use conventional methods for showing vertical responses of a simple structure, e.g. beam, induced by a moving mass. Increase of structural damping can reduce the values of increased structure responses which are acompanied with achieving high performances at cranes. General remarks are given in this paper because of many factors that influence the dynamic behaviour of complex structure, such as frame-gantry structure. However, the aim of this work is to present mathematical model who is more accurate then models of moving force. Also, frequency spectar is more detailed with this model which can conclude that innertial effects of moving bodies should be taken in calculation of cranes in modern researches.

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NOMENCLATURE

$\mathbf{M_{st}}$	structural mass matrix
$\mathbf{C_{st}}$	structural damping matrix
$\mathbf{K_{st}}$	structural stiffness matrix
$\mathbf{U}$	nodal displacement vector
$\dot{\mathbf{U}}$	nodal velocity vector
$\ddot{\mathbf{U}}$	nodal acceleration vector
$\mathbf{P}(t)$	external force vector
m_{ss}	mass of the trolley
ω	angular frequency
$\xi_{i,j}$	damping ratios
f	frequency
$f_i^s(t)$	nodal forces of the element s
N_{i}	shape functions
l	element length
$x_m(t)$	position of the moving mass from the left end
P_x, P_y	projections of the interaction force
g	gravitational acceleration
τ	overall time
A	element cross-sectional area
I	moment of inertia
E	Young's modulus
ρ	mass density