Conditional optimization of computer automatic control system of an selected plant at arbitrary initial conditions

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The aim of this paper is to present effect of initial conditions on choice of optimal parameters in computer automatic control system of an selected actual plant. Considering is made in a parameter plane wherein the area of formerly guaranteed relative damping coefficient of all closed-loop poles is separated. A performance index is chosen to be sum of error squared (SSE), taking into account of arbitrary initial conditions.

Experimental results obtained on coupled-tanks plant are provided.

Keywords: conditional optimization, performance index, computer control system

1. INTRODUCTION

Over 95 % of coexisting practical industrial applications use PID (or PI as a special case) control algorithms and thus the suitable PID control design is still very actual, especially for systems under some nonlinearities, perturbations, or time-variant behaviour. Without any doubts, the ultimately primary and fundamental requirement of all applications is the stability of closed control loop, [1].

For the continuous-time PID controllers, in many works collected in [2], the stability regions in the space of the gains of the PID controllers are determined. A nice and simple procedure is also given in [3] that requires less numerical computations. One procedure is presented in [4], where the result of the parameter space approach [5] is used to derive the stability domain of PID controllers.

Nevertheless, PID controllers are very often implemented digitally using microprocessors [6]. Research on the stability of digital control system goes back to early 1960’s, when stability of such system was investigated. In [7, 8] the results of [2] are generalized to the case of digital PID controllers.

The aim of this work is to show that integral criterion has different value when we take into account non-zero initial conditions from value of the same integral criterion at zero initial conditions. At the same time, and more important, optimal controller gains can have drastically different values when we take into account nonzero initial conditions from values that we get at zero initial conditions.

In many papers, as in [9, 10, 11], are assumed zero initial conditions for calculation of actual system output. The past and the present of a dynamic system are contained in a initial conditions. That same initial conditions together with the external input fully determine the output of the system. Initial conditions cannot be chosen and they are, usually, totally unpredictable. Complete analysis of continious linear time invariant systems with nonzero initial conditions is presented in [12].

2. EXPERIMENTAL SETUP

The Coupled Tanks plant is a “Two-Tank” module made up of a pump with a water basin and two tanks. The two tanks are built in the front panel such that flow from the first (i.e. upper) tank can flow, through an outlet orifice located at the bottom of the tank, into the second (i.e. lower) tank. Flow from the second tank flows into the main water basin. To name a few, practical industrial applications of such Coupled-Tank structure can be found in the processing system of petro-chemical, paper making, and/or water treatment plants, [13].

Two experiments will be performed. Goal in the first experiment will be to manage the level of water in the second tank whereby the optimal controller parameters K_p and K_I will be derived at zero initial conditions. In a second experiment our aim is also to maintain a level of water in the second tank, only in that case the optimum parameters of the controller K_p and K_I will be obtained at nonzero initial conditions. While the optimal parameters K_p and K_I will be different in first and second experiment, both experiments will be conducted with the same nonzero initial conditions.

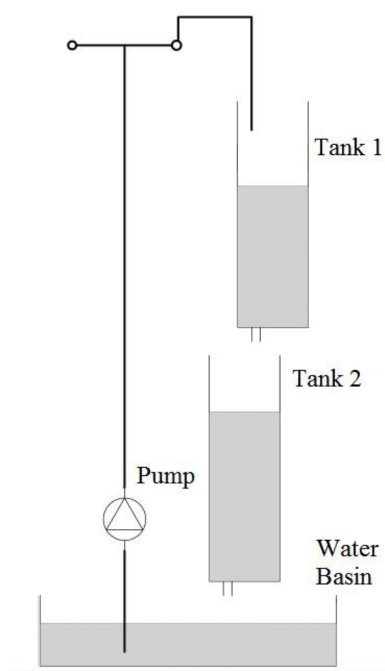

Figure 1: The Coupled Tanks plant

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3. MATHEMATICAL MODELING

In order to obtain the mathematical model of Coupled-Tank system it is necessary to bring to mind that the pump feeds into tank 1 and that tank 2 is not considered. Thus, the input to the process is the voltage to the pump V_p and its output is the water level in tank 1, H_1 (i.e. top tank). The purpose of the present modelling stage is to assure us with the system's open-loop transfer function $W_1(z)$ and $W_2(z)$ which in turn will be used to design an appropriate level controller. In obtaining the tank 1 equation of motion the mass balance principle can be applied to the water level in tank 1, i.e.

$$A_{t1}\frac{\mathrm{d}H_1}{\mathrm{d}t} = Q_{i1} - Q_{o1}, \tag{1}$$

where A_{t1} is the area of tank 1 while Q_{i1} and Q_{o1} are the inflow rate and outflow rate, respectively. The volumetric inflow rate to tank 1 is supposed to be directly proportional to the applied pump voltage, such that

$$Q_{i1} = KV_p. \tag{2}$$

Applying Bernoulli's equation for small orifices, the outflow velocity from tank 1, V_{o1}, can be expressed by the succeeding relationship

$$V_{o1} = \sqrt{2gH_1}. \tag{3}$$

In order to design and implement a linear level controller for the tank 1 system, the open-loop Laplace transfer function should be obtained. Nevertheless, by definition, such a transfer function can only express the system's dynamics from a linear differential equation. Because of that, the nonlinear equation of motion of tank 1 should be linearized around a nominal point of operation. By definition, static equilibrium at a nominal operating point (V_{pnom}, H_{1nom}) is presented by the tank 1 level being at a constant position H_{1nom} due to a constant water flow generated by constant pump voltage V_{pnom}. In the case of the water level in tank 1, the operating range corresponds to small deviations heights, h_1, and small deviations voltages, v_p, from the desired nominal point (V_{pnom}, H_{1nom}). Therefore, h_1 and v_p can be expressed as shown below

$$v_p = V_p - V_{pnom}, \tag{4}$$

$$h_1 = H_1 - H_{1nom}. \tag{5}$$

The derived linearized equation of motion should be a function of the system's small deviations about its nominal point (V_{pnom}, H_{1nom}). After linearization we get

$$W_1(s) = \frac{H_1(s)}{V_p(s)} = \frac{K_{dc1}}{\tau_1 s + 1}, \tag{6}$$

where K_{dc1} and τ_1 are tank 1's gain and time constant, respectively. Expression (6) represents tank 1's voltage-to-level 1 transfer function.

The water level equation of motion in tank 2 still needs to be obtained. The input to the tank 2 process is the water level, H_1, in tank 1 (generating the outflow feeding tank 2) and its output variable is the water level, H_2, in tank 2 (i.e. bottom tank). The obtained equation of motion should be a function of the system's input and output, as previously defined.

By implementing a similar procedure by which we obtain tank 1's transfer function, now we get

$$W_2(s) = \frac{H_2(s)}{H_1(s)} = \frac{K_{dc2}}{\tau_2 s + 1}, \tag{7}$$

where K_{dc2} and τ_2 are tank 2's gain and time constant, respectively. Expression (7) represents tank 2's level 1-to-level 2 transfer function.

Discretization of continuous time system is showed on figure 2.

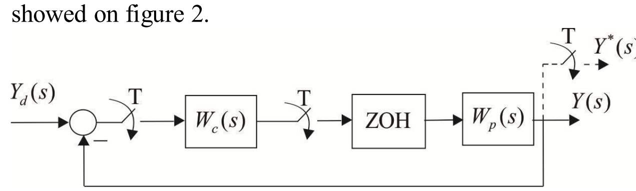

Figure 2: s-block diagram of discrete system

Zero-order hold assumes the control inputs are piecewise constant over the sampling period T. Applying zero-order hold method for finding z-transform of plant's s-transfer function we get following results

$$W_1(z) = \frac{c_2}{z - c_1}, \tag{8}$$

and

$$W_2(z) = \frac{c_4}{z - c_3}, \tag{9}$$

where c_1, c_2, c_3 and c_4 are corresponding real constants. Model verification of first tank discrete model is showed on figure 3.

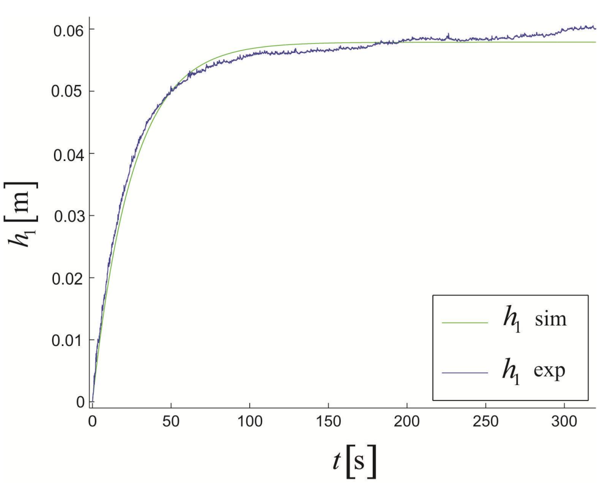

Figure 3: Verification of first tank discrete model

The transfer function of whole process is obtained by following relationship

$$W_p(z) = W_1(z)W_2(z). \tag{10}$$

Verification of whole plant's discrete transfer function model is showed on figure 4.

Transfer function of discrete PI controller is obtained by using trapezoid rule

$$W_c(z) = \frac{(2K_p + K_I \mathrm{T})z + K_I \mathrm{T} - 2K_p}{2z - 2}, \tag{11}$$

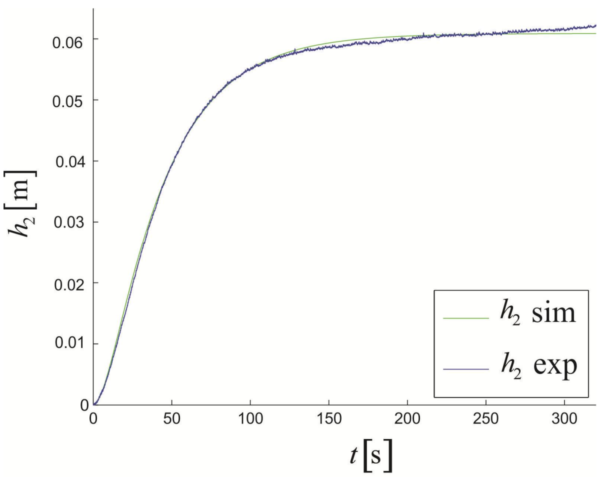

Figure 4: Verification of coupled tanks discrete model

where K_p and K_I are constants of proportional and integral gain, while T is the sampling period. Now we have z-block diagram of the same system.

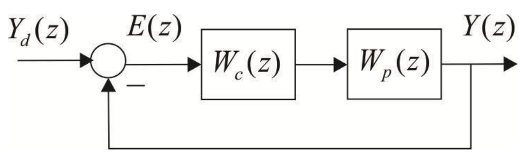

Figure 5: z-block diagram of discrete system

Transfer function of system showed on figure 5 is equal to

$$W(z)=\frac{W_c(z)W_p(z)}{1+W_c(z)W_p(z)}, \tag{12}$$

and this transfer function is used from now on.

4. RELATIVE STABILITY

In the design of sampled-data control system the characteristic polynomial have parameter dependent coefficients, and it is essential to determine the ranges of parameter values which ensure the system relative stability. Sampled-data control systems can be analyzed in both the s and z planes. Siljak [14] proposed the following procedure for the determination of the parameters α and β which are actually in our case K_p and K_I, respectively.

If the analysis is to be carried out in the s plane, the characteristic equation is given as

$$f\left(e^{s\mathrm{T}}\right)=\sum_{k=0}^{n}a_k e^{ks\mathrm{T}}=0, \tag{13}$$

where a_k $(k=0,1,...,n)$ are real coefficients. In order to discuss sampled-data systems in the z plane, it is essential to introduce the substitution

$$z=e^{s\mathrm{T}}, \tag{14}$$

and rewrite (13) as

$$f(z)=\sum_{k=0}^{n}a_k z^k=0. \tag{15}$$

By substituting

$$z=\varsigma_z\omega_z+j\omega_z\sqrt{1-\varsigma_z^{\,2}}, \tag{16}$$

into (15), (15) can be rewritten as following two equations

$$\sum_{k=0}^{n}a_k\omega_z^k T_k\left(\varsigma_z\right)=0, \tag{17}$$

$$\sum_{k=0}^{n}a_k\omega_z^k U_k\left(\varsigma_z\right)=0, \tag{18}$$

where $T_k\left(\varsigma_z\right)$ and $U_k\left(\varsigma_z\right)$ are the Chebyshev functions of the first and the second kinds, respectively. The argument of these functions is denoted by ς_z $\left(0\le\varsigma_z\le1\right)$. By using the relation

$$T_k\left(\varsigma_z\right)=\varsigma_z U_k\left(\varsigma_z\right)-U_{k-1}\left(\varsigma_z\right), \tag{19}$$

(17) and (18) can be further rewritten as

$$\sum_{k=0}^{n}-a_k\omega_z^k U_{k-1}\left(\varsigma_z\right)=0, \tag{20}$$

$$\sum_{k=0}^{n}a_k\omega_z^k U_k\left(\varsigma_z\right)=0. \tag{21}$$

Equations (20), (21) have an advantage over equations (17), (18) in that they use only one kind of the Chebyshev functions and the design procedure is much easier.

Now, we consider the case when the coefficients a_k $(k=0,1,...,n)$ are linear functions of system parameters α and β

$$a_k=b_k\alpha+c_k\beta+d_k. \tag{22}$$

Equations (20), (21) may be written in the form

$$\alpha B_1\left(\omega_z,\varsigma_z\right)+\beta C_1\left(\omega_z,\varsigma_z\right)+D_1\left(\omega_z,\varsigma_z\right)=0, \tag{23}$$

$$\alpha B_2\left(\omega_z,\varsigma_z\right)+\beta C_2\left(\omega_z,\varsigma_z\right)+D_2\left(\omega_z,\varsigma_z\right)=0, \tag{24}$$

where

$$B_1=\sum_{k=0}^{n}-b_k\omega_z^k U_{k-1}\left(\varsigma_z\right), \tag{25}$$

$$B_2=\sum_{k=0}^{n}b_k\omega_z^k U_k\left(\varsigma_z\right), \tag{26}$$

$$C_1=\sum_{k=0}^{n}-c_k\omega_z^k U_{k-1}\left(\varsigma_z\right), \tag{27}$$

$$C_2=\sum_{k=0}^{n}c_k\omega_z^k U_k\left(\varsigma_z\right), \tag{28}$$

$$D_1=\sum_{k=0}^{n}-d_k\omega_z^k U_{k-1}\left(\varsigma_z\right), \tag{29}$$

$$D_2=\sum_{k=0}^{n}d_k\omega_z^k U_k\left(\varsigma_z\right). \tag{30}$$

Equations (23), (24) may be solved for unknowns α and β which gives us following expressions for parameters α and β

$$\alpha=\frac{C_1D_2-C_2D_1}{B_1C_2-B_2C_1}, \tag{31}$$

$$\beta=\frac{B_2D_1-B_1D_2}{B_1C_2-B_2C_1}. \tag{32}$$

Because attention is primary focused on the relative damping coefficient ς, or the undamped (natural) frequency ω_n, it is important to replace the complex variable s in (13) with following expression

$$s=-\varsigma\omega_n+j\omega_n\sqrt{1-\varsigma^2}. \tag{33}$$

Then from (14), (15), (16) and (33) ω_z and ς_z may be expressed as follows

$$\omega_z = e^{-\varsigma\omega_n \mathrm{T}}, \tag{34}$$

$$\varsigma_z = \cos\omega_n \mathrm{T}\sqrt{1-\varsigma^2}. \tag{35}$$

Substituting (34), (35) into (25)-(30), gives us following expressions

$$B_1 = \sum_{k=0}^{n} -b_k e^{-k\mathrm{T}\omega_n\varsigma} U_{k-1}\left(\cos\omega_n \mathrm{T}\sqrt{1-\varsigma^2}\right), \tag{36}$$

$$B_2 = \sum_{k=0}^{n} b_k e^{-k\mathrm{T}\omega_n\varsigma} U_{k}\left(\cos\omega_n \mathrm{T}\sqrt{1-\varsigma^2}\right), \tag{37}$$

$$C_1 = \sum_{k=0}^{n} -c_k e^{-k\mathrm{T}\omega_n\varsigma} U_{k-1}\left(\cos\omega_n \mathrm{T}\sqrt{1-\varsigma^2}\right), \tag{38}$$

$$C_2 = \sum_{k=0}^{n} c_k e^{-k\mathrm{T}\omega_n\varsigma} U_{k}\left(\cos\omega_n \mathrm{T}\sqrt{1-\varsigma^2}\right), \tag{39}$$

$$D_1 = \sum_{k=0}^{n} -d_k e^{-k\mathrm{T}\omega_n\varsigma} U_{k-1}\left(\cos\omega_n \mathrm{T}\sqrt{1-\varsigma^2}\right), \tag{40}$$

$$D_2 = \sum_{k=0}^{n} d_k e^{-k\mathrm{T}\omega_n\varsigma} U_{k}\left(\cos\omega_n \mathrm{T}\sqrt{1-\varsigma^2}\right). \tag{41}$$

If these equations are used in (31), (32) α and β are expressed as functions of ω_n and ς. Thus, (31) and (32) may represent in the parameter plane the loci of points corresponding to the roots with constant damping coefficient (ς curve), with constant undamped (natural) frequency (ω_n curve), or with constant settling time (ω_z curve), depending on which variable among ς, ω_n and ω_z is considered constant. The loci of points corresponding to constant real roots, which represent the real-root boundaries in the parameter plane, are derived from the characteristic equation. If the z plane is considered, the substitution

$$z = \sigma_z, \tag{42}$$

in (13) yields the real-root boundary as

$$\alpha\sum_{k=0}^{n} b_k\sigma_z^k + \beta\sum_{k=0}^{n} c_k\sigma_z^k + \sum_{k=0}^{n} d_k\sigma_z^k = 0. \tag{43}$$

For a given σ_z this equation represents a straight line in the $\alpha\beta$ plane which will be called σ_z line. A linear sampled-data control system is stable if there are no roots of the characteristic equation (13) outside the unit circle. To investigate the absolute stability in the parameter plane, it is necessary to map the unit circle onto the $\alpha\beta$ plane by using (31), (32) and (43).

If we take into consideration $\sigma_z = 1$, that means that we wish all roots of characteristic equation to lie in unit circle. In that case radius is equal to one. When the radius is chosen small enough, the roots of characteristic equation are located close to the origin of the z-plane and in that case the control system behaves like a dead-beat control system, whose transient response settles down fast, [15].

Applying all previous analysis we get region in parameter plane where system have damping coefficient $\varsigma = 0.7$ or bigger, which is showed on figure 6. Denotation $\Gamma_{0.7}(3,0)$ means that all three roots of characteristic equation (15) lie within unit circle in z plane.

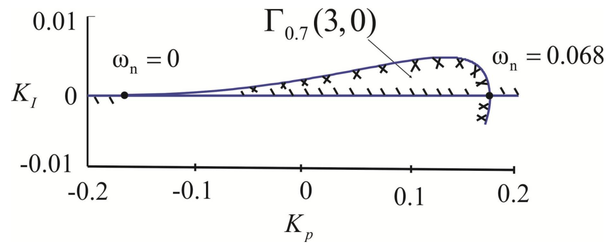

Figure 6: Region of relative stability

5. SYSTEM OPTIMIZATION

In general, the idea behind time domain optimization methods is to choose the PID controller parameters to minimize an integral cost functional, [16].

A design is performed on the basis of conditional optimization. The goal is to find a position of working point which assure minimum value of performance index and at the same time, relative stability to be satisfied. A position of working point in parameter plane determines relative stability and also minimal value of performance index in these cases.

In general, it is requested for a system output change $y(k\mathrm{T})$ to vary as little as possible from the change of the wanted output $y_d(k\mathrm{T})$ during a time interval. Integral square error may be expressed as function of the z-transform of quantity $e(k\mathrm{T}) = y_d(k\mathrm{T}) - y(k\mathrm{T})$ depending on the means by which system is investigated

$$I = \sum_{k=0}^{\infty} e^2(k\mathrm{T}) = \sum_{k=0}^{\infty} e(k\mathrm{T})e(k\mathrm{T}). \tag{44}$$

As we know, inverse z-transform may be found using following expression

$$e(k\mathrm{T}) = \mathrm{Z}^{-1}\{E(z)\} = \frac{1}{2\pi j}\oint_C E(z)z^{k-1}\mathrm{d}z, \tag{45}$$

where C is circle in z-plane. So, expression (44) becomes

$$I = \sum_{k=0}^{\infty} e(k\mathrm{T})\frac{1}{2\pi j}\oint_C E(z)z^{k-1}\mathrm{d}z, \tag{46}$$

$$I = \frac{1}{2\pi j}\oint_C E(z)\sum_{k=0}^{\infty} e(k\mathrm{T})z^{k-1}\mathrm{d}z, \tag{47}$$

$$I = \frac{1}{2\pi j}\oint_C E(z)\sum_{k=0}^{\infty} e(k\mathrm{T})z^{-(1-k)}\mathrm{d}z, \tag{48}$$

$$I = \frac{1}{2\pi j}\oint_C E(z)\sum_{k=0}^{\infty} e(k\mathrm{T})z^{-1}\mathrm{z}^{-(-k)}\mathrm{d}z, \tag{49}$$

By definition of z-transform we have

$$E(z) = \sum_{k=0}^{\infty} e(k\mathrm{T})z^{-k}. \tag{50}$$

In order to minimize integral cost functions (49), Parseval's theorem can be invoked to express the time functions in terms of their z-transforms, [17].

$$I = \frac{1}{2\pi j}\oint_C E(z)E(z^{-1})z^{-1}\mathrm{d}z. \tag{51}$$

Taking into account Cauchy residue theorem a value of the preceding expression is determined by sum of integrand residues for its poles enclosed by the contour C. As parameters α and β are determined so that all poles of $W(z)$ and $E(z)$ lie in the unit circle of the plane z, it means that no one pole of $E\left(z^{-1}\right)$ is encircled by contour C, and then performance index can be calculated as follows

$$I=\sum_{r=1}^{n}\operatorname{Res}\left[E(z)E(z^{-1})z^{-1}\right]_{|z=z_r^*}, \qquad (52)$$

where z_r^* is root from $z^{-1}E(z)$. It means calculation of integral along the contour C is substituted to calculation of integrand residue in its poles enclosed by this contour. Residues in complex poles of some rational function with real coefficients appear in conjugate-complex pairs. After summation of these residues their imaginary parts annul and real parts only remain. It is phisicaly clear as sum of residues represents SSE, that performance index I must be positive real quantity.

After shading decomposition curve $\Gamma_{0.7}$ and calculating performance index I for span of $\omega_n \in]0, 0.068[$ we find optimal values $K_p = 0.1576$, $K_I = 0.005121$ for zero initial conditions $h_1(0)=0$, $h_2(0)=0$.

When we want to take into account nonzero initial conditions $h_1(0)=-0.003$, $h_2(0)=-0.005$ then we do z-transform of system difference equation by using following well known formula

$$\mathrm{Z}\left\{x(k+n)\right\}=z^n X(z)-z^n\sum_{k=0}^{n-1}x(k)z^{-k}\ . \qquad (53)$$

Now we find new $E(z)$ in which exists nonzero initial conditions. Using this new $E(z)$, and calculating performance index I using (52) new optimal controller gain values are $K_p = 0.1476$, $K_I = 0.004921$.

Figure 7 shows the experimental results obtained for the two sets of parameters K_p and K_I at the zero and nonzero initial conditions.

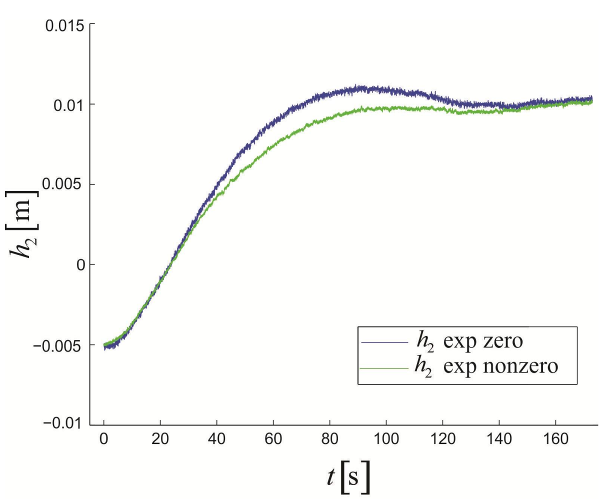

Figure 7: Experimental results

6. CONCLUSION

In many papers choice of optimal controller gains was carried out only on the basis of effect of inputs on behaviour of dynamic system, while the effect of initial conditions were neglected.

Effect of initial conditions on behaviour of every dynamic system must be taken into account in the process of finding optimal values for controller gains.

This paper presented methods for design of standard discrete PI controller with two adjustable parameters K_p and K_I taking into account of arbitrary initial conditions. Values of controller gains K_p and K_I which guarantee damping ratio of all closed loop poles to be greater or equal to 0.7 have been extracted.

Experimental results obtained on coupled tanks plant clearly shows that when initial conditions of plant are not zero, controller parameters derived taking into consideration initial conditions will provide optimal behaviour of system which is not guaranteed with controller parameters obtained at the zero initial conditions.

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