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ANALYTICAL MODELING OF BALLISTIC PERFORATION IN PLUG FORMATION MODE

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Abstract:

The paper considers ballistic perforation of ductile metal plates by rigid projectiles in plug formation regime. Analytical modeling of the penetration process is based on the application of momentum and energy conservation laws for the penetrator/plug system. After outlining of the well-known Recht-Ipson model, the new analytical model has been formulated. The model considers the plug compression and shear – two most important resistance forces acting on the projectile. The final result of the model is the ballistic curve – a simple relation between the projectile and plug residual velocity and the projectile impact velocity. The important parameters in the relation are the ballistic limit velocity and the relative mass of the plug. For low values of the relative plug mass, the ballistic curves of the proposed model and the Recht-Ipson model are very close to each other, but for higher values of this parameter the results of two models significantly deviates. Comparison with available experimental data indicates good agreement between measured velocities and model predictions. However, further experiments are needed for validation of the model, especially in the case of high values of relative plug thickness.

Key words: ballistic perforation, plugging, analytical model, experimental investigation

1. Introduction

Penetration mechanics is a field of applied mechanics that studies the interaction process of a projectile (penetrator) with an obstacle (target). This discipline is of significant interest in military, aerospace, civil and nuclear applications [1, 2].

Depending on the basic "input" parameters, i.e. properties of penetrator and target and impact conditions, penetration process may have a completely different nature. The basic regimes (modes) of penetration are: plug formation (plugging), ductile hole formation (piercing), petaling, brittle fracture, fragmentation and erosion [1-5]. The present paper considers the plugging mode of perforation. This regime is typical for a penetrator with a blunt nose and ductile targets with low to moderate thickness. The main feature of this regime is the formation of a cylindrical cut-out of target material (plug) with a diameter approximately equal to the diameter of the penetrator. Dominant loads on the target during the penetration process are compression and shear, which at a certain moment can lead loaded zone in an unstable state, causing a fracture of the material due to shear and separation of the plug. The plugging process has been analyzed in numerous studies, e.g. [6-11].

There are generally four approaches to the investigation of penetration mechanics phenomena: experimental test, semi-empirical data fits, numerical and analytical modeling. The analytical modeling, as a "compromise" approach between empirical and numerical models, will be treated in the present study. The fact that analytical models are efficient, that can provide a satisfactory level of accuracy of the results, as well as their adaptability and suitability for parametric studies make them very useful predictive tool to be applied in the analysis of penetration, and therefore in the design of projectiles and/or armor protection.

The purpose of the present paper is to present a new analytical model for perforation in plugging regime and compare the results with experimental data.

2. Analytical modeling of perforation in plugging mode

The perforation process in plugging mode is illustrated in Fig. 1. Normal impact of rigid, cylindrical penetrator with mass m and impact velocity V_0 into a ductile metal target plate with thickness H is considered. After perforation both the penetrator and the created plug of mass m_{pl} have the same residual velocity V_{r}. First, the derivation of the well-known Recht-Ipson (RI) model [9] will be outlined.

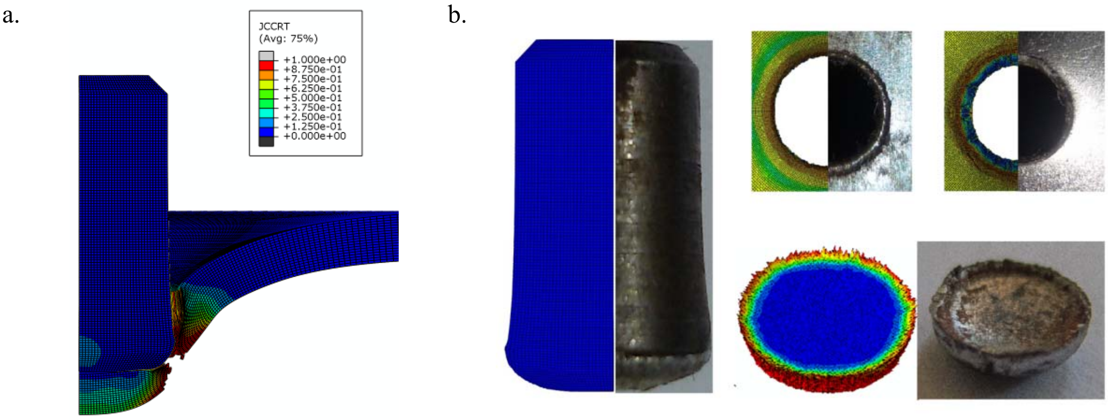

Fig.1. Plate perforation in plug formation mode: a) penetration process, results of simulation [13], b) clockwise: penetrator shape, front side of the plate, rear side of the target plate, plug – comparison between experimental data and simulation results [13]

2.1 Recht-Ipson model

The RI approach is one of the most important contributions to the theoretical modeling of the penetration process. The model is based on the application of the basic laws of mechanics – the laws of conservation of energy and momentum.

The energy conservation in the penetration process can be written in the form:

$$\frac{mV_0^2}{2} = W + A + \frac{(m + m_{\mathrm{pl}})V_{\mathrm{r}}^2}{2}. \tag{1}$$

It will be assumed that the total kinetic energy of the penetrator is transformed into:

- work W, which is spent on heating and deformation of the plug, at perfectly inelastic collision (assuming that the plug is not coherent with the rest of the target material), i.e. it is the loss of the penetrator's kinetic energy due to inelastic impact with a "free" plug,

- work A, which is spent on separating the plug from the secondary target zone and on corresponding target deformation (primarily shear and compression)
- the kinetic energy of the penetrator and the plug after the perforation.

The work W can be determined by calculating the hypothetical common velocity of the penetrator and the plug after inelastic impact, applying the momentum conservation law:

$$V_{r1} = \frac{m}{m + m_{pl}} V_0 . \tag{2}$$

The required work W is the difference of the kinetic energy of the penetrator/target system before and after the impact:

$$W = \frac{mV_0^2}{2} - \frac{(m + m_{pl})V_{r1}^2}{2} = \frac{m_{pl}}{m + m_{pl}} \frac{mV_0^2}{2} . \tag{3}$$

Introducing eq. (3) in starting eq. (1), we get:

$$\frac{mV_0^2}{2} = \frac{m_{pl}}{m + m_{pl}} \frac{mV_0^2}{2} + A + \frac{(m + m_{pl})V_r^2}{2} . \tag{4}$$

Equation (4) is also valid in the limiting case, at the ballistic limit velocity V_0=V_{BL}, when the residual velocity V_r=0. The corresponding value of work A_{BL} in that case is:

$$A_{BL} = \frac{m}{m + m_{pl}} \frac{mV_{BL}^2}{2} . \tag{5}$$

If we additionally assume that the value of this work is the characteristic of the process independent on the impact velocity, A=const.=A_{BL}, then from eq. (4) the final formula for the residual velocity arises:

$$V_r = \frac{\sqrt{V_0^2 - V_{BL}^2}}{1 + \dfrac{m_{pl}}{m}} . \tag{6}$$

The mass of plug is usually determined by $m_{pl}=\rho AH$, (ρ - target plate density, A - area of the hole formed equal to the penetrator's cross-section area) or experimentally, and the ballistic limit velocity can be measured or calculated by empirical formulas.

2.2 New model

The distribution of the initial impact energy can be treated in a different way, taking into account dominant resistance forces. The energy conservation equation for the perforation process can be written in the form:

$$\frac{mV_0^2}{2} = \frac{(m + m_{pl})V_r^2}{2} + \int_0^H (F_c + F_s) \mathrm{d}x \tag{7}$$

where the integral defines the total work of two dominant resistance forces that act on the penetrator – compression force F_c and shear force F_s, and x is the current displacement of the penetrator, Fig. 2. The compressive force is responsible for plug deformation due to compression and the corresponding work is:

$$W_{\text{def}} = \int_0^H F_{\text{c}} \text{d}x \,. \tag{8}$$

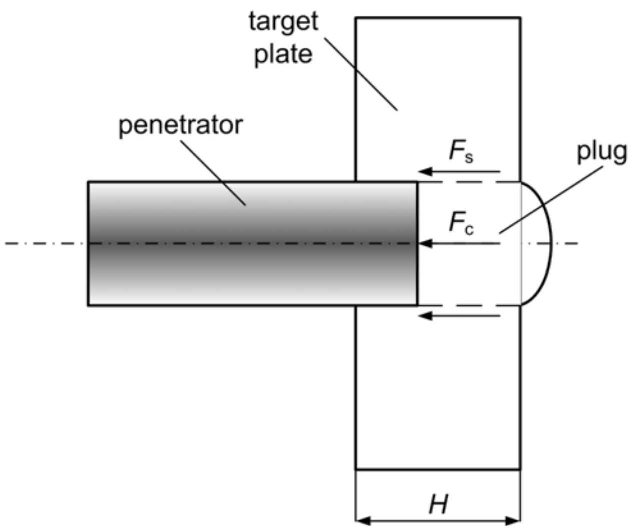

Fig. 2. Scheme of the plugging mode of perforation and characteristic resistant forces

The second term is related with the plug separation from the target and will be identified as the minimum energy required for target perforation:

$$W_{\text{BL}} = \int_0^H F_{\text{s}} dx = \frac{m V_{\text{BL}}^2}{2} \,. \tag{9}$$

Now, eq. (7) can be expressed in the form:

$$\frac{m V_0^2}{2} = \frac{(m + m_{\text{pl}}) V_{\text{r}}^2}{2} + W_{\text{def}} + W_{\text{BL}} \,. \tag{10}$$

The momentum conservation law applied to the system penetrator/plug reads:

$$m V_0 = (m + m_{\text{pl}}) V_{\text{r}} + I \,, \tag{11}$$

where I is the impulse imparted to the target material, which is equivalent to the impulse of the shear force:

$$I = I_{\text{BL}} = \int_0^T F_{\text{s}} \text{d}t \,. \tag{12}$$

Assuming that the average values of the shear resistance force are approximately equal in the time and distance domain, one can obtain the following relation between the critical values of energy and impulse:

$$\frac{W_{\text{BL}}}{I_{\text{BL}}} = \frac{\int_0^H F_{\text{s}} dx}{\int_0^T F_{\text{s}} dt} = \frac{\left(\bar{F}_{\text{s}}\right)_x H}{\left(\bar{F}_{\text{s}}\right)_t T} = \frac{V_0 + V_{\text{r}}}{2} \,. \tag{13}$$

Using the energy and momentum eqs. (10) and (11), and relation (13), the deformation work can be determined as:

$$W_{\text{def}} = \frac{1}{2} m_{\text{pl}} V_0 V_{\text{r}} \,. \tag{14}$$

Also, the quadratic equation is obtained for determination of the residual velocity of the penetrator and plug:

$$\left(1+\frac{m_{\text{pl}}}{m}\right)\left(\frac{V_{\text{r}}}{V_{\text{BL}}}\right)^2 + \frac{m_{\text{pl}}}{m}\frac{V_0}{V_{\text{BL}}}\frac{V_{\text{r}}}{V_{\text{BL}}} + \left(1-\frac{V_0^2}{V_{\text{BL}}^2}\right) = 0 \,. \tag{15}$$

If we introduce the parameter *a*, which defines the ratio of masses of the plug and penetrator:

$$a = \frac{m_{\text{pl}}}{m} \,, \tag{16}$$

and normalized velocities:

$$v_0 = \frac{V_0}{V_{\text{BL}}}, \quad v_{\text{r}} = \frac{V_{\text{r}}}{V_{\text{BL}}} \,, \tag{17}$$

then eq. (15) gets the form:

$$(1+a)v_{\text{r}}^2 + a v_0 v_{\text{r}} + \left(1-v_0^2\right) = 0 \,. \tag{18}$$

Finally, the normalized residual velocity of the projectile and plug is:

$$v_{\text{r}} = \frac{\sqrt{(2+a)^2 v_0^2 - 4(1+a)} - a v_0}{2(1+a)} \,. \tag{19}$$

The last formula defines the ballistic curve, the most important property of a penetrator/target system – a dependence of the residual velocity on impact velocity. The final formula of the RI model, eq. (6), expressed in normalized velocities, has the simple form:

$$v_{\text{r}} = \frac{\sqrt{v_0^2 - 1}}{1+a} \,. \tag{20}$$

3. Analysis and comparison with experiments

The comparison between the ballistic curves obtained by the RI approach and the present model is given in Fig. 3. Generally, the following conclusions can be drawn from the analysis of the two ballistic curves:

i. It can be easily shown that $v_{\text{r}} \geq 0$, where $v_{\text{r}}=0$ only if $v_0=1$, i.e. $V_0=V_{\text{BL}}$.

ii. Also, residual velocity calculated by the proposed model is lower than in the case of RI model $v_{\text{r}} \leq \left(v_{\text{r}}\right)_{\text{RI}}$; equality holds only for $v_0=1$.

iii. Both ballistic curves have the same asymptote, whose slope is defined as:

$$\lim_{v_0 \to \infty} \frac{v_r}{v_0} = \frac{1}{1+a} \tag{21}$$

iv. The slope of the ballistic curves in the point $v_0=1$ is different:

$$\left.\frac{\mathrm{d}v_\mathrm{r}}{\mathrm{d}v_0}\right|_{v_0=1} = \begin{cases} \infty, & \text{RI model} \\ \dfrac{2}{a}, & \text{present model} \end{cases} \tag{22}$$

v. When the parameter $a=m_{\mathrm{pl}}/m$ approaches to zero, the two curves become coincident.

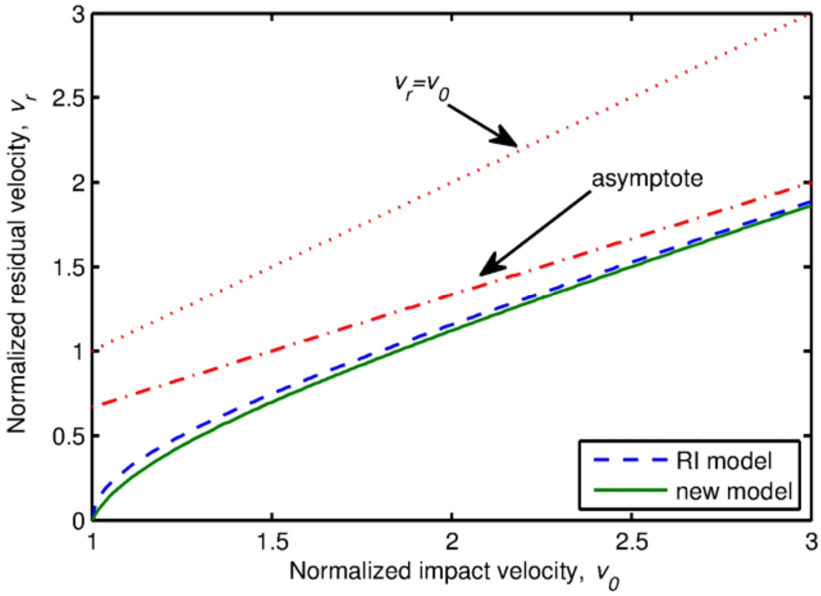

Fig. 3. A comparison between the ballistic curves obtained by the Recht-Ipson approach and the present model

A comparison between theoretical models and experimental results is shown in Fig. 4. The comprehensive experimental study [11] considered perforation of steel plates by the rigid cylindrical penetrator with diameter of 20 mm and mass of 0.2 kg. Impact velocities were in sub-ordnance domain and perforation process was clearly in the plugging mode. For both target thicknesses (10 and 12 mm), the parameter a is relatively small (0.11 and 0.15) and the curves are close to each other. Both models predict the experimental results very well.

Figure 5 presents the comparison of analytical model with experimental results of perforation of steel plates by fragment simulating projectiles [12, 13]. The penetrators were steel cylinders with mass of 1.090 g and diameter 4.70 mm. Two thicknesses of the target were used: 1.25 mm and 2.20 mm. Corresponding values of average plug masses of 0.21 g and 0.41 g were measured. The dominant perforation regime in all cases was plugging and impact velocity was in the range from 450 m/s to 950 m/s. Ballistic limit velocities were determined by numerical simulations [13]: 261.0 m/s and 506.0 m/s for thinner and thicker target plate, respectively. In the case of thinner target plate (1.25 mm) relative plug mass is low (a=0.19) and the RI model and the suggested curve are very close to each other and both provide good fit to the experimental data. For the thicker plate (2.20 mm, parameter a=0.38), the agreement between the measured data and the proposed model is even better than with the RI model. It should be noted that for highest impact velocities in this case, significant deformation (mushrooming) and damage of the penetrator occurs, which deviates from the conditions under which both models are valid.

Obviously, further experiments are needed, especially with thicker targets (higher values of parameter a), in order to provide the answer which model has better prediction capabilities.

a. b.

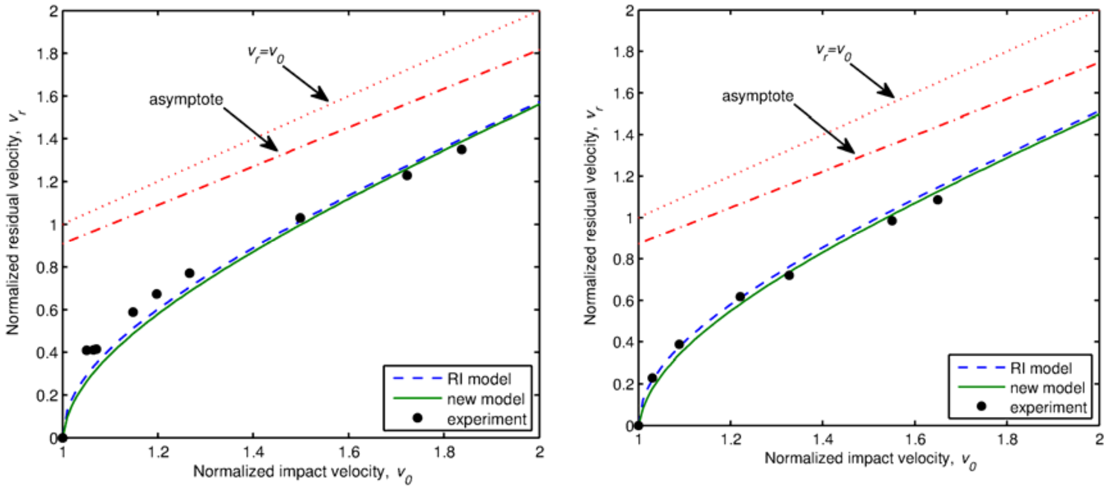

Fig. 4. Comparison of analytical models' predictions and experimental data [11]. Rigid penetrator with mass m=0.2 kg and diameter 20 mm perforates Weldox 460 E steel plates: a) plate thickness 10 mm, average plug mass 22.2 g, ballistic limit velocity 161.2 m/s, b) plate thickness 12 mm, average plug mass 28.7 g, ballistic limit velocity 184.0 m/s

a. b.

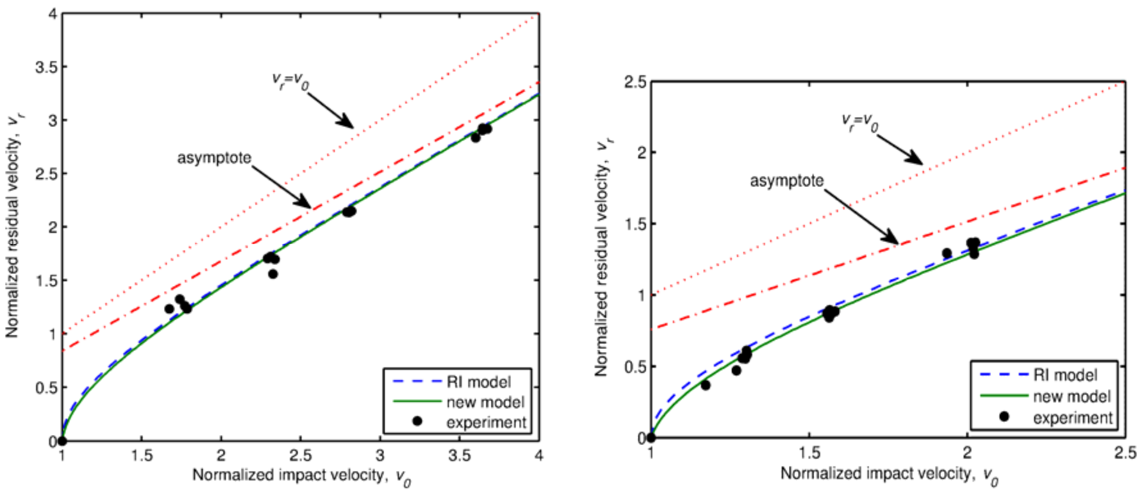

Fig. 5. Analytical models' predictions vs. experimental results [12, 13]. Cylindrical penetrators with mass m=1.090 g and diameter 4.70 mm perforated steel plates: a) plate thickness 1.25 mm, average plug mass 0.21 g, ballistic limit velocity 261 m/s, b) plate thickness 2.20 mm, average plug mass 0.41 g, ballistic limit velocity 506 m/s

4. Conclusions

After consideration of the plugging mode of ballistic perforation and its analytical modeling, the following conclusions can be drawn:

- a new analytical model has been suggested, based on conservation of momentum and energy of penetrator/plug system, as well as on consideration of compressive and shear resistance forces,

- the model resulted with the ballistic curve which exhibits similar properties to the well-known Recht-Ipson model,
- the two curves are almost coincident for low values of relative plug mass, but significantly deviates for higher masses of the plug,
- comparison with available experiments shows that both models provide good agreement with experimental data,
- further experiments are needed for validation of the model and its comparison with other models.

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