Comments on "An exact dynamic stiffness method for multibody systems consisting of beams and rigid-bodies, Mechanical Systems and Signal Processing 150 (2021) 107264, by X. Liu, Ch. Sun, J.R. Banerjee, H-Ch. Dan, L. Chang"

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Abstract

This article contains some comments on "An exact dynamic stiffness method for multibody systems consisting of beams and rigid-bodies", X. Liu, Ch. Sun, J.R. Banerjee, H-Ch. Dan, L. Chang. Mechanical Systems and Signal Processing, 150, 107264 (2021).

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1. Introduction

In Section 3.1 of paper [1] authors use the example from Section 4.1 of our paper [2]. Namely, considerations involve the elastic uniform hybrid beam composed of two elastic Euler-Bernoulli beam segments of circular cross-section with the diameter $D = 0.05$ m and one rigid body in the form of a thin rigid plate, as it is shown in Fig.1.

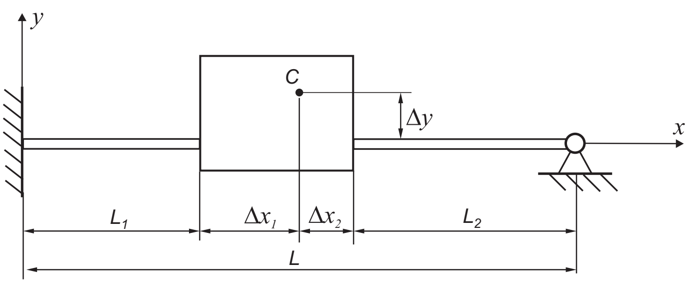

Figure 1: Elastic beam carrying a rigid body

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The Young's modulus and mass density of the beam segments are, respectively, $E = 2.069 \times 10^{11}\,\mathrm{N/m^2}$ and $\rho = 7.8367 \times 10^3\,\mathrm{kg/m^3}$ whereas the mass and the centroidal moment of inertia of the rigid plate are, respectively, $M = 15.387\,\mathrm{kg}$ and $I_C = 12.31\,\mathrm{kgm^2}$. Also, the geometrical parameters shown in Fig. 1 have the following values: $L = 2.6\,\mathrm{m}$, $L_1 = 0.8\,\mathrm{m}$, $L_2 = 1.2\,\mathrm{m}$, $\Delta x_1 = 0.4\,\mathrm{m}$, $\Delta x_2 = 0.2\,\mathrm{m}$, and $\Delta y = 0.2\,\mathrm{m}$. Note that C in Fig.1 denotes the mass center of the plate. In [1], for the considered example, the first three dimensionless frequency coefficients were obtained by using both the dynamic stiffness method and the finite element method (FEM), as it is shown in Table 2 of paper [1]. In the same table the corresponding results from [2] are also shown. Based on the results given in Table 2 of paper [1], authors of the mentioned work present the following observations (see the first paragraph on page 12 in [1]):

"It can be clearly seen from the results in Table 2 that the theory of this paper is significantly more accurate than the transfer matrix method (TMM) of Ref. [46] when compared with the FE results using a very refined mesh. The error between the present theory and the FE results are within 0.04% whilst the error between the TMM and the FEM is larger than 14%. The reason for this is probably due to the fact that the transfer matrix method uses intensive matrix inversions one after another, which introduce numerical problems leading to errors in the results."

We cannot agree with the presented statements in [1] for the following reasons:

- In [1] it is taken that $\Delta y = 0.02\,\mathrm{m}$, whereas in [2] the value $\Delta y = 0.2\,\mathrm{m}$ is used
- In [1] dimensionless frequency coefficients are calculated based on the expression $\lambda_i = \sqrt[4]{\omega_i \varrho A L^2/(EI)}$, whereas in [2] the expression $\lambda_i = \sqrt[4]{\omega_i^2 \varrho A L^4/(EI)}$ is used. In Eq. (37) instead of $\lambda = \sqrt[4]{m\omega^2 L^2/(EI)}$ it should be $\lambda = \sqrt[4]{\varrho A \omega^2 L^4/(EI)}$ (see [3, 4])

Also:

- In Eq. (29) instead of $a_2 = \frac{EA}{l}\mu csc\mu$ it should be $a_2 = -\frac{EA}{l}\mu csc\mu$ (see [3, 4])

Besides, in [1] not enough data is given on used parameters of the FEM model, especially on the manner of creating a rigid plate in the framework of the FEM so as to check the values of the frequency coefficients given in the fourth column of Table 2 in [1]. Further, confirmation of our findings is given through verification of the frequency coefficients values obtained by the approach given in [2] using the finite element package ANSYS.

2. Verification of the results by using the FEM

Figure 2 shows a dynamic equivalent model of the considered rigid plate that will be used in the finite element analysis of the considered vibration problem. The presented rigid plate is massless and three concentrated masses, $m_1 = m_3 = 6.155\,\mathrm{kg}$ and $m_2 = 3.068\,\mathrm{kg}$, are attached to it at positions as it is shown in Fig. 2. In this way, the considered system composed of a massless plate and concentrated masses has the same total mass, the same position of the center of masses relative to the frame Oxy and the same centroid moment of inertia like the plate shown in Fig. 1.

Further, the finite element model of the considered system consists of 6119 elements and 18 662 nodes. It was made in Ansys Workbench software, version 2020R2. The solid elements

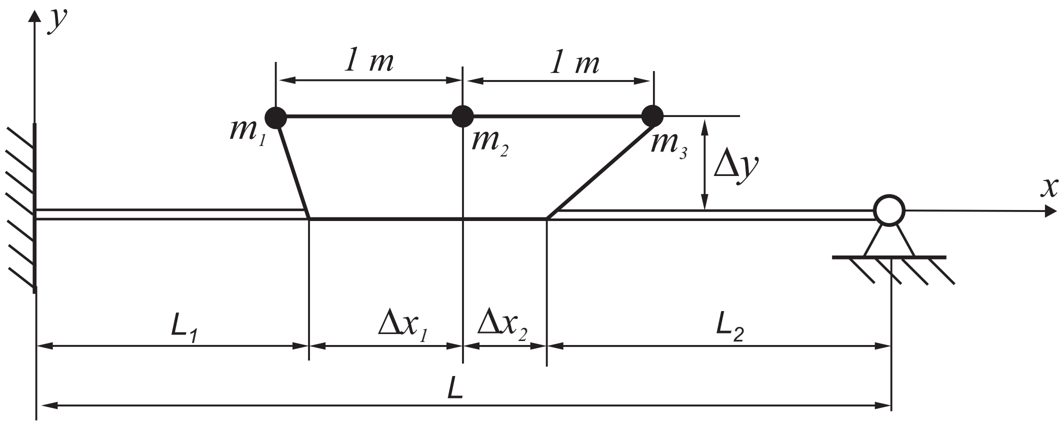

Figure 2: A model of the rigid plate in the frame of the FEM

used for beams were BEAM3 and BEAM189 (in total 2064 elements have been created), while element PLANE183 was used for modeling of the rigid plate (in total 4052 elements have been created). Point masses were introduced using element MASS21. Calculations assume element mass at element centroid. Fourteen modes were extracted using the eigenvalue extraction method for natural vibration analysis based on a block Lanczos algorithm.

The values of the first three dimensionless frequency coefficients obtained by using the finite element analysis are shown in Table 1. The same table also displays the percentage relative difference, Δ, between the FEM results and the corresponding results obtained in [2].

The quantity Δ is determined by the following expression:

$$\Delta = \frac{\lambda_{FEM} - \lambda_{[2]}}{\lambda_{FEM}} \times 100\%$$

The results from Table 2 confirm the accuracy of our approach described in [2] and refute above mentioned statements on accuracy presented in [1].

Table 1: The lowest three dimensionless frequency coefficients

	λ_1	λ_2	λ_3
[2]	2.81093	4.68603	6.99522
FEM	2.81078	4.67851	6.97587
Δ (%)	0.005	0.161	0.277

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