Generalized Forces of the Robotic System with Fractional Order Thermoviscoelastic Element

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Abstract – **In this paper, we analyze generalized forces of a discrete fractional order Kelvin-Voigt thermoviscoelasic element connected into a multibody robotic system. An efficient numerical approximation scheme is used to approximate fractional order derivative. The effects of fractional order derivative and temperature change on generalized forces are examined through the numerical example of simple three rigid body robotic system.**

Keywords – **Fractional calculus, thermoviscoelasticity, multibody dynamics.**

I. Introduction

Thermal effects are present in the exploitation of robots under various conditions. Influence of such effects on rigid robotic parts as well as on the flexible one should not be neglected in the modelling and design procedure. In addition, these effects are having influence on the motion of the system and can change positioning trajectory of robot manipulators. However, available rigid body system modelling procedures are limiting our study to consideration of only discrete deformable elements in the system accounting the thermal effects. In the sense of Lagrange motion equations of the second kind, we consider a discrete rigid multibody system coupled with a fractional order thermoviscoelastic element. We derived generalized forces of the element based on the principle of virtual work (see [1] and [2]). In the literature, one can find many papers applying the fractional order viscoelasticity for the problems in structural mechanics or other applications in mechanics or control where use of fractional order derivatives or integrals is justified. By performing experiments, it has been concluded that fractional order viscoelastic models represent viscoelastic material behavior much better than classical viscoelastic models and in some cases, much less parameters are needed to fit experimental curves. In [3], one can find several fractional order models for different applications. Numerical approximation methods for fractional order derivatives one can find in [4] and [5].

In this paper, we introduce fractional order thermovicoelastic element into a robotic system composed of multiple rigid bodies using generalized forces of the element. Element is connected to two different bodies in the system (see Fig. 1). For a numerical simulation, we use simple robotic system composed of three rigid bodies.

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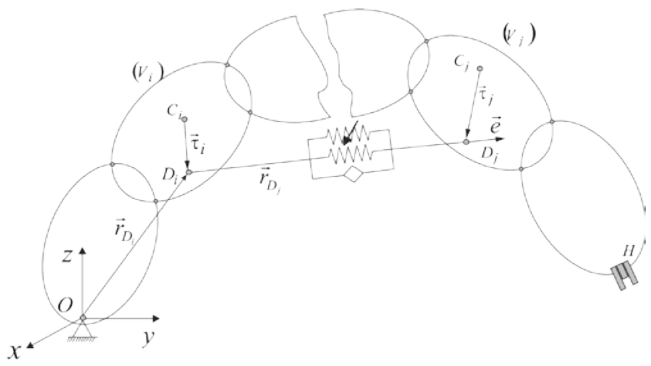

Fig. 1. Multibody robotic system with thermoviscoelastic element

II. Formulation of the problem

First, we give Lagrange motion equations of arbitrary multibody system in the covariant form as

$$\sum_{\alpha}^{n} a_{\alpha\gamma}\ddot{q}^{\alpha} + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\Gamma_{\alpha\beta,\gamma}\dot{q}^{\alpha}\dot{q}^{\beta} = Q_{\gamma}, \quad \alpha,\beta,\gamma = 1,...,n. \qquad (1)$$

where with q^{α} and q^{β} we denote generalized coordinates, $\Gamma_{\alpha\beta,\gamma}$ are Christoffel symbols of the first kind, $a_{\alpha\gamma}$ are elements of the basic metric tensor and with Q_{γ} we denote generalized forces. In the above equation, we can introduce generalized forces such as gravitational forces, generalized control forces, semi dry friction forces etc. In our case, we will introduce generalized forces of the fractional order thermoviscoelastic element, which can be deformed only in the direction of the unit vector lying on the line which connects two different points on two different bodies in the system (see Fig. 1). Further, we propose a force-displacement relation of the element in the form

$$F_{KV} = \left(E_c + E_{KV}\right)x + E_d\, {}_0^{RL}D_t^{\lambda}\left(x\right) - a_{KV}E_{KV}T\ , \qquad (2)$$

where x is dilatation of the element, F_{KV} is force of the element, E_c and E_{KV} are stiffness's of the element, E_d is relaxation time, a_{KV} is thermo-dilatation constant and T is temperature of the element. The operator ${}_0^{RL}D_t^{\lambda}$ is fractional order derivative operator denoting the left Riemann-Liouville definition and which is given in the form

$${}_0^{RL}D_t^{\lambda}\left(x\right) = \frac{1}{\Gamma\left(n-\alpha\right)}\frac{d^n}{dt^n}\int_0^t \left(t-\tau\right)^{n-\alpha-1} x\, d\tau\ . \qquad (3)$$

In Eq. (3) $x\left(q^i\left(t\right), q^{i+1}\left(t\right), ..., q^k\left(t\right)\right)$ is displacement of the element. It is equal to the difference of the instant length l of the element and constant initial length l_0 and it is a composite function depending of the generalized coordinates which are time dependent functions. Using the principle of virtual work we can derive generalized forces of the element where we obtain

$$Q_\beta^w = -\Bigg[(E_{KV} + E_c)\cdot x \frac{\vec{l}}{l}\frac{\partial\left(\vec{l}\right)}{\partial q^\alpha} + E_d \cdot \frac{\vec{l}}{l}\frac{\partial\left(\vec{l}\right)}{\partial q^\alpha}\cdot D^\lambda\left(x\right)$$

$$-\alpha_{KV} E_{KV} T \frac{\vec{l}}{l}\frac{\partial\left(\vec{l}\right)}{\partial q^\alpha}\Bigg] \quad . \tag{4}$$

III. NUMERICAL RESULTS

In our numerical simulation, we use robotic system composed of three rigid bodies and thermoviscoelastic element connected to the first and the third body in the system. We assumed the following generalized coordinates $q^1 = 0$, $q^2 = 0.1\sin\left(8t\right)$ and $q^3 = 0.1t$. Since displacement of the element is a composite function of generalized coordinates we cannot use classical chain rule for fractional derivative of a displacement. Thus, we use numerical approximation scheme proposed in [4] and [5] but for the case of a composite function. For the numerical calculations we used the following values of parameters: $E_c = 30\left[\mathrm{N/m}\right]$, $E_c = 20\left[\mathrm{N/m}\right]$, $E_d = 50\left[\mathrm{s}^\alpha\right]$, $a_{KV} = 0.0007\left[\mathrm{m/C}\right]$ and $l_0 = 0.7\left[\mathrm{m}\right]$.

Values of second generalized force of the thermovicoelastic element at room temperature and for different values of fractional order parameter are shown in Fig. 2. It can be noticed that decrease of the fractional order parameter softens the curve amplitude i.e. difference between values of generalized forces in time is less for lower fractional order derivatives than for the higher one. In Fig. 3, one can see the difference between values of the second generalized force for changes of temperature T in the range 0-60 [C^O]. It is obvious that in general, an increase of the temperature decreases the value of generalized force. This effect depends on the temperature of the element and value of the thermo-dilatation constant a_{KV}, which is different for each material.

IV. CONCLUSION

Here, we analyzed generalized forces of the robotic system with thermoviscoelastic element. We showed that generalized forces depend of the element temperature but this influence is conditioned with the value of thermo-dilatation constant, which depends of the material properties. In addition, we showed that different viscoelastic properties of the element change the values of generalized forces significantly. By choosing the appropriate parameters of the model and fractional order parameter, which are normally obtained from experimental results, we can simulate real viscoelastic behavior much better than with classical integer order derivative models. The model proposed in this work can be used for future investigation to develop a proper position control algorithm of a three body robotic system.

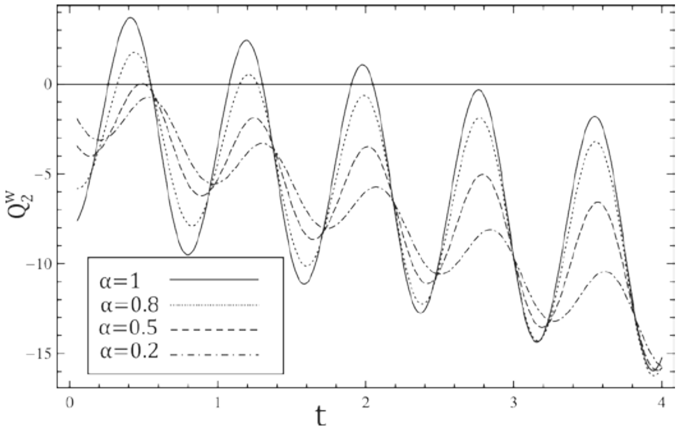

Fig. 2. Q_2^w for different α and $T = 25$ [C^O]

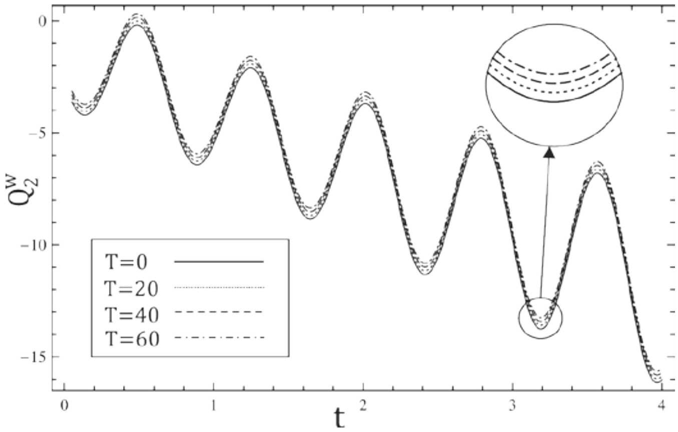

Fig. 3. Q_2^w for different T and $\alpha = 0.5$

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