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A FRACTIONAL ORDER VISCOUS FRICTION MODEL IN ROBOTIC JOINTS

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Abstract:
The fractional calculus and the calculus of variations are utilized to model and control complex dynamical systems. Those systems are presented more accurately by means of fractional models. In this paper, dynamic model of robot manipulator is introduced using the Rodriquez approach. Then, a viscous friction term is extended to be proportional to the fractional derivative of the angular displacement. This model is introduced into dynamic equations via generalized forces which are derived by using the principle of virtual work. Numerical example for the robotic system with three degrees of freedom is presented. The results obtained for generalized forces are compared for different values of parameters in the fractional order derivative model.

Key words: fractional calculus, rigid multibody dynamics, viscous friction model

1. Introduction

Fractional calculus is an emerging field in the area of applied mathematics and mathematical physics and it has many applications in many areas of science and engineering. Researchers have recently proved that the physical systems with dissipation can be modeled more accurately by using fractional representations [1]. Friction plays a dominant role in limiting the quality of robot performance. Non-compensated friction produces static error, delay, and limit cycle behavior. Many works have been devoted to studying friction torque in the joint and transmission systems, and various friction models have been proposed in the literature [2]. The viscous friction is generally represented as being proportional to the velocity. In this paper, the viscous friction is modeled to be proportional to the fractional derivative of the relative angle of rotation q (in case of revolute joints), i.e.

$$M_w = -\beta_w \frac{d^\mu q}{dt^\mu}, \tag{1}$$

wherein M_w denotes friction torque at robot joint, β_w is the viscous friction parameter, and $d^\mu(\bullet)/dt^\mu$ is the fractional derivative (with respect to time) of real order μ. For $\mu = 1$ we obtain classical viscous friction model.

2. Generalized fractional order friction forces

The mechanical structure of a robot manipulator consists of a sequence of rigid bodies (V_1), (V_2), … , (V_n) interconnected by means of joints. In this paper, only robot configurations with revolute joints are considered. Two neighboring bodies (V_{i-1}) and (V_i) are connected together with a joint (i), which allows rotation of the body (V_i) in respect to body (V_{i-1}). The values q^i represent generalized coordinates. Unit vector $\vec{e}_i$, i=1,2, … , n is describing the axis of rotation of the i-th segment with respect to the previous segment. Mathematical model of robot manipulator can be derived using the Rodriquez approach [3], which gives equations of motion of a multibody system in the following form

$$\sum_{\alpha=1}^{n} a_{\gamma\alpha}(q)\ddot{q}^{\alpha} + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} \Gamma_{\alpha\beta,\gamma}(q)\dot{q}^{\alpha}\dot{q}^{\beta} = Q_{\gamma}^{g} + Q_{\gamma}^{m} + Q_{\gamma}^{w}, \quad \gamma = 1,\ldots,n. \tag{2}$$

Herein, n is a number of bodies in the system, $a_{\gamma\alpha} = a_{\alpha\gamma}$ are coefficients of the basic metric tensor, and $\Gamma_{\alpha\beta,\gamma}$ are Cristoffel symbols of the first kind. On the right hand side of (2) we have Q_{γ}^{g}, Q_{γ}^{m} and Q_{γ}^{w} which denote the generalized gravitational forces, motor torques and friction forces, respectively. By using the principle of virtual work, we can calculate the generalized dissipative forces described with (1). First, we define a torque vector that acts on the body (V_i) in the direction of the unit vector $\vec{e}_i$ as

$$\overrightarrow{M}_{wi} = -\beta_{wi}\frac{d^{\mu_i}q^i}{dt^{\mu_i}}\vec{e}_i. \tag{3}$$

According to the third Newton law, a torque vector $\overrightarrow{M}_{wi}^{*} = -\overrightarrow{M}_{wi}$ is acting on the body (V_{i-1}). Then, equation for the virtual work can be given by

$$\delta A_{wi} = \delta A\left(\overrightarrow{M}_{wi}\right) + \delta A\left(\overrightarrow{M}_{wi}^{*}\right) = \overrightarrow{M}_{wi}\sum_{\alpha=1}^{i}\vec{e}_{\alpha}\delta q^{\alpha} + \overrightarrow{M}_{wi}^{*}\sum_{\alpha=1}^{i-1}\vec{e}_{\alpha}\delta q^{\alpha}, \tag{4}$$

which leads to $\delta A_{wi} = \overrightarrow{M}_{wi}\vec{e}_i\delta q^i$. Now, virtual work δA_w of all the friction torques acting on the system can be calculated as $\delta A_w = \sum_{i=1}^{n}\delta A_{wi} = \sum_{i=1}^{n}\overrightarrow{M}_{wi}\vec{e}_i\delta q^i$. Having in mind that $\delta A_w = \sum_{i=1}^{n} Q_{wi}\delta q^i$ and (3), we finally obtain expression for generalized fractional order friction forces in the form

$$Q_i^w = \overrightarrow{M}_{wi}\vec{e}_i = -\beta_{wi}\frac{d^{\mu_i}q^i}{dt^{\mu_i}}. \tag{5}$$

3. Numerical results

For the purpose of performing the numerical simulations we used the robotic system with three degrees of freedom. We presumed generalized coordinates as $q^1 = t,\ q^2 = \sin(2\pi t)$ and $q^3 = t^2$. Figure 1 shows the generalized forces $Q_i^w,\ i = 1,2,3$ with respect to time for different values of fractional order parameter μ_i, and for $\beta_{wi} = 7.7,\ i = 1,2,3$ [2].

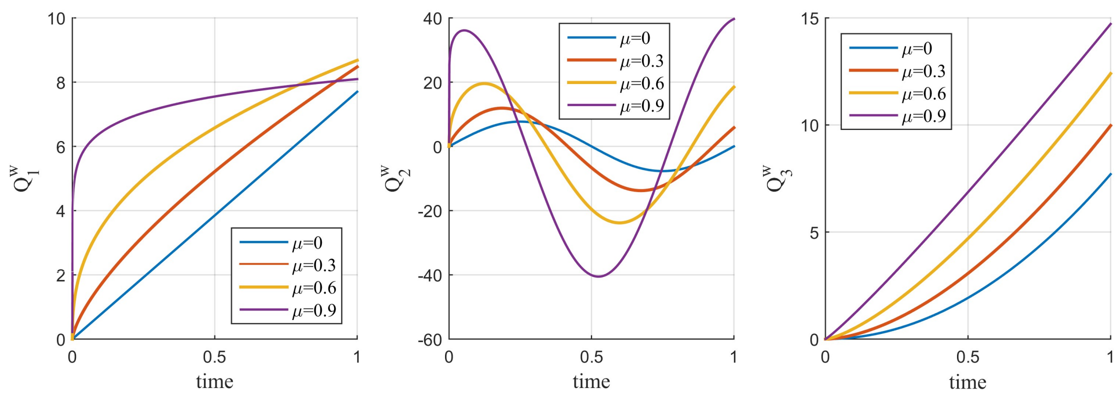

Fig. 1 Generalized forces $Q_i^w,\ i = 1,2,3$ for different values of fractional order μ.

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