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Determining the Risk Level in Client Analysis by Applying Fuzzy Logic in Insurance Sector

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Abstract: The aim of the paper is to determine the risk level of a contract extension with the existing policyholders, which is further propagated to the business effectiveness and long-term sustainability of the company. The uncertainties in the relative importance of risk factors, their values, and risk levels are described by the linguistic forms, which are modeled by using the fuzzy sets theory. The evaluations of the relative importance of risk factors are stated as a fuzzy group decision-making problem. The weights of risk factors are obtained by using a fuzzy analytic hierarchy process. The determination of production rules for the assessment of the risk level is based on fuzzy IF-THAN rules. The verification of the model is performed by using real-life data originating from the insurance company which operates in the Republic of Serbia.

Keywords: risk level; fuzzy data; FAHP; fuzzy logic; production rules

MSC: 97M30

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1. Introduction

Changes in the business world, primarily in the domain of politics and economy, might lead to business uncertainties in all organizations, especially insurance companies. The enhancement of business efficiency is one of the most important tasks for operational and strategic management. To determine whether there will be an extension of the insurance contract for each insured client, it is necessary to anticipate the possibility of further damage to the insurer. Insurance companies have different policies and business strategies, which are based either on high levels of management or acceptable risk levels; it is necessary to analyze the evidence of risk factor values (RFs) and then decide to extend contracts with clients. In practice, it can be seen, that these two RFs need to be analyzed: the amount of money and the number of claims. It is also worth considering that these two RFs, alongside with receivables ratio responsible for measuring premium payments, have a significant impact on clients. Uncertainties in the relative importance of the RFs and their values cannot be accurately determined when the conditions persistently change, due to difficulty in determining the complexity involved in the risk of extension of the contract with the insured. Different types of vagueness, imprecision, and uncertainties are described by linguistic forms that are assigned with different numerical values as a certain degree of affiliation [1–3]. The development in some areas of mathematics such as fuzzy sets theory [4,5] allows uncertainties to be quantitatively represented in a fairly proper way. The basic characteristic of a fuzzy number is a membership function which can take different forms. In the literature, the triangular and trapezoidal membership functions are widely used [6] because they do not require complex calculations; it should be emphasized that the domains of fuzzy numbers are defined on a real line with respect to the nature of linguistic expressions and estimates of DMs.

According to the experience of the best practice, it is known that decision makers have different assessments of the relative importance of RFs. Therefore, it is fully justified to introduce the assumption that determining their relative importance should be posed as a fuzzy group decision-making problem. Decision makers can make significantly better judgments if they look at each pair of RFs separately, by analogy [6]. In accordance with the introduced assumptions, the weight vector of RFs is given by using the Fuzzy Analytic Hierarchy Process (FAHP).

Complex problems, such as those that can determine the level of risk, can be successfully solved by applying fuzzy logic [4,5]. The theory of fuzzy logic has been usefully proven when it is necessary to decide based on experience, intuition, and subjective assessments of individual parameters by decision-makers. Zadeh [2] emphasized the use of fuzzy logic: (1) the mathematical concept is very simple, (2) it is flexible, (3) fuzzy logic tolerates the imprecise data, (4) it can incorporate into the decisions the experience of DM who know and understand the problem, (5) Fuzzy logic is based on the native language, which presents the best way for communication. In addition, this concept has certain shortcomings, such as a large number of production rules. Thus, one of the basic goals is to reduce the number of rules so that they can be effectively used in solving real problems.

The assessment of the level of risk in practice is performed in relation to these two RF: the amount and the frequency of claims; it is believed that RFs are equally important. The motive of this research can be defined as the extent to which the business of the insured, as an individual, may affect the risk level assessment of the insurance company. The given fuzzy logic model is for determining whether it is a risky business for the insurance company to extend the contract with existing clients.

The motivation for this research comes from the fact that there are no research papers that treat the problem of determining the level of business risk based on fuzzy logic rules. In addition, there are no guides or developed methodologies for the company to assess the level of risk of doing business with a client before signing a contract with him.

The research challenges, motivations and the scientific research area is the application of a fuzzy model in the field of determining critical workflow processes to improve business management and risk transfer; this research investigates risk forecasting and management by forming a fuzzy model to determine whether it is risky for an insurance company to extend contracts with existing policyholders based on the flow, the amount, and the number of their claims. Predicting the financial result gives the basic concept of development and business characteristics of the insurance company; it achieves the stabilization of the company's business, and then the growth, development, and improvement of the insurance market, as well as full protection of the interests of policyholders.

Decision-making on the extension of contracts with existing clients in the Republic of Serbia is greatly influenced by changes in the business world, especially in the domain of politics. In addition, the decision makers responsible for the extension of contractual obligation with the specific insured, being also managers within insurance companies are often described as inexperienced, relatively incompetent and dependent, with a lack of tendency to take risks. Weaknesses and failures of managers can lead to wrong business decisions, because of which there can be immediate and long-term consequences for the business and positioning of the insurer in the market.

Many authors believe that the basis of the problem of insolvency of insurance companies lies in low-quality and unprofessional management, while insufficient premium is the ultimate manifestation of this problem. Quality management, i.e., management of DM is of essential importance for the stability of each DM. In addition, lately increasing competition in the insurance field has put focus on the use of new methodologies based on fuzzy logic [7].

These are the goals and main reasons why the authors developed a mathematical model that would be employed to determine the exact level of risk for each client. The model gives significantly better results compared to the used risk matrices in an insurance company. In this regard, there is a possibility of real application of this model because its great importance can be seen in increasing the objectivity of management for decision-making. The obtained results are important for determining a more precise strategy, which leads to an increase in business efficiency.

Integration of the fuzzy sets theory and risk assessment approach can be marked as the aim of this research: (a) the assessment of the relative importance of RFs by using FAHP; (b) modeling of RFs values by fuzzy sets theory; (c) determining the overall index by using the fuzzy operators; (d) defining production rules that can easily and simply determine the level of business risk for each client, in the field of non-life insurance.

The paper is organized as follows: In Section 2, there is a detailed literature review related to the applied fuzzy sets theory for modeling uncertainties and fuzzy logic for the determination of production rules. The proposed methodology is presented in Section 3. In Section 4, the proposed model is illustrated by real-life data which comes from domestic insurance companies which exist in the Republic of Serbia. The discussion of the obtained results and Conclusion is given in Section 5.

2. Literature Review

For the purposeful presentation of the literature review, this section is divided into 3 sub-sections: (1) basic consideration of management problems in the field of insurance, (2) modeling of uncertainties into the relative importance of RFs and their values as well as risk levels, and (3) determining of production rules.

2.1. Some Management Problems in the Insurance Domain

In the literature, many papers consider the problem of assessing the level of the business risk of an insurance company. Many authors suggest that it is necessary to combine risk level assessment methods with fuzzy sets theory as there are a lot of uncertainties in the considered problem. Shapiro [8] analyzed and discussed the benefits of applying fuzzy sets theory and fuzzy logic in solving management problems in the field of insurance.

The problems of investment management in the field of insurance, scheduling of liabilities, as well as cash flow management, are solved by applying the fuzzy logic by Shapiro [9]. Determining the time [10] structure of interest rates, in the field of life insurance, is given by using the fuzzy regression analysis in [10,11]. Berry-Stölzle, et al. [12] suggest that the assessment of the required solvency in property insurance can be successfully performed by using fuzzy regression analysis. Shapiro [13] suggests that annuity damage modeling should be based on the fuzzy set theory. Abul-Haggad and Barakat [14] have developed a fuzzy risk matrix combined with the Mamdani method. In this way, it is possible to accept and process expert knowledge in a much more intuitive way that is closer to human thinking. Markowski and Mannan [15] propose the procedure for determining three types of fuzzy risk matrices (low-cost, standard, and high-cost) that can be used for different safety analyses in the chemical industry. The problem of determining the identification of an insurance company can be successfully solved by applying fuzzy logic rules, according to the opinion of Zapa and Cogollo [16].

2.2. Modelling of Existing Uncertainties

The uncertain and imprecise data (in this paper these are the relative importance of RFs and their values as well as the risk level) can be adequately represented by linguistic expressions. The choice of membership function can be considered a problem in itself. Triangular or trapezoidal membership functions are most often used because they do not require great computation complexity. Furthermore, there is no guideline or recommendation in the relevant literature for the determination of the bounds in the domain of fuzzy numbers. Hence, it can be said that the number and type of linguistic variables are determined by DMs, depending on the type and complexity of the problem; it should be mentioned that linguistic variables can be modeled by self-confidence interval (LIT) [17].

The relative importance of considered RFs is not equal and does not change over time. The assessment of relative importance depends on the knowledge and experience of DMs.

In general, the relative importance of items can be determined: (a) in a direct way [18] in the direct method of processing, decision-makers associate pre-defined linguistic terms to each RFs that describe their weight, and (b) by setting up a fuzzy pair-wise comparison matrix [19–21]; it is considered that in this way DMs can make a better and more accurate assessment. The fuzzy rating of the relative importance of items is, more or less, burdened by DMs' errors. Therefore, it is necessary to check the extent to which these errors affect the accuracy of the vector weights. In conventional AHP [22], the consistency estimate of DMs is based on applying the Eigenvector method. There are many procedures for handling FAHP, their advantages, and disadvantages, which are analyzed by Kahraman, et al. [23]. The similarities and differences between the proposed FAHP are presented in Table 1.

Table 1. FAHP.

Author's	Type Variable/ Granularity/Domain	Group Decision-Making Problem/ Aggregation Method	Pair-Wise Comparison Matrix/Consistency Checking	Handling of Uncertainties in FAHP	The Weights Vector	Application Domain
Chen, et al. [24]	TFNs/5/[1–3.5]	Yes/the proposed procedure	Concept equal possibilities/ Eigenvector [22]	Extent analysis [25]	crisp	Evaluation performance in the education domain
Sultana, et al. [26]	TFNs/5/[1–9]	-	Yes	Extent analysis [25]	crisp	
Sirisawat and Kiatcharoenpol [27]	TFNs/9/[1–10]	-	-	Extent analysis [25]	crisp	
Jakšić, et al. [20]	TFNs/5/[1–5]	-	-	Extent analysis [25]	crisp	Ranking of banks
Banduka, et al. [28]	TFNs/5/[1–5]	-	-	Extent analysis [25]	crisp	Extension of FMEA in automotive industry
Lyu, et al. [29]	Defined procedure for determination of fuzzy elements of fuzzy pair-wise comparison matrix/the elements of pair-wise comparison matrix is given by applying the ranking of fuzzy numbers	-	Eigenvector [22]	Extent analysis [25]	crisp	Risk assessment in civil engineering
Bakır and Atalık [21]	TFNs/9/[1–9]	Yes/fuzzy geometric mean	-	The proposed method by Buckley [30]	Crisp is given by applying the center of area method [4] and linear normalization procedure TFNs/5 [31]	Assessment of quality in the air industry
Calabrese, et al. [32]	TFNs/5/[1–3.5]	-	The defuzzification procedure for TFNs [33]/ Eigenvector [22]	Extent analysis [25]	crisp	ranking of ISO sustainability subjects
The proposed model	TFNs/3/[1–5]	Yes/the proposed procedure	The center of area method [4]/ Eigenvector [22]	Extent analysis [25]	crisp	Assessment of RFs in the insurance sector

By comparing papers that deal with the proposed procedure certain similarities can be noticed. In the analyzed papers, elements of fuzzy pair-wise comparison matrix are described by TFNs, as in this research. As it is noted, the granulation of used fuzzy numbers depends on the size and complexity of the considered problem. There are no recommendations on how to determine granulation. The nine-point scale has been proposed by [21,27]. Most authors suggest a five-point scale [20,24,26,29,33]. The point scale is introduced in this research which represents one of the differences between this and the other analyzed papers.

In the literature, many authors are determining the relative importance of fuzzy group decision-making problems [21,24] as in this research. The authors are of the opinion that a more accurate assessment of the relative importance can be more accurately determined if more DMs participate in the decision-making process. The aggregation of the opinions of DMs into unique marks can be performed by applying the different aggregation operators such as: fuzzy geometric mean [21], and (ii) the proposed procedure [24], as in this research.

In conventional AHP [22] it was emphasized that it is necessary to check the consistency of estimates of DMs. A fuzzy pair-wise comparison matrix can be considered consistent if the corresponding crisp matrix is consistent. The fuzzy pair-wise comparison matrix can be transformed into a correspondent pair-wise comparison matrix by using different defuzzification procedures, such as: (i) simple defuzzification is applied in [33], (ii) α cut level applied in [24], and (iii) the center of the area method which is applied in this research. There are many developed methods for checking the consistency of a pair-wise comparison matrix [25]. One of the most widely used methods is the Eigenvector used in the analyzed papers (see Table 1), as in this research.

The determination of the weights vector can be based on: (i) the method proposed by Buckey [30] and (ii) the method of extended analysis [25]. Some authors [21] consider that the method proposed by Buckey [30] has certain advantages over the method of extended analysis. On the other hand, the method of extended analysis [25] is easy to understand, and therefore, the method of extended analysis is widely used for handling FAHP (see Table 1) as well as in this research.

2.3. Fuzzy Production Rules

Assume that the output variable depends on several input variables that have different values. The number of possible values that can be assigned to an output variable is equal to the number of combinations with repetition. The solution obtained in this way is not applicable in practice. Therefore, the application of IF-THEN logic rules can lead to solutions that practitioners can easily understand and apply. The problem becomes significantly more complex if there are multiple input variables whose values can be described using several linguistic terms. The solution to such complex problems can be successfully obtained through the experience and knowledge of DMs, which are formalized by fuzzy IF-THAN rules (see Table 2).

Table 2. Fuzzy IF-THAN rules.

Author's	Number of Input Variables	Number, Type and Domain of Linguistic Terms	Normalized Input Variable Values/the Weights Vector of Input Variables/the Weighted Input Variables Values	The Overall Index/Defuzzification	Number of Decision Rules and its Type	Application Domain
Sii, et al. [34]	2	6/TrFNs/[0–10]	-	-	4/Fuzzy rules made by experts	safety of the marine system
Gentile, et al. [35]	3	5/Gausian and TFNs/[100–500] and [0–1]	-	Fuzzy union/moment method	5/TFNs and TrFNS	safety principles to plant design and operating plants
Tadić, et al. [36]	15	7/TFNs/[0–1]	-	Fuzzy union/moment method	7/TrFNs	Customer satisfaction with banking service quality
Aleksić, et al. [37]	7	5/TFNs/[0–1]	The linear normalization procedure/FAHP combined with FOWA and fuzzy union/product of fuzzy numbers	Arithmetic mean	5/TrFNs	Assessment of organization's vulnerability
Tadić, et al. [38]	3	5/TFNs/[0–1]	-/FAHP/dilatation operator	Fuzzy cut/moment method	5/TFNs	Inherent safety index for food industry
The proposed model	3	4/TFNs/different measurement scales	Yes/linear normalization procedure-	Fuzzy union of the TFNs describing the weighted normalized input variable values/moment method	4/TrFNs	Assessment risk level in insurance companies

According to the overall index value, production rules can be defined. In this way, the number of production rules is significantly reduced, and at the same time, the effectiveness of solving complex problems is significantly increased. A brief retrospective of these papers is given below. The similarities and differences between the proposed fuzzy IF-THAN rules are presented in Table 2.

The authors have used a different number of input variables whose membership functions have different shapes. In the analyzed papers, the authors assumed that input, as well as output variables, are described by uncertain numbers whose domains are defined on real lines into intervals [0–1]. If the values of the input variables are defined on other measurement scales (as in this paper), then it is necessary to perform their normalization. In this research, the linear normalization procedure is performed. There are many papers in which the overall index depends on the values and weights of input variables [38,39]. With respect to the results published in the literature in the field of risk analysis, it can be considered that the risk level can describe with not less than 3 and not more than 5 linguistic terms. Basically, all authors (see Table 2) discuss those linguistic expressions that describe the values of output variables that can be modeled with sufficient accuracy by using TrFNs, as in this research. As is well known, TrFNs capture uncertainty better than TFNs. In that case, it is necessary to normalize the values of input variables in Aleksić, et al. [37] developed a procedure for determining the overall index, which is described by precise numbers. [38] or fuzzy union in the rest analyzed papers, as in this research. By applying the defuzzification procedure, the fuzzy overall index value is presented by a precise value. There are many defuzzification procedures that can be found in the relevant literature [5]. The widely used defuzzification procedure is the current method in this research.

The results obtained by analyzing the relevant literature show that the determination of the risk level of a contract extension with the clients in the field of insurance is based on: (i) respecting two RFs (the claim amount and the claim frequency) which have the same relative importance and (ii) subjective assessment of the DMs. The best practice experience shows that it is necessary to consider the Claims ratio, which is included as the third RF in this research; it is assumed that the considered RFs do not have equal importance and that it is determined in an exact manner. In the analyzed literature, the RF values are described as crisp. Due to the significant economic and political changes which are happening in the region, it can be said that describing RF values by precise numbers, and especially the claim amount, is not appropriate; it considerably makes it difficult for DMs to estimate the risk of a contract extension with the clients. In this research, the RF values are modeled by using the fuzzy sets theory which allows for them to be described in a sufficient enough manner. Determination of the risk level is based on the proposed model which significantly decreases the subjectivity of the DMs.

3. The Proposed Model

One of the important problems in any insurance company is the risk level analysis of business due to the extension of contracts with clients. The solution to this problem greatly affects the achievement of business goals, primarily the survival and the development of the insurance company.

It is known that the risk level of insurance policy extensions is affected by numerous RFs that can be formally represented by a set of indexes $\{1, \ldots, i, \ldots, I\}$ where I is the total number of RFs and i, $i = 1, \ldots, I$ is the index of RF. The number and the type of RFs are determined by DMs according to their experience and knowledge as well as the results of the best practice. In this research, an insurance extension risk assessment is considered with respect to three RFs: the amount of claims $(i = 1)$, the number of claims $(i = 2)$, and the claims ratio $(i = 3)$.

In this research, based on the results from the most successful insurance companies, we have introduced RFs that do not have the same relative importance. The relative importance of RFs is assessed by DMs. The DMs are presented by sets of indices $\{1, \ldots, e, \ldots, E\}$. The total number of DMs is denoted as E and e, $e = 1, \ldots, E$ is the index of DM. In this paper, DMs are underwriters, as one of the most important functions of insurers in the decision-making process. The underwriter needs to match earned premium with the claims with an eye on profitability. If the premium is not sufficient to cover the claims, the insurer is confronted with the probability of loss, and the underwriting risk arises; this risk could include the underestimated liabilities arising from unpaid business written in previous years, for example. The relative importance of RFs is stated by a fuzzy pair-wise comparison matrix at the level of each DM.

In general, RFs values can be adequately described by using K different linguistic expressions which are modeled by TFNs, $\widetilde{v}_{ji}, i = 1, \ldots I; j = 1, \ldots J$. The domains of these TFNs belong to different intervals on the real line and have different measurement units. Interval limits are determined according to DMs estimates; they base their estimates on evidence data and experience. The weighted normalized RFs values are given by using the fuzzy algebra rules.

The procedure for determining the level of risk of contract renewal for each client separately, considering RFs values as well as their weights, is further briefly described. Firstly, the fuzzy overall risk index values, as well as their representative scalars are calculated by using fuzzy algebra rules.

The total number of output rules N is given according to the following logistic rule $N = J^K$. In this manuscript, respecting the introduced assumption $N = 3^4 = 81$; it can be clearly concluded that the use of the principle of approximate reasoning is not justified in practice. Reducing the number of output variables, which at the same time leads to increased decision-making efficiency, can be achieved by using fuzzy IF-THAN rules based on the Mamdani method [18,40], as well as fuzzy preference relation (LIT 2) [41]. Mamdani method is widely accepted for identifying the level of risk in insurance for the collection of expert knowledge because it allows expertise in a more intuitive way [8]. Each rule can be represented by a classical implication where the logical operator cut in the logic phase is replaced by taking the minimum value under certain conditions. Because of that, it can be said that the system is simplified by discarding the least significant rules. In this way, the practical applicability of the developed model is significantly increased and at the same time, sufficient inference accuracy is achieved. Representation of the considered problem can be presented in black box form, as it is shown on Figure 1.

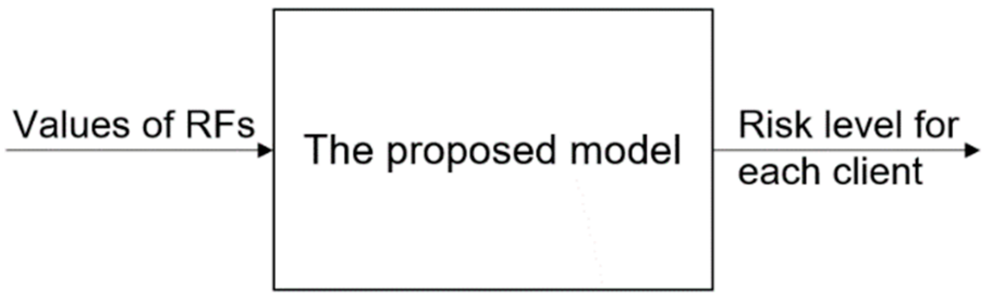

Figure 1. Black box form of the considered problem.

A flowchart of the research methodology is presented in Figure 2.

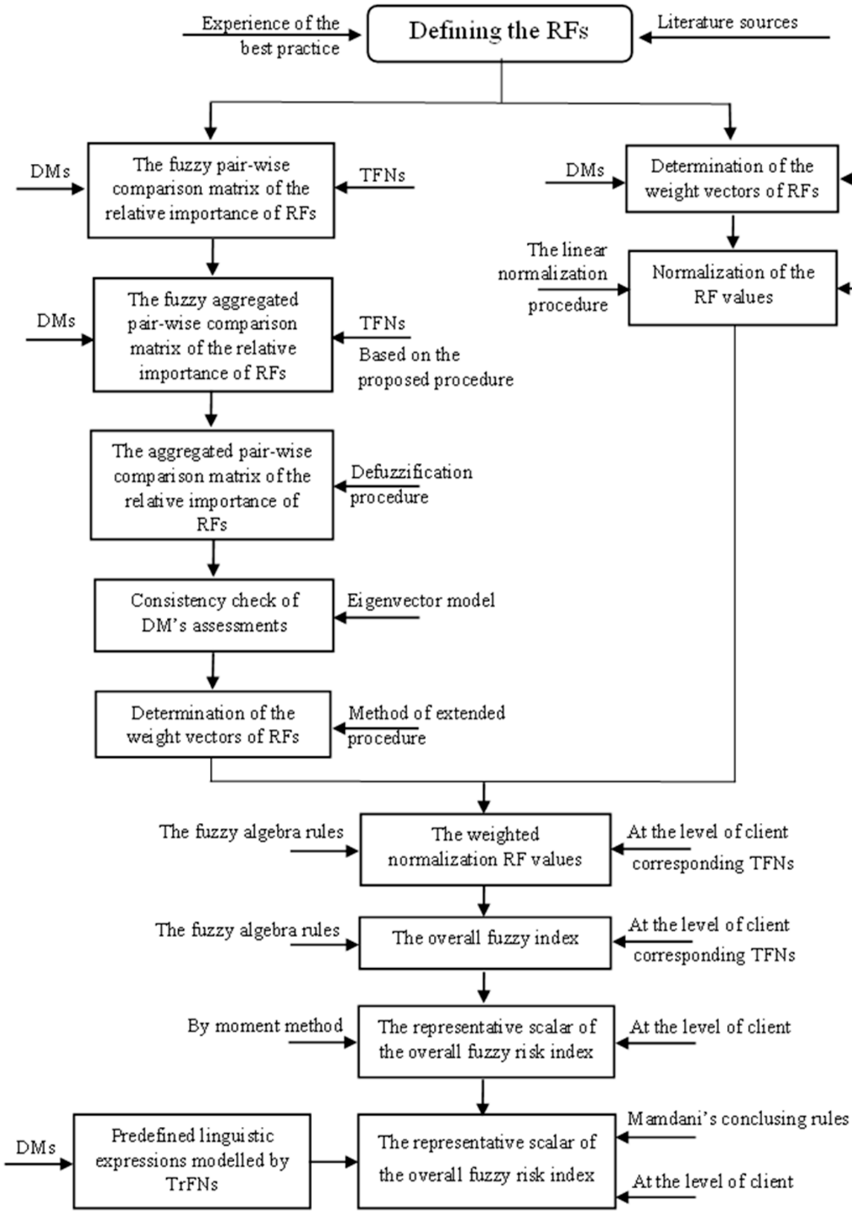

Figure 2. The proposed methodology.

3.1. The Modelling of the Relative Importance of the RFs

The relative importance of RFs is not equal, and it does not change over time; they involve a high degree of subjective judgments, knowledge, and experience of DMs; they use pre-defined linguistic expressions which are modeled by TFNs, $\widetilde{W}^e_{ii'} = \left(l^e_{ii'}, m^e_{ii'}, u^e_{ii'}\right)$ with the lower and upper bounds $l^e_{ii'}, u^e_{ii'}$ and modal value $m^e_{ii'}$, respectively. Values in the domain of these TFNs belong to a real set within the interval [1–5]. A value of 1 or 5 means that the relative importance of RF over RF is very small, or extremely large, respectively.

If the strong relative importance RF i' over RF i holds, then the pair-wise comparison scale can be represented by the TFN

$$\widetilde{W}^e_{ii'} = \left(\widetilde{W}^e_{i'i}\right)^{-1} = \left(\frac{1}{u^e_{ii'}}, \frac{1}{m^e_{ii'}}, \frac{1}{l^e_{ii'}}\right)$$

If $i = i'$ then the relative importance of RF i over RF i' is represented by a single point 1, which is a TFN (1, 1, 1).

These TFNs are given in the following way and presented in Figure 3:

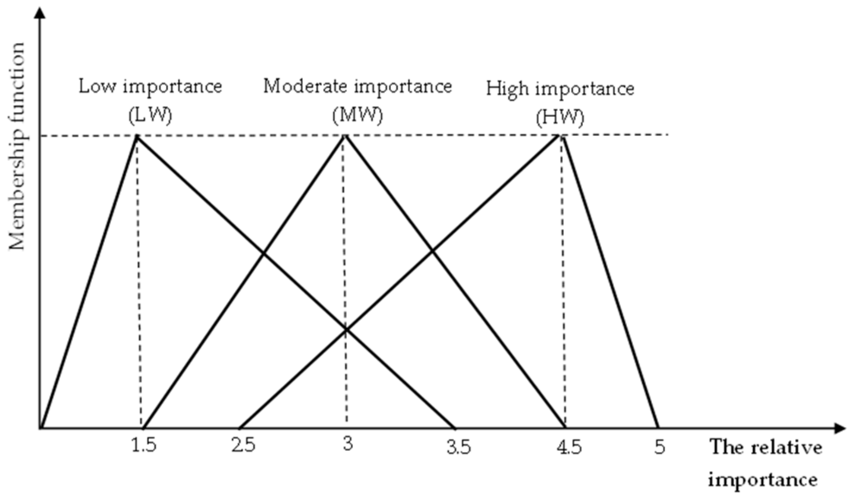

Figure 3. The TFNs describing the relative importance of RFs.

low importance (LW)—(1, 1.5, 3.5)
moderate importance (MW)—(1.5, 3, 4.5)
high importance (HW)—(2.5, 4.5, 5)

Evaluation and relative importance of RFs is based on the consideration of the probability of the possible event, the outcome (amount of the claim incurred), and frequency (number of claims over the period of time). Decision makers, managers, will manage by analyzing the relative importance of RFs, frequency and the size of a claim for one client. The linguistic domain scales are determined by the number and amount of liquidated damage claims for the clients in the insurance company. The domain of the TFN is made on the basis of an analysis of the total number and amount of claims, the control environment, the inherent risks and the measurement in terms of impact and probability.

3.2. The Modeling of RFs Values and Risk Levels

DMs have defined linguistic expressions that can be used to describe the values of treated RFs; these linguistic expressions and their corresponding TFNs are presented in the following.

RF- the claim amount: Claims covered by property insurance (things) are, as a rule, only pecuniary damage claims, incurred on an insured thing or object, which can be partial or total considering the claim intensity. Based on the actual database, it is known that the amount of claim (expressed in thousands of monetary units) is neither less than zero nor more than 300. Values of these RFs can be described by four linguistic forms: *Small (L1), Medium (L2), Large (L3), and Total (L4)*; these linguistic forms are modeled by using TFNs (see Figure 4); it should be noted that the values in the domain were determined by DMs based on their experience. Figure 4b shows these normalized TFNs.

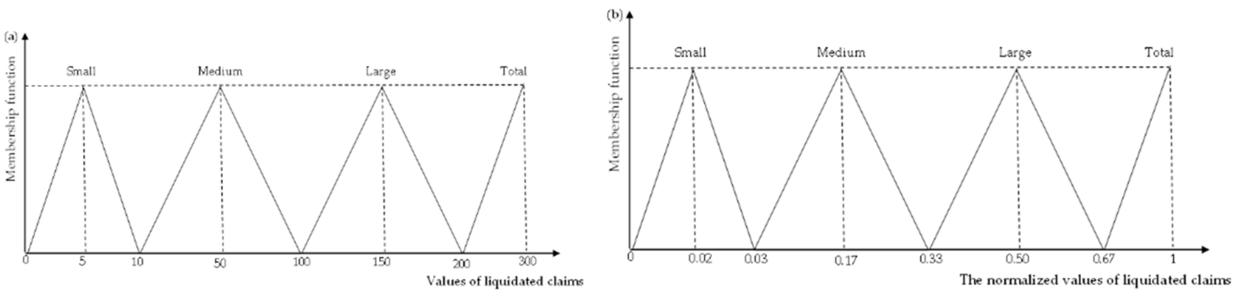

Figure 4. Linguistic forms for describing: (**a**) values of settled claims and (**b**) corresponding the normalized TFNs.

Property insurance claims, considering the amount of claim, can be partial or total. The domain of the TFN for the amount of claims is determined by defining the limit values of the claim. Limited amounts are defined by expert experience according to the actual movement of the amount of claims all clients in the insurance company in the case of property insurance. Based on expert experience, the limit values are defined by the domain of the TFN based on the average claim amount for all insureds in the case of property insurance; it therefore seems that the claims exceeding RSD 300,000 are considered to be total, and all under this amount of partial claims. Fuzzy sets that describe the input variable 'claim amount' do not cover equal intervals, as a result of the fact that these fuzzy sets are defined on the basis of the empirical dana of the real insurance company portfolio.

RF- the claim frequency: The number of incurred claims by one client in the observed period defines the frequency of claims. The value of this RF can be described by four linguistic forms joined by TFNs correspondents: *Negligible (M1), Moderate (M2), High (M3), and Extremely high (M4)* (see Figure 5). The values in the domain of these linguistic expressions are determined by respecting the number of incurred adverse events of each client under each contract. By using the normalization procedure, the normalized TFNs are presented in Figure 5b.

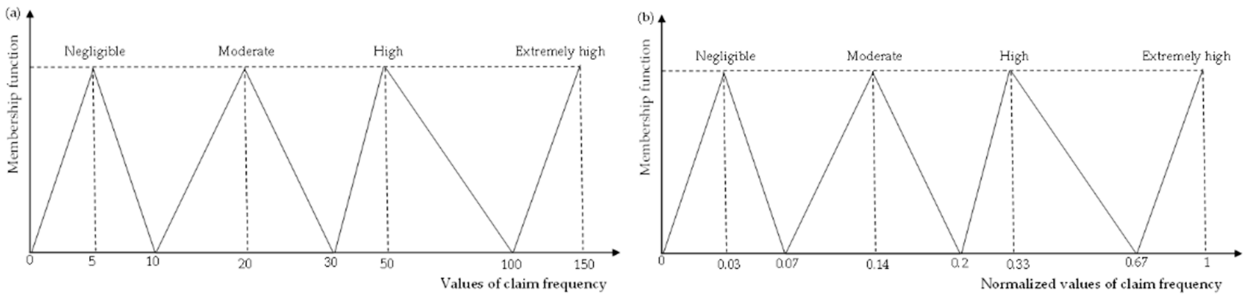

Figure 5. Linguistic forms for describing (**a**) the claim frequency and (**b**) corresponding the normalized TFNs.

The values of RF number of claims depend on the claim frequency of one client. The input variable claim frequency refers to the number of claims caused by one client over the observed period. As well as with the first input variable, it is necessary to determine its domain. Based on the actual movement of the number of claims in the case of property insurance for all clients in the insurance company, the experts will define the interval to which the linguistic scales belong. The second input variable, claim frequency, refers to the number of claims incurred by one client during the insurance period. The parameter assessment is defined on the basis of expert experience and the actual database about number of claims incurred by each client individually in the insurance company for property insurance. The claim frequency is estimated according to the number of harmful events caused by one client, observed through all his property insurance contracts. Because of the ranking of frequency, which is based on the real database expertise, this input variable is shown by 4 fuzzy sets that do not cover equal intervals.

RF- Claim ratio: The claim ratio can be defined as the ratio of incurred claims and earned premiums in the observed period (year) and it is considered the simplest measure of premium adequacy in the field of non-life insurance; it can be mentioned that the value of this RF may affect the company's profit for the entire insurance period of the observed client. As a percentage share of incurred claims in the earned premiums, it is necessary to consider the claim ratio when determining the risk level in the insurance company; it is an indicator of the sufficiency of the premium to cover insurance liabilities. Based on the value of this RF, it can be determined whether the premium is sufficient to cover policy liabilities. Based on the best practice experience, the values of RF can be described by four linguistic forms which are modeled by TFNs: *Optimal (K1), Very good (K2), Conditionally acceptable (K3), and Unacceptable (K4)*, which are presented in Figure 6.

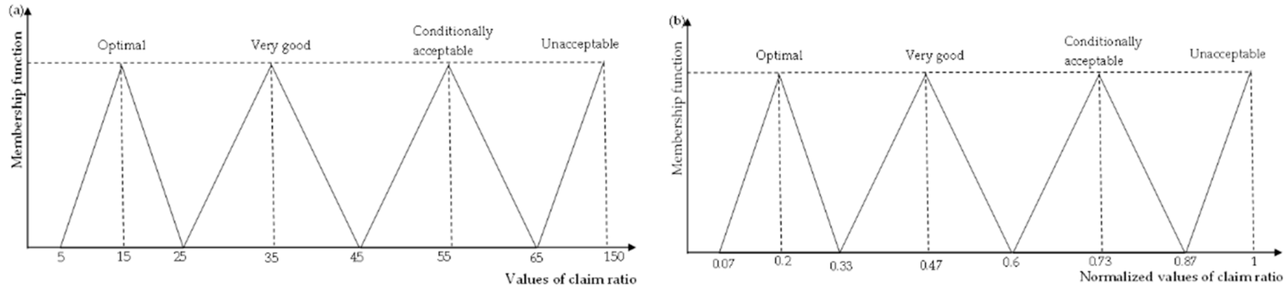

Figure 6. Linguistic forms for describing (**a**) claim ratio and (**b**) corresponding the normalized TFNs.

Fuzzy sets that describe the input variable 'claim amount' do not cover equal intervals, as a result of the fact that these fuzzy sets are defined on the basis of empirical data of the real portfolio of the insurance company. Earned premium depends on each contract with each client. The linguistic variables corresponding to the TFN ratio claims this input variable shown by 4 fuzzy sets that do not cover equal intervals, because the ratio claims represent the percentage share of the damage in the premium.

3.3. Risk Levels

The management of insurance companies may define different levels of risk. For instance, a risk level may refer to the maximum percentage of change given the worst-case level of RFs values. Based on the best practice results from the insurance domain, the risk level can be modeled by one of the four predetermined linguistic terms which are modeled by trapezoidal fuzzy numbers (TrFNs) which are presented in Figure 7:

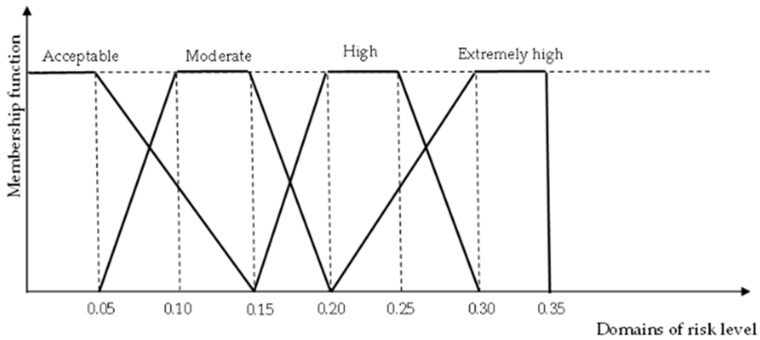

Figure 7. Linguistic expressions and corresponding TrFNs for describing risk levels.

Acceptable (Q1)—(0, 0, 0.05, 0.15)
Moderate (Q2)—(0.05, 0.1, 0.15, 0.2)
High (Q3)—(0.15, 0.2, 0.25, 0.3)
Extremly high (Q4)—(0.2, 0.3, 0.35, 0.35)

The domains of these TFNs are defined into a set of real line intervals [0–0.35]. The upper bound of this interval was determined by the assumption that the considered RFs have different weights. If the overlap from one TrFNs to the other TrFNs is very high, it obviously indicates that there is a lack of knowledge about the risk level or a lack of sufficient partitioning. The proposed values of the defined risk level represent the initial draft assessed by DMs' opinion in the insurance companies in the Republic of Serbia.

3.4. The Proposed Algorithm

In this Section the proposed Algorithm is presented and carried out in the following steps:

Step1. Fuzzy rating of the relative importance of each paiI If RI, $i = 1, \ldots, I$ is performed by DMs, so that:

$$\widetilde{W}^{e}_{ii'}, i, i' = 1, \ldots, I; i \neq i'$$

Step 2. Fuzzy aggregated pair-wise comparison matrix of the relative importance of RFs is $\left[\widetilde{W}_{ii'}\right]_{I\times I}$.
where:

$$l_{ii'} = \min_{e=1,\ldots,E} l^{e}_{ii'},\ m_{ii'} = \sqrt[E]{m^{e}_{ii'}},\ u_{ii'} = \max_{e=1,\ldots,E} u^{e}_{ii'},$$

Step 3. Transform the fuzzy pair-wise comparison matrix into the pair-wise comparison matrix of the relative importance RFs:

$$[\theta_{ii'}]_{I\times I}$$

where:

$\theta_{ii'}$ is the representative scalar of the TFN of $\widetilde{W}_{ii'}$, which is obtained by the moment method [4].

The consistency of the pairwise comparison matrix, is verified by applying the eigenvector method [22].

Step 4. Calculate the normalized weights vector, of treated RFs by using the method of extended analysis [25]:

$$[\omega_i]_{I\times 1}$$

Step 5. Each RF can be described by using K_j predefined linguistic forms modeled by TFN,

$$\widetilde{v}_{ij}, i = 1,\ldots I;\ j = 1,\ldots J$$

Step 6. Normalized RFs values, $\widetilde{r}_{ji}, i = 1,\ldots I;\ j = 1,\ldots J$, were obtained by linear normalization procedure [38].

Step 7. The weighted normalized RFs values are given by using fuzzy algebra rules [9]:

$$\widetilde{z}_{ij} = \omega_i \cdot \widetilde{r}_{ij}\ ,\ i = 1,\ldots I;\ j = 1,\ldots J$$

Step 8. Determine the overall fuzzy risk index, $\widetilde{\rho}_j$:

$$\widetilde{\rho}_j = \bigcup_i \widetilde{z}_{ji}$$

where I is the overall output variables (in this case the overall number of RFs).

Step 9. The representative scalar of the TFN, $\widetilde{\rho}_j, \rho_j$ is calculated by the moment method [9]:

$$\rho_j = defuzz\ \widetilde{\rho}_j$$

Step 10. There are several manners for determining the IF-THEN rules. In this paper, rules are built from the DMs' knowledge and experience by analogy to Mamdani's concluding rules. There are four production rules modeled by the TFNs $\widetilde{s}_q, q = 1,\ldots,4$.

The region of risk in the observed insurance company can be defined according to the rule:

IF the value of "the overall risk index value" equals ρ_j, THEN the region of risk is described by the linguistic form where

$$\max_{q=1,\ldots,4} \mu_{\widetilde{s}_q}(\rho_j) = \mu_{\widetilde{s}^*_q}$$

In this way, the fuzzy risk matrix is constructed.

Step 11. The proposed model is verified by real life data.

4. Illustrative Example

The proposed model is tested on real-life data obtained in the period from 2009 to 2019 and comes from the domain of property insurance, one of the most common types of non-life insurance, from one of the largest insurance companies in the Republic of Serbia. In insurance companies, risk management is the responsibility of actuaries and underwriters, so the assessment of the importance of the treated RF is obtained from the actuaries and underwriters. By applying the interview method, we have obtained a fuzzy rating of actuaries and underwriters. RF values at the level of each client can be based on data evidence. Validation of the model has been performed on a sample of 100 clients from the group of clients who were observed over the period of 10 years and had claims for at least 7 years. The sample was determined randomly without repetition.

The insured in advance pays the premium, and the insurance company pays off compensation to the client, if, and when an insured adverse event occurs. If DMs do not determine the level of the client's risk from the aspect of a contract extension for the next period, that could lead to the inability of the company to settle its obligations to other policyholders. The consequence of a bad or insufficiently good decision of DMs may jeopardize the liquidity and survival of the insurance company.

The procedure of the proposed Algorithm is shown below.

To reduce the number of calculations, the example was formed on decisions of three DMs, respectively, the assumption is E = 3.

According to the proposed algorithm (Step 1) fuzzy pair-wise comparison matrix at the level of each DM is constructed

$$\begin{bmatrix} (1,1,1) & LW, 1/MW, (1,1,1) & HW, MW, MW \\ & (1,1,1) & MW, HW, LW \\ & & (1,1,1) \end{bmatrix}$$

The proposed process of aggregation is illustrated by the following example (Step 2 of the proposed Algorithm):

$$l_{12} = \min_{e=1,\ldots,E}(1,\ 0.22,\ 1) = 0.22$$

$$m_{12} = \sqrt[3]{(1.5 \cdot 0.33 \cdot 1)} = 0.79$$

$$u_{12} = \max_{e=1,\ldots,3}(3.5,\ 0.67,\ 1) = 3.50$$

The aggregate values of the other elements of the unclear aggregate comparison matrix are determined in a similar way (Step 2 of the proposed Algorithm), so that:

$$\begin{bmatrix} (1,1,1) & (0.22,\ 0.79,\ 3.5) & (1.5,\ 3.39,\ 5) \\ (0.29,\ 1.27,\ 4.55) & (1,1,1) & (1,\ 2.69,\ 5) \\ (0.2,\ 0.29,\ 0.67) & (0.2,\ 0.37,\ 1) & (1,1,1) \end{bmatrix}$$

The pair-wise comparison matrix of the relative importance of RFs (Step 3 of the proposed algorithm):

$$\begin{bmatrix} 1 & 0.79 & 3.39 \\ 1.27 & 1 & 2.69 \\ 0.29 & 0.37 & 1 \end{bmatrix},\ C.I. = 0.064$$

By applying the concept of extent analysis (Step 4 of the proposed Algorithm), the weights vector is calculated:

$$W_p = (1,\ 0.97,\ 0.5)$$

The normalized weights vector ω:

$$\omega = \begin{pmatrix} 0.41 & 0.39 & 0.20 \end{pmatrix}$$

The proposed procedure (Step 5 to Step 9 of the proposed Algorithm) is illustrated in the next example. Let the considered RFs be described by the linguistic characterization: *Medium (L1), High (M3) and Unacceptable (K4).* The weighted normalized values of these RFs are shown in Figure 8.

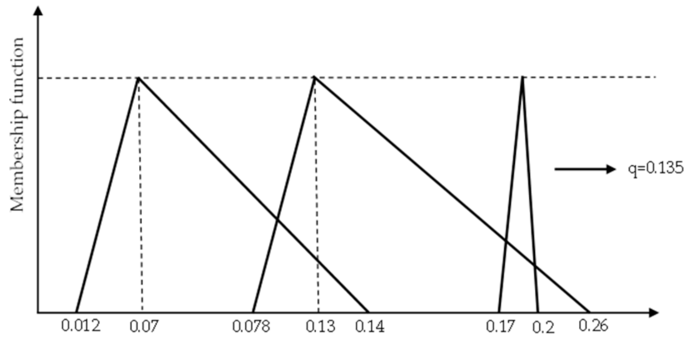

Figure 8. The weighted normalized values of linguistic expressions L1, M3 and L4.

The region of risk for the treated example is obtained by using the procedure (Step 10 of the proposed Algorithm):

max(0.15, 1) = 1, so, it follows, that the level of risk can be described as a moderate risk level.

In a similar way, the level of risk is determined for all combinations of RF values, so that the fuzzy risk matrix is constructed, presented in Table 3:

Table 3. Fuzzy risk matrix.

	Risk Level		**Risk Level**		**Risk Level**	
L1-M1-K1	Q1	L2-M1-K1	Q2	L3-M1-K1	Q2	L4-M1-K1
L1-M1-K2	Q1	L2-M1-K2	Q2	L3-M1-K2	Q2	L4-M1-K2
L1-M1-K3	Q2	L2-M1-K3	Q2	L3-M1-K3	Q2	L4-M1-K3
L1-M1-K4	Q2	L2-M1-K4	Q2	L3-M1-K4	Q2	L4-M1-K4
L1-M2-K1	Q2	L2-M2-K1	Q2	L3-M2-K1	Q2	L4-M2-K1
L1-M2-K2	Q2	L2-M2-K2	Q2	L3-M2-K2	Q2	L4-M2-K2
L1-M2-K3	Q2	L2-M2-K3	Q2	L3-M2-K3	Q2	L4-M2-K3
L1-M2-K4	Q2	L2-M2-K4	Q2	L3-M2-K4	Q2	L4-M2-K4
L1-M3-K1	Q1	L2-M3-K1	Q2	L3-M3-K1	Q2	L4-M3-K1
L1-M3-K2	Q2	L2-M3-K2	Q2	L3-M3-K2	Q2	L4-M3-K2
L1-M3-K3	Q2	L2-M3-K3	Q2	L3-M3-K3	Q2	L4-M3-K3
L1-M3-K4	Q2	L2-M3-K4	Q2	L3-M3-K4	Q3	L4-M3-K4
L1-M4-K1	Q2	L2-M4-K1	Q3	L3-M4-K1	Q3	L4-M4-K1
L1-M4-K2	Q2	L2-M4-K2	Q3	L3-M4-K2	Q3	L4-M4-K2
L1-M4-K3	Q3	L2-M4-K3	Q3	L3-M4-K3	Q3	L4-M4-K3
L1-M4-K4	Q3	L2-M4-K4	Q3	L3-M4-K4	Q3	L4-M4-K4

The proposed model is verified by a sample consisting of 100 clients and presented in Table 3 (Step 11 of the proposed Algorithm).

It should be noted that, in the considered insurance company, there is a good record of the values of RFs, at the level of each client. For the purposes of this research, the considered period was last 10 years. The values of these RFs were calculated by using an arithmetic mean operator. Based on thus obtained values each RF, at the level of each client, appropriate linguistic characterizations were joined. Respecting the constructed fuzzy risk matrix, the risk level for a contract extension was determined and presented in Table 3. Furthermore, the risk level determined by the assessment of DMs of the insurance company was presented in the same table.

5. Discussion and Conclusions

The management practice shows that evaluation and enhancement of business effectiveness in the insurance domain represent some of the most relevant issues of competitiveness and sustainability over a long period. The definition of an enhancement strategy should be based on the assessment of the level of risk for a contract extension for each of the insured clients. Insurance companies have different policies and strategies, which depend on the extent to which experts are willing to accept a certain level of risk; it is necessary to determine the current "behavior" of the insured, which includes records of all claims by the same policy (number and the number of claims in an accident year), fulfillment of financial obligations from previous contracts (premium), how often and by which terms the client violated previous contracts, the ratio of premiums and claims.

In practice, risk assessment is mainly based on the application of the risk matrix. The elements of the risk matrix are average values that depend on expert judgment and opinion; it should be noted that DMs can be characterized by a lack of experience, competence, autonomy, as well as a tendency to take risks. Weaknesses and omissions of managers may lead to wrong business decisions that can be immediate but also have long-term consequences for the business and position of the insurer in the market.

Determining the solvency of clients by using the exact method would significantly contribute to reducing the business risk of an insurance company that operates in a changing and competitive environment.

The main contributions of the presented research are:

1. Determines the lists of RFs in compliance with the best practice
2. With respect to the human way of thinking, modeling of existing uncertainties is based on TFNs and TrFNs
3. The aggregated fuzzy pair-wise comparison matrix of the relative importance of RFs is constructed by using the proposed method
4. The weights vector of RFs is determined by FAHP [25]
5. The fuzzy overall risk index at the level of each insured client is calculated by applying fuzzy algebra rules
6. Risk matrix is constructed with respect to all RFs, their weights, and values by using the fuzzy IF-THAN rules.

The proposed model is tested and verified on real-life obtained based on data from 100 clients. The values of RFs were obtained from the data basis within the period from 2009 to 2019.

The practical implications of the proposed methodology are oriented to DMs who need to make a decision about which should enable the liquidity of the company. Based on the results obtained, it can be concluded that 1% of clients have an acceptable risk level. 46% of clients have a Moderate risk level. Based on the results obtained, for about 50% of the insured clients, the company's management may conclude that it could extend the contract. An unacceptable level of risk occurs for 12% of policyholders, which further means that the extension contracts with them may cause a decrease in the liquidity of the company. To extend the contract with these insured clients, it is necessary to do additional research.

The main advantage of the proposed model can be emphasized through the fact that DMs easily extend to the analysis of other management decision-making problems in different areas. The main limitation of the proposed model is that it can make decisions quickly, and at the same time, the obtained decision is less encumbered by the subjectivities of DMs so, this could make it more accurate.

Finally, further research could focus on the development of a software based on the proposed model.

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References

1. Zhang, C.L. Risk assessment of supply chain finance with intuitionistic fuzzy information. *J. Intell. Fuzzy Syst.* **2016**, *31*, 1967–1975. [CrossRef]
2. Zadeh, L.A. The concept of a linguistic variable and its application to approximate reasoning—I. *Inf. Sci.* **1975**, *8*, 199–249. [CrossRef]
3. Nestic, S.; Lampón, J.F.; Aleksic, A.; Cabanelas, P.; Tadic, D. Ranking manufacturing processes from the quality management perspective in the automotive industry. *Expert Syst.* **2019**, *36*, e12451. [CrossRef]
4. Dubois, D.J. *Fuzzy Sets and Systems: Theory and Applications*; Academic Press: Boston, MA, USA, 1980; Volume 144.
5. Zimmermann, H.J. Fuzzy set theory. *Wiley Interdiscip. Rev. Comput. Stat.* **2010**, *2*, 317–332. [CrossRef]
6. Paksoy, T.; Pehlivan, N.Y.; Kahraman, C. Organizational strategy development in distribution channel management using fuzzy AHP and hierarchical fuzzy TOPSIS. *Expert Syst. Appl.* **2012**, *39*, 2822–2841. [CrossRef]
7. Xu, Y.; Li, M.; Cabrerizo, F.J.; Chiclana, F.; Herrera-Viedma, E. Multiplicative Consistency Ascertaining, Inconsistency Re-pairing, and Weights Derivation of Hesitant Multiplicative Preference Relations. *IEEE Trans. Syst. Man Cybern. Syst.* **2021**, 1–16. [CrossRef]
8. Shapiro, A.F. Fuzzy logic in insurance. *Insur. Math. Econ.* **2004**, *35*, 399–424. [CrossRef]
9. Shapiro, A.F. An overview of insurance uses of fuzzy logic. In *Computational Intelligence in Economics and Finance*; Springer: Berlin/Heidelberg, Germany, 2007; pp. 25–61.
10. De Andres Sanchez, J.; Terceño Gómez, A. Applications of fuzzy regression in actuarial analysis. *J. Risk Insur.* **2003**, *70*, 665–699. [CrossRef]
11. Koissi, M.C.; Shapiro, A.F. Fuzzy Regression and the Term Structure of Interest Rates-A Least Squares Approach. *Actuar. Res. Clear. House Newsl.* **2009**. Available online: https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.98.6550&rep=rep1&type=pdf (accessed on 8 July 2022).
12. Berry-Stölzle, T.R.; Koissi, M.C.; Shapiro, A.F. Detecting fuzzy relationships in regression models: The case of insurer solvency surveillance in Germany. *Insur. Math. Econ.* **2010**, *46*, 554–567. [CrossRef]
13. Shapiro, A.F. Modeling future lifetime as a fuzzy random variable. *Insur. Math. Econ.* **2013**, *53*, 864–870. [CrossRef]
14. Abul-Haggag, O.Y.; Barakat, W. Application of fuzzy logic for risk assessment using risk matrix. *Int. J. Emerg. Technol. Adv. Eng.* **2013**, *3*, 49–54.
15. Markowski, A.S.; Mannan, M.S. Fuzzy risk matrix. *J. Hazard. Mater.* **2008**, *159*, 152–157. [CrossRef]
16. Zapa, E.; Cogollo, J. Fuzzy-BSC methodology for decision making in indemnity area of insurance companies. *IEEE Lat. Am. Trans.* **2018**, *16*, 2539–2546. [CrossRef]
17. Zhu, S.; Huang, J.; Xu, Y. A consensus model for group decision making with self-confident linguistic preference relations. *Int. J. Intell. Syst.* **2021**, *36*, 6360–6386. [CrossRef]
18. Wang, C. *A Study of Membership Functions on Mamdani-Type Fuzzy Inference System for Industrial Decision-Making*; Lehigh University: Bethlehem, PA, USA, 2015.
19. Ho, W.; Ma, X. The state-of-the-art integrations and applications of the analytic hierarchy process. *Eur. J. Oper. Res.* **2018**, *267*, 399–414. [CrossRef]
20. Jakšić, M.; Moljević, S.; Aleksić, A.; Misita, M.; Arsovski, S.; Tadić, D.; Mimović, P. Fuzzy approach in ranking of banks ac-cording to financial performances. *Math. Probl. Eng.* **2016**, *2016*, 6169586. [CrossRef]
21. Bakır, M.; Atalık, Ö. Application of fuzzy AHP and fuzzy MARCOS approach for the evaluation of e-service quality in the airline industry. *Decis. Mak. Appl. Manag. Eng.* **2021**, *4*, 127–152. [CrossRef]
22. Saaty, T.L. The modern science of multicriteria decision making and its practical applications: The AHP/ANP approach. *Oper. Res.* **2013**, *61*, 1101–1118. [CrossRef]
23. Kahraman, C. A brief literature review for fuzzy AHP. *Int. J. Anal. Hierarchy Process* **2018**, *10*. [CrossRef]
24. Chen, J.F.; Hsieh, H.N.; Do, Q. HEvaluating teaching performance based on fuzzy AHP and comprehensive evaluation approach. *Appl. Soft Comput.* **2015**, *28*, 100–108. [CrossRef]
25. Chang, D.Y. Applications of the extent analysis method on fuzzy AHP. *Eur. J. Oper. Res.* **1996**, *95*, 649–655. [CrossRef]
26. Sultana, I.; Ahmed, I.; Azeem, A. An integrated approach for multiple criteria supplier selection combining Fuzzy Delphi, Fuzzy AHP & Fuzzy TOPSIS. *J. Intell. Fuzzy Syst.* **2015**, *29*, 1273–1287.
27. Sirisawat, P.; Kiatcharoenpol, T. Fuzzy AHP-TOPSIS approaches to prioritizing solutions for reverse logistics barriers. *Comput. Ind. Eng.* **2018**, *117*, 303–318. [CrossRef]
28. Banduka, N.; Tadić, D.; Macuzic, I.; Crnjac Zizic, M. Extended process failure mode and effect analysis (PFMEA) for the automotive industry: The FSQC-PFMEA. *Adv. Prod. Eng. Manag.* **2018**, *2*, 206–215. [CrossRef]
29. Lyu, H.M.; Sun, W.J.; Shen, S.L.; Zhou, A.N. Risk assessment using a new consulting process in fuzzy AHP. *J. Constr. Eng. Manag.* **2020**, *146*, 04019112. [CrossRef]
30. Buckley, J.J. Fuzzy hierarchical analysis. *Fuzzy Sets Syst.* **1985**, *17*, 233–247. [CrossRef]
31. Pomerol, J.C.; Barba-Romero, S. *Multicriterion Decision in Management: Principles and Practice*; Springer Science & Business Media: Berlin/Heidelberg, Germany, 2000; Volume 25.
32. Calabrese, A.; Costa, R.; Levialdi, N.; Menichini, T. Integrating sustainability into strategic decision-making: A fuzzy AHP method for the selection of relevant sustainability issues. *Technol. Forecast. Soc. Change* **2019**, *139*, 155–168. [CrossRef]

33. Wang, Y.M.; Elhag, T.M. A fuzzy group decision making approach for bridge risk assessment. *Comput. Ind. Eng.* **2007**, *53*, 137–148. [CrossRef]
34. Sii, H.S.; Ruxton, T.; Wang, J. A fuzzy-logic-based approach to qualitative safety modelling for marine systems. *Reliab. Eng. Syst. Saf.* **2001**, *73*, 19–34. [CrossRef]
35. Gentile, M.R.W.M.M.; Rogers, W.J.; Mannan, M.S. Development of a fuzzy logic-based inherent safety index. *Process Saf. Environ. Prot.* **2003**, *81*, 444–456. [CrossRef]
36. Tadic, D.; Aleksic, A.; Mimovic, P.; Puskaric, H.; Misita, M. A model for evaluation of customer satisfaction with banking service quality in an uncertain environment. *Total Qual. Manag. Bus. Excell.* **2018**, *29*, 1342–1361. [CrossRef]
37. Aleksić, A.; Stefanović, M.; Tadić, D.; Arsovski, S. A fuzzy model for assessment of organization vulnerability. *Measurement* **2014**, *51*, 214–223. [CrossRef]
38. Tadic, D.; Savovic, I.; Misita, M.; Arsovski, S.; Milanovic, D.D. Development of a fuzzy logic-based inherent safety index for food industries. *Proc. Inst. Mech. Eng. Part E J. Process Mech. Eng.* **2014**, *228*, 3–13. [CrossRef]
39. Zhou, Q.; Thai, V.V. Fuzzy and grey theories in failure mode and effect analysis for tanker equipment failure prediction. *Saf. Sci.* **2016**, *83*, 74–79. [CrossRef]
40. Shariff, A.M.; Leong, C.T.; Zaini, D. Using process stream index (PSI) to assess inherent safety level during preliminary design stage. *Saf. Sci.* **2012**, *50*, 1098–1103. [CrossRef]
41. Xu, Y.; Li, M.; Cabrerizo, F.J.; Chiclana, F.; Herrera-Viedma, E. Algorithms to detect and rectify multiplicative and ordinal inconsistencies of fuzzy preference relations. *IEEE Trans. Syst. Man Cybern. Syst.* **2021**, *51*, 3498–3511. [CrossRef]