MULTI-OBJECTIVE OPTIMIZATION OF PIEZOELECTRIC SENSOR AND ACTUATOR PLACEMENT AND SIZING FOR ACTIVE VIBRATION CONTROL

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Abstract. Piezoelectric sensors and actuators have wide range of applications in suppression of vibrations and shape control. Design of thin-walled composite structures with integrated piezoelectric sensors and actuators is complex process, and it requires taking into account various design variables. The placement and sizing of the actuators and sensors are based on control effectiveness and observability, but these piezoelectric patches affect the host structure's mass and original dynamics properties and in the case of failure of active system, this change of dynamics properties becomes significant. This paper deals with optimal placement and sizing of piezoelectric sensor and actuator on composite beam for maximum active vibration control effectiveness and observability considering residual modes to limit spill-over effects with minimal change in original system dynamics. The problem is formulated using finite element method (FEM) based on third-order shear deformation theory (TSDT). Particle swarm optimization method is used to find optimal configuration. Numerical example is presented for cantilever beam.

1. Introduction

Active vibration control of thin-walled structures is an important subject of research in mechanical and aerospace engineering. Vibration control can be passive, with passive constrained layer damping and active with active damping and active constrained layer damping. In active vibration control, the use of piezoelectric sensors and actuators has significant research interests. The original approach of analysis of active vibration control with piezoelectric patches is presented in [1], where polyvinylidene fluoride (PVDF) was used for vibration control of cantilever beam.

Optimization of sizing and placement piezoelectric sensors and actuators has been shown as the most important issue in design of active structures. An exhaustive review of optimization problems until 2001 is presented in [2], and review of various optimization criteria is presented in [3]. The optimization problem can be divided in two approaches. The first approach consists of combination of optimal location and size of sensors and actuators and controller parameters. A few studies [4,5,6] propose a quadratic cost function which is used to taking into account the measurement error and control energy. The second approach deals with optimal location and size of sensors and actuators independently of controller definition. Optimization using objective function based on grammian matrix is presented in [7,8]. Ref. [9] presents an optimal placement method using H_2. In [10] modal controllability index based on singular value analysis of control vector was developed.

It can be seen that the main focus of these investigations is sizing and location of piezoelectric actuators and sensors for maximum controllability and observabillity. But, these piezoelectric patch cause modification of dynamics properties of host structures changing natural frequencies and in the case of failure of active system, this change of dynamics properties becomes significant. In [11], change of natural frequencies has been considered through objective function in multi-objective optimization.

Many times, an active structure is discretized into finite number of elements for vibration analysis and control. For practical implementation, this model needs to be truncated, where only first few modes are taken into account. But, state feedback control law can based on reduced model may excite the residual modes which results into spillover instability for even simple beam problem [12]. Ref. [13] takes into account the residuals modes in optimization problem.

This paper deals with optimal placement and sizing of collocated piezoelectric sensors and actuators on cantilever composite beam for maximum active vibration control effectiveness and observability considering residual modes to limit spillover effects with minimal change in original system dynamics. The problem is formulated using finite element method (FEM) based on third-order shear deformation theory (TSDT). Due to complexity of the problem, classical optimization methods which apply gradient-based search techniques can not be used. Because of that, stochastic optimization method is used: Particle Swarm Optimization. Maximizations of controllability and observability, as minimization of spillover effects are involved through objective functions, and changes of natural frequencies are involved through constraint functions.

2. Finite element model

A laminated composite beam with integrated piezoelectric sensors and actuators is considered (Fig. 1.).

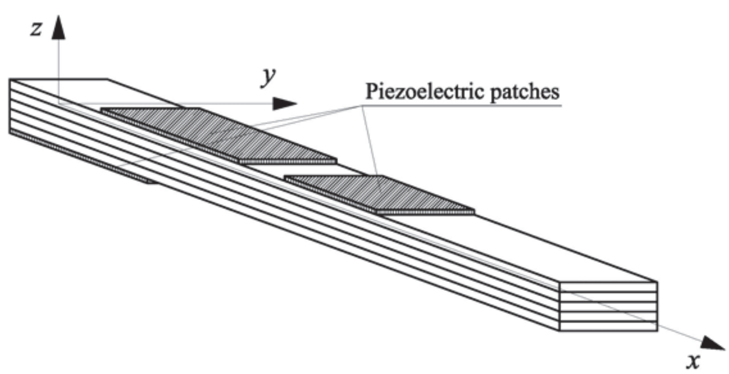

Figure 1. Laminated composite beam with piezoelectric sensors and actuators

The coordinate x is coincident with the beam axis, the $x-y$ plane coincides with mid-plane of the beam and z axis is defined as normal to the mid-plane according to the righthand rule. Both elastic and piezoelectric layers are supposed to be thin, such that a plane stress state can be assumed. The sensors and actuators are perfectly bonded on upper and lower surfaces at different locations along the length of beam. It is assumed that they span the entire width of beam. Elastic layers are obtained by setting their piezoelectric coefficients to zero. The equivalent single layer theory is used, so the same displacement field is considered for all layers of beam. The formulation results in a coupled finite element model with mechanical (displacement) and electrical (potentials of piezoelectric patches) degrees of freedom. Also, only specially orthotropic layers are considered.

2.1. Mechanical displacements and strains

The displacement field based on third-order shear deformation theory of Reddy [14] is given

$$\begin{aligned} u(x,z,t) &= u_0(x,t) + z\phi_x(x,t) - c_1 z^3\left(\phi_x + \frac{\partial w_0}{\partial x}\right) \\ w(x,z,t) &= w_0(x,t) \end{aligned} \tag{1}$$

where u and w are displacement components in the x and z directions respectively, u_0, w_0 are mid-plane ($z = 0$) displacement and ϕ_x is bending rotation of mid-plane, $c_1 = 4/(3h^2)$, where h is total thickness of the beam. The strains associated with above displacement field are

$$\begin{aligned} \varepsilon_{xx} &= \varepsilon_{xx}^0 + z\varepsilon_{xx}^1 - c_1 z^3 \varepsilon_{xx}^3 \\ \gamma_{xz} &= \gamma_{xz}^0 - c_2 z^2 \gamma_{xz}^2 \end{aligned} \tag{2}$$

where

$$\begin{aligned} &\varepsilon_{xx}^0 = \frac{\partial u_0}{\partial x} \quad \varepsilon_{xx}^1 = \frac{\partial \phi_x}{\partial x} \quad \varepsilon_{xx}^3 = \frac{\partial \phi_x}{\partial x} + \frac{\partial^2 w_0}{\partial x^2} \\ &\gamma_{xz}^0 = \gamma_{xz}^2 = \phi_x + \frac{\partial w_0}{\partial x} \end{aligned} \tag{3}$$

and

$$c_2 = \frac{4}{h^2}. \tag{4}$$

2.2. Piezoelectric constitutive equations

Specific electric enthalpy density of a piezoelectric layer is given in [15]

$$h=\frac{1}{2}\{\varepsilon\}^T[Q]\{\varepsilon\}-\{\varepsilon\}^T[e]\{E\}-\frac{1}{2}\{E\}^T[k]\{E\}, \tag{5}$$

where $[Q]$ is the elastic stiffness matrix, $\{\varepsilon\}$ is the strain vector, $[e]$ is the piezoelectric constant matrix, $\{E\}$ is the electric field vector and $\{k\}$ is the permittivity matrix. The constitutive equations for each piezoelectric layer can be obtained

$$\begin{aligned}\{\sigma\}&=\frac{\partial h}{\partial\{\varepsilon\}}=[Q]\{\varepsilon\}-[e]^T\{E\}\\ \{D\}&=-\frac{\partial h}{\partial\{E\}}=[e]\{\varepsilon\}+[k]\{E\}\end{aligned} \tag{6}$$

where $\{\sigma\}$ is the stress vector and $\{D\}$ is the electric displacement vector. For a one-dimensional composite beam, the width (y-direction) is free of stresses. Therefore $\sigma_y=\sigma_{yz}=\sigma_{xy}=0$ while $\varepsilon_y\neq\gamma_{yz}\neq\gamma_{xy}\neq 0$. Using the above constraints, assuming that $E_1=E_2=0$ and taking into account orthotropic lamina, equation (6) can be written in following form

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{xz}\\ D_3\end{Bmatrix}^{(k)}=\begin{bmatrix}\overline{Q}_{11} & 0 & -\overline{e}_{31}\\ 0 & \overline{Q}_{55} & 0\\ -\overline{e}_{31} & 0 & \overline{k}_{33}\end{bmatrix}^{(k)}\begin{Bmatrix}\varepsilon_{xx}\\ \gamma_{xz}\\ E_3\end{Bmatrix}^{(k)} \tag{7}$$

where superscript k denotes number of layer, and material constants are expressed in principal directions. Relations between material constants expressed in principal directions and material directions for orthotropic lamina are

$$\begin{aligned}\overline{Q}_{11}&=Q_{11}\cos^4\Theta+Q_{22}\sin^4\Theta\\ \overline{Q}_{55}&=Q_{55}\cos^2\Theta+Q_{44}\sin^2\Theta\\ \overline{e}_{31}&=e_{31}\cos^2\Theta+e_{32}\sin^2\Theta\\ \overline{k}_{33}&=k_{33},\end{aligned} \tag{8}$$

where Θ presents angle between material directions of layer and principal direction of lamina.

2.3. Variational formulation

The governing equations of motions are derived using Hamilton's principle

$$\int_{t_1}^{t_2}(\delta T-\delta H+\delta W)dt=0, \tag{9}$$

where t_1 and t_2 are two arbitrary time instants, δT is variation of kinetic energy, δH is variation of electric enthalpy and δW is variation of work done by external forces. According to the equation (5), variation of electric enthalpy can be written in following form

$$\delta H = \int_V \delta h dV = \delta H_m - \delta H_{me} - \delta H_{em} - \delta H_e, \tag{10}$$

where

$$\begin{aligned} \delta H_m &= \int_V \delta\{\varepsilon\}^T [\overline{Q}]\{\varepsilon\} dV \\ \delta H_{me} &= \int_V \delta\{\varepsilon\}^T [\overline{e}]^T \{E\} dV \\ \delta H_{em} &= \int_V \delta\{E\}^T [\overline{e}]\{\varepsilon\} dV \\ \delta H_e &= \int_V \delta\{E\}^T [\overline{k}]\{E\} dV. \end{aligned} \tag{11}$$

The variation of kinetic energy, written in terms of mechanical displacements, is

$$\delta T = \int_V \rho \delta\{\dot{u}\}^T \{\dot{u}\} dV, \tag{12}$$

where $\{\dot{u}\}$ is velocity vector, and ρ is density of layer.

2.4. Finite element discretization and coupled equations of motions

After finite element discretization [16,17,18], assembling obtained equations into Hamilton's principle (11), it can be obtain global equations of motions written in terms of the nodal mechanical and electrical degrees of freedom

$$\begin{aligned} &[M]\{\ddot{u}\} + [K_m]\{u\} + [K_{me}]_A\{\phi\}_A + [K_{me}]_S\{\phi\}_S = \{F_m\} \\ &[K_{me}]_A^T\{u\} - [K_e]_A\{\phi\}_A = [K_e]_A\{\phi\}_{AA} \\ &[K_{me}]_S^T\{u\} - [K_e]_S\{\phi\}_S = 0, \end{aligned} \tag{13}$$

where $[M]$ presents mass matrix, $[K_m]$ presents elastic stiffness matrix, $[K_{me}]_A$ and $[K_{me}]_S$ are piezoelectric stiffness matrices of actuator and sensor, respectively, $[K_e]_A$ and $[K_e]_S$ are dielectric stiffness matrices of actuator and sensor, respectively, $\{u\}$ presents vector of mechanical nodal variables, $\{\phi\}_A$ and $\{\phi\}_S$ are electric potentials on actuators and sensors, respectively and $\{\phi\}_{AA}$ is vector of external applied electric potentials on actuators. Replacing electric potentials of actuators and sensors expressed form last two equation of (13) in first one, it can be obtained following equation of motion

$$[M]\{\ddot{u}\} + [K^*]\{u\} = \{F_m\} + [K_{me}]_A\{\phi\}_{AA}, \tag{14}$$

where

$$\left[K^*\right]=\left[K_m\right]+\left[K_{me}\right]_A\left[K_e\right]_A^{-1}\left[K_{me}\right]_A^T+\left[K_{me}\right]_S\left[K_e\right]_S^{-1}\left[K_{me}\right]_S^T . \quad (15)$$

Equations of motion in modal coordinates. The equation (16) can be converted to modal-space as

$$\{\ddot{\eta}\}+\left[\omega^2\right]\{\eta\}=\left[\Psi\right]^T\{F_m\}+\left[\Psi\right]^T\left[K_{me}\right]_A\{\phi\}_{AA}, \quad (16)$$

where $\left[\Psi\right]$ presents modal matrix which has been normalized with respect to mass, $\{\eta\}$ vector of modal coordinates and $\left[\omega^2\right]$ diagonal matrix of squares natural frequencies obtained as following way

$$\left[\omega^2\right]=\left[\Psi\right]^T\left[K^*\right]\left[\Psi\right]. \quad (17)$$

State-space representation. Lower ordered modes are the most easily excitable because they have lower energy associated. Due to that, controller has been designed for active control only first few modes. Thus, equation (16), expressed in truncated modal-space, becomes

$$\{\ddot{\eta}\}+\left[\omega^2\right]_{Tr}\{\eta\}=\left[\Psi\right]_{Tr}^T\{F_m\}+\left[\Psi\right]_{Tr}^T\left[K_{me}\right]_A\{\phi\}_{AA}, \quad (18)$$

where matrix $\left[\omega^2\right]_{Tr}$ is consisted of first few controllable eigen-modes. For residual modes, equation (17) becomes

$$\{\ddot{\eta}\}+\left[\omega^2\right]_{\mathrm{Rez}}\{\eta\}=\left[\Psi\right]_{\mathrm{Rez}}^T\{F_m\}+\left[\Psi\right]_{\mathrm{Rez}}^T\left[K_{me}\right]_A\{\phi\}_{AA} . \quad (19)$$

Equations (18) and (19) can be presented in state-space form as

$$\{\dot{X}\}=\left[A\right]\{X\}+\left[B\right]\{\phi\}_{AA}+\{d\}, \quad (20)$$

where

$$\{X\}=\begin{Bmatrix}\{\eta\}\\ \{\dot{\eta}\}\end{Bmatrix} \quad (21)$$

present state vector,

$$\left[A\right]=\begin{bmatrix}[0] & [0] & [I] & [0]\\ [0] & [0] & [0] & [I]\\ -\left[\omega^2\right]_{Tr} & [0] & [0] & [0]\\ [0] & -\left[\omega^2\right]_{\mathrm{Rez}} & [0] & [0]\end{bmatrix} \quad (22)$$

present state matrix,

$$[B] = \begin{bmatrix} [0] \\ [0] \\ [B]_{tr} \\ [B]_{\mathrm{Re}z} \end{bmatrix} = \begin{bmatrix} [0] \\ [0] \\ [\psi]_{Tr}^{T}[K_{me}]_{A} \\ [\psi]_{\mathrm{Re}z}^{T}[K_{me}]_{A} \end{bmatrix} \quad (23)$$

present control matrix, and

$$\{d\} = \begin{Bmatrix} \{0\} \\ \{0\} \\ [\psi]_{Tr}^{T}\{F_{m}\} \\ [\psi]_{\mathrm{Re}z}^{T}\{F_{m}\} \end{Bmatrix} \quad (24)$$

is disturbance vector. From third equation in (13) it can be expressed sensor output in state-space form

$$\{Y\} = [C]\{X\}, \quad (25)$$

where $[C]$ presents output matrix given as

$$[C] = \left[[C]_{Tr} \quad [C]_{\mathrm{Re}z} \quad [0] \quad [0] \right] = \left[[K_{me}]_{S}^{T}[\Psi]_{Tr} \quad [K_{me}]_{S}^{T}[\Psi]_{\mathrm{Re}z} \quad [0] \quad [0] \right]. \quad (26)$$

3. Optimization criteria for piezoelectric actuators and sensors sizing and location

In [10], a controllability index for actuator is proposed, which is obtained by maximizing global control force. The modal control force applied to the system can be written as

$$\{f_{C}\} = [B]\{\phi\}_{AA}. \quad (27)$$

It follows from (27) that

$$\{f_{C}\}^{T}\{f_{C}\} = \{\phi\}_{AA}^{T}[B]^{T}[B]\{\phi\}_{AA}. \quad (28)$$

Using singular value analysis, $[B]$ can be written as $[B] = [M][S][N]^{T}$, where $[M]^{T}[M] = [I]$, $[N]^{T}[N] = [I]$ and

$$[S] = \begin{bmatrix} \sigma_{1} & 0 & \cdots & 0 \\ 0 & \sigma_{2} & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{Na} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \quad (29)$$

where N_{a} presents number of actuators. Equation (29) can be written as

$$\{f_C\}^T\{f_C\} = \{\phi\}_{AA}^T[N][S]^T[S][N]^T\{\phi\}_{AA}, \quad (30)$$

or

$$\|f_C\|^2 = \|\{\phi\}_{AA}\|^2\|S\|^2. \quad (31)$$

Thus, maximizing this norm independently of the applied voltage $\{\phi\}_{AA}$ induces maximizing $\|S\|^2$. The magnitude of σ_i is a function of location and size of piezoelectric actuator. In [10] is proposed controllability index which is defined by

$$\Omega_C = \prod_{i=1}^{Na}\sigma_i. \quad (32)$$

The higher controllability index indices the smaller electrical potential will be required for control. The observabillity index can be obtained on similar way

$$\Omega_O = \prod_{i=1}^{Ns}\sigma_i, \quad (33)$$

Controllability index can be obtained in more suitable form for models based on finite element, where, instead of maximizing the norm $\|S\|^2$, applied force for each mode have been maximized independently of $\{\phi\}_{AA}$. According to the (23), controllability index can be written on following way

$$\bar{\sigma}_{Ci}^2 = (B_i)_{Tr}(B_i)_{Tr}^T \quad (34)$$

for controlled mode, and

$$\left(\bar{\sigma}_{Ci}^{\mathrm{Re}z}\right)^2 = (B_i)_{\mathrm{Re}z}(B_i)_{\mathrm{Re}z}^T \quad (35)$$

for residual mode. $(B_i)_{Tr}$ and $(B_i)_{\mathrm{Re}z}$ present i-th row of matrices $[B]_{Tr}$ and $[B]_{\mathrm{Re}z}$, respectively. Simillary, observabillity index is

$$\bar{\sigma}_{Oi}^2 = \{C_i\}_{Tr}^T\{C_i\}_{Tr} \quad (36)$$

for observable mode, and

$$\left(\bar{\sigma}_{Oi}^{\mathrm{Re}z}\right)^2 = \{C_i\}_{\mathrm{Re}z}^T\{C_i\}_{\mathrm{Re}z} \quad (37)$$

where $\{C_i\}_{Tr}$ and $\{C_i\}_{\mathrm{Re}z}$ present i-th column of matrices $[C]_{Tr}$ and $[C]_{\mathrm{Re}z}$, respectively.

4. Optimization implementation using particle swarm optimization

The particle swarm optimization (PSO) has been inspired by the social behavior of animals such as fish schooling, insect swarming and birds flocking. It was first introduced in [19]. The system is initialized with a population of random solutions (called particle position in

PSO). Every particle is affected by three factors: its own velocity, the best position it has achieved (best local position) which is determined by the highest value of the objective function encountered by this particle in all previous iteration and overall best position achieved by all particles (best global position), which is determined by the highest value of the objective function encountered in all the previous iteration. A particle changes its velocity (v) and position (p) on following way

$$v_{id}^{k+1} = wv_{id}^{k} + c_1 r_1 \left(plbest_{id} - p_{id}^{k}\right) + c_2 r_2 \left(pgbest_d - p_{id}^{k}\right), \tag{38}$$

$$p_{id}^{k+1} = p_{id}^{k} + v_{id}^{k+1}, \quad i = 1,..,n \quad d = 1,...,m \tag{39}$$

where W is inertia weight, c_1 is cognition factor, c_2 is social learning factor, r_1 and r_2 are random numbers between 0 and 1, the superscript k denotes iterative generation, n is population size, m is particle's dimension, $plbest_{id}$ and $pgbest_d$ are best local and global position. Cognition factor and social learning factor are usually set as $c_1 = c_2 = 1.5$. Upper and lower limits of inertia weight for structural design are given in [20].

In this work, beam is discretized in finite elements. Each piezoelectric patch is determined with two coordinates: left node, which defines position and number of elements covered by this patch which defines size of piezoelectric patch (Fig. 2), so, dimension of particle is twice then number of piezoelectric patches.

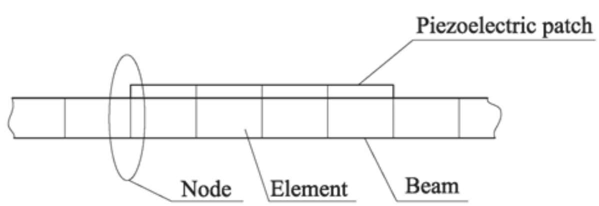

Figure 2.

It is obvious that coordinates of particle and corresponding velocity are integer number. Because of that, discrete method must be used. According to the [21], velocity is updated by following equation on every iteration

$$v_{id}^{k+1} = \mathrm{int}\left(wv_{id}^{k} + c_1 r_1 \left(plbest_{id} - p_{id}^{k}\right) + c_2 r_2 \left(pgbest_d - p_{id}^{k}\right)\right), \tag{40}$$

in which $\mathrm{int}(f)$ is getting integer part of f.

4.1. Objective functions

As mentioned earlier, the piezoelectric patches sizing and location should be such that those should give good controllability and observability with minimization of spillover effects. According to the (34-37), objective functions can be written as

$$\text{maximize} \quad f_i = \frac{\overline{\sigma}_{Ci}^2}{\overline{\sigma}_{Ci\max}^2} \cdot 100 \quad i = 1,.., N_{TrA}\,, \tag{41}$$

$$\text{minimize} \quad f_i = \frac{\left(\overline{\sigma}_{Ci}^{\mathrm{Re}z}\right)^2}{\left(\overline{\sigma}_{Ci\max}^{\mathrm{Re}z}\right)^2} \cdot 100 \quad i = 1,.., N_{\mathrm{Re}zA}\,, \tag{42}$$

$$\text{maximize} \quad f_i = \frac{\overline{\sigma}_{Oi}^2}{\overline{\sigma}_{Oi\max}^2} \cdot 100 \quad i = 1,.., N_{TrS}\,, \tag{43}$$

$$\text{minimize} \quad f_i = \frac{\left(\overline{\sigma}_{Oi}^{\mathrm{Re}z}\right)^2}{\left(\overline{\sigma}_{Oi\max}^{\mathrm{Re}z}\right)^2} \cdot 100 \quad i = 1,.., N_{\mathrm{Re}zS}\,, \tag{44}$$

where $\overline{\sigma}_{Ci\max}$, $\overline{\sigma}_{Ci\max}^{\mathrm{Re}z}$, denote maximum controllability index of controlled modes and residual modes and $\overline{\sigma}_{Oi\max}$, $\overline{\sigma}_{Oi\max}^{\mathrm{Re}z}$ denote maximum observability index of observed modes and residual modes. These functions are called degree of controllability and observability. It can be seen that there are $N_{TrA} + N_{\mathrm{Re}zA} + N_{TrS} + N_{\mathrm{Re}zS}$ objective functions. In this work, only collocated sensors and actuators will be considered. Actuator and its corresponding sensor have the equal length and they are set symmetrically: sensor at the bottom surface, and actuator on the top surface of the beam. In [22] states that it is not necessary to search for maximum controllability and observability index separately. The same result can be obtained by maximizing only one index. According to that, only equations (41) and (42) will be considered. $N_{TrA} + N_{\mathrm{Re}zA}$ objective functions (41) and (42) can be transformed in two objective functions using weighted aggregation approaches

$$\text{maximize} \quad f_{Tr} = \sum_{i=1}^{N_{TrA}} w_i^{Tr} \frac{\overline{\sigma}_{Ci}^2}{\overline{\sigma}_{Ci\max}^2} \cdot 100 \tag{45}$$

and

$$\text{minimize} \quad f_{\mathrm{Re}z} = \sum_{j=1}^{N_{\mathrm{Re}zA}} w_j^{\mathrm{Re}z} \frac{\overline{\sigma}_{Cj}^2}{\overline{\sigma}_{Cj\max}^2} \cdot 100\,, \tag{46}$$

where w_i^{Tr} and $w_j^{\mathrm{Re}z}$ are the weighting coefficients which express the relative importance of the objective function. Two obtained objective functions can be transformed in one objective function in following way

$$\textit{maximize} \quad f = f_{Tr} - \gamma\, f_{\mathrm{Re}z} \tag{47}$$

where γ denotes weighting constant which expresses relative importance of minimization controllability of residual modes related to maximization of controllability of controlled modes. So, multi-objective optimization problem which consist of $N_{TrA} + N_{\mathrm{Re}zA} + N_{TrS} + N_{\mathrm{Re}zS}$ objective functions are transformed in single-objective problem.

4.2. Constraints

The first type of constraint used in optimization problem in this work is related to the geometry of the piezoelectric patches. These geometric constraints are

1. no one of the coordinate of the particle must not be nonpositive number
2. minimum distance between two patches is 1
3. last piezoelectric patch must not to be outside of beam

The problem of violation of geometric constraint is solved on following way:

1. generate given number of particles in feasible space which every of them is placed between the particle in previous iteration, which is placed in feasible space, and obtained particle in current iteration, which is placed in nonfeasible space.
2. calculate Euclidian distances between generated particles and particle in current iteration
3. new particle is one which distance is minimum
4. modify velocity on following way

$$v_{id}^{k+1} = p_{id}^{k+1*} - p_{id}^{k}, \tag{48}$$

where p_{id}^{k+1*} is particle obtained in 3.

Second type of constraints are related to change of natural frequencies. It can be written on following

$$\left|\omega_i^{New} - \omega_i^{Old}\right| / \omega_i^{Old} < \varepsilon_i \quad i = 1,.., N_{\mathrm{mod}\, s}, \tag{49}$$

where ω_i^{New} denotes natural frequency of i-th mode of the beam with piezoelectric patches, ω_i^{Old} denotes natural frequency of i-th mode of the beam without piezoelectric patches and i-th ε_i is tolerance. These constraints will be treated with penalty function technique, so objective function to be maximized is modified to

$$maximize \quad f' = f - k \cdot |f| \cdot g_j / g \tag{50}$$

where

$$g_j = \sum_{i=1}^{N_{\mathrm{mod}\, s}} \left[\left|\omega_{ij}^{New} - \omega_{ij}^{Old}\right| / \omega_{ij}^{Old} - \varepsilon_{ij}\right], \tag{51}$$

presents constraint violation of j-th particle which left feasible design space, g is average of all g_j in current iteration and k presents number of current iteration.

5. Numerical example

In the example, cantilever beam is considered. The length of beam is $L = 0.5m$, and its width is $b = 0.025m$. Laminated layers are made of Graphite-Epoxy. Their thicknesses is given as $h = [0.0005 \quad 0.001 \quad 0.002]$, and its orientations are $\Theta = [90 \quad 0 \quad 90]$. Piezoelectric sensors and actuators are made of PZT. Their thickness is $0.35mm$. Material properties of Graphite-Epoxy and PZT are given in Table 1.

Material properties	Graphite-Epoxy	PZT
$E_1(GPa)$	174	63
$E_2(GPa)$	10.3	63
$G_{13}(GPa)$	7.17	24.6
$G_{23}(GPa)$	6.21	24.6
ν_{12}	0.25	0.28
$\rho(kg/m^3)$	1389.23	7600
$e_{31}(C/m^2)$	/	10.62
$k_{33}(F/m)$	/	0.1555

Table 1. Material properties of Graphite-Epoxy and PZT.

In the example are involved three sensors and actuators. The objective is good controllability the 4 first modes ($N_{Tr} = 4$) and less controllability the of the 5th to 8th modes ($N_{Rez} = 4$). The constraint is that changes of the four first modes are less then 10%. The beam is divided into 50 finite elements. Cognition factor and social learning factor are set as $c_1 = c_2 = 1.5$, and inertia weight are linearly varied form 1 to 0.5, number of population is 20 particles, and number of iteration is 50. Weighting coefficients are given as follows: $w^{Tr} = (0.25 \quad 0.25 \quad 0.25 \quad 0.25)$ and $w^{Rez} = (0.25 \quad 0.25 \quad 0.25 \quad 0.25)$.

5.1. First case - $\gamma = 0$ (residual modes are neglected)

The optimal solution of this case is given as $pgbest = (6 \quad 14 \quad 24 \quad 16 \quad 48 \quad 3)$. Model of the beam with piezoelectric patches which corresponds to the optimal solution is presented at Fig. 3. Fig. 4 presents convergence of objective functions. Changes of natural frequencies and degree of controllability of each controlled and residual mode are presented in Table 2 and Table 3.

Mode	$\omega^{Old}(rad/s)$	$\omega^{New}(rad/s)$	NF change(%)	Deg. of Cont. (%)
1	52.36	55.23	5.48	13.81
2	328.18	348.56	6.09	20.64
3	919.11	1008.28	9.7	29.28
4	1801.67	1978.09	9.79	33.38

Table 2.

Res. mode	$\omega^{Old}(rad/s)$	$\omega^{New}(rad/s)$	NF Change(%)	Deg. of Cont. (%)
1	2979.53	3367.66	13.03	36.07
2	4453.2	5303	19.08	43.31
3	6223.58	7216.81	15.96	34.61
4	8291.74	9611	15.91	36.71

Table 3.

Figure 3. Beam with piezoelectric sensors and actuators which corresponds to the first case

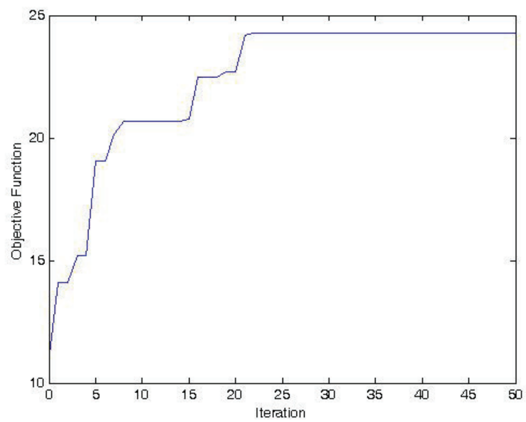

Figure 4. Convergence of objective function

5.2. Second case - $\gamma = 1$

The optimal solution of this case is given as $pgbest = (2 \quad 4 \quad 35 \quad 1 \quad 40 \quad 2)$. Beam with piezoelectric patches which corresponds to the optimal solution is presented at Fig. 5. Fig. 6 presents convergence of objective functions. Changes of natural frequencies and degree of controllability of each controlled and residual mode are presented in Table 4 and Table 5.

Mode	$\omega^{Old}(rad/s)$	$\omega^{New}(rad/s)$	NF change(%)	Deg. of Cont. (%)
1	52.36	55.46	5.92	4.13
2	328.18	360.81	9.94	4.44
3	919.11	988.97	7.49	4.24
4	1801.67	1873.68	4	3.08

Table 4.

Res. mode	$\omega^{Old}(rad/s)$	$\omega^{New}(rad/s)$	NF Change(%)	Deg. of Cont. (%)
1	2979.53	3004	0.79	1.73
2	4453.2	4403.64	1.11	1
3	6223.58	6138.47	1.37	0.92
4	8291.74	8247.69	0.54	1.98

Table 5.

Figure 5. Beam with piezoelectric sensors and actuators which corresponds to the second case

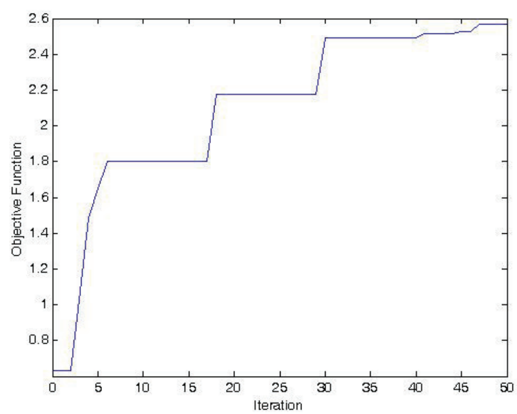

Figure 6. Convergence of objective function

6. Conclusion

In this paper, the problem of sensors and actuators sizing and placement on laminated beam with minimum changes of original dynamics properties is considered. Multi-objective problem is transformed into single-objective using weighted aggregation approach. Optimization problem is solved using Particle swarm optimization which is shown up as suitable algorithm for this case of optimization. Several applications presented in the case of cantilever beam using three collocated sensors and actuators. In the first case, residual modes are neglected. The degree of controllability of residual modes is higher compared to the controlled modes. Minimizing of degrees of controllability of residual modes affects on degrees of controllability of controlled modes, causing their reduction, which can be seen for second and third case. One of solution for avoiding this problem is placing large number of smaller sensors and actuators [8].

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