Free Vibrations of the Planar Gantry-like Structures

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This paper deals with eigenfrequencies of distributed-parameter system within the form of gantry-like structure with cantilever part. The individual members of the structure-framework are assumed to be governed by the transverse vibration theory of Euler-Bernoulli beam. It is obtained postulation of frequency equation while solutions are obtained numerically with in-house software, for several cases of structures. Also, it is done finite element postulation of the gantry-like structure as discrete-parameter system for analyzing the free undamped vibrations. Thus, it stands for two folded presentation. It is done verification of postulated algorithms.

Keywords: Gantry crane, Free vibration, Modal analysis

1. INTRODUCTION

Structural dynamics is always needed when complete behaviour of structure has to be analyzed. It is very important for design of bridges, buildings and high-performance cranes. Nowadays, we have a permanent tendency towards constant improvement of performances of machines and systems in general, including their increase in size [1]. However, the mass and the stiffness of the structure are not always in suitable proportion which requires good understanding of structural dynamic characteristics.

The orientation here is towards gantry-like structures as at gantry cranes. Especially, this group of cranes are important for container terminals because of importance of container transportation in world economy.

Figure 1: Rail mounted gantry crane at container terminal

From the main producers of the container RMG cranes (Konecranes, Liebherr, Kuenz) one can found the current level of main performances, Table 1.

Table 1: RMG container crane performances

Span	22...70 m
Cantilever	up to 21 m
Height	up to 28 m
Capacity	up to 50 t

The first step in structural dynamic is always modal analysis. Approximate expressions for fundamental symmetric and antisymmetric frequencies of symmetric portal frame can be obtained buy the Reyleigh method [2], useful for simplifying the vibration formulation of beams. Laura, Filipich [3] dealt with the determination of the fundamental frequency in the case of antisymmetric modes of a frame elastically restrained against translation and rotation, carrying concentrated masses. Blevins [4] presented formulas for determination of fundamental frequencies for symmetric portal frame, for first symmetric and first antisymmetric mode, according to frequency equation presented with trigonometric-hyperbolic functions. Furthermore, frequencies of non-regular frames were investigated by Bolotin, Kiselev [5,6], with slope-deflection method. But, even that process of gaining frequency equation was defined, finding solutions were difficult because of its transcendental nature involving trigonometric and hyperbolic functions. Nowadays, state-of-the-art computer routines enable solution of frequency equation of in-plane vibrations of structural system of portal crane i.e. non-regular frame. Such routine is given here symbolically with software Mathematica, Wolfram. Also, nowadays, modal analysis with commercial FEM software are widely used for determination of frequencies of various structures [7]. The most common structural dynamics problems include vibration excitation, blast and shock, wind and earthquake loads. In vibration excitation analysis one is primarily concerned with avoiding resonance, usually at a few frequencies. In this case, modal analysis is generally used to calculate a small number of eigenfrequencies, e.g. vibrations of machine parts and machine foundations.

This paper deals with analysis of in-plane vibrations of the structural system of gantry-like structures with cantilever part, which is improvement of the model given in [8]. First, it is obtained frequency equation for distributed-parameter system of the structure and then the eigenfrequencies and mode shapes with finite element model in basic form. Solutions are verified against each other.

2. MATHEMATICAL MODEL

The main structural parts of gantry-like structure are main girder, pier leg and shear leg. The main girder has span part of length L and cantilever part with length Lp. The different heights for legs, H,h, are used solely for generalization of the framework, despite the fact that in

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almost all the cases legs of the gantry cranes are mounted on the same level. The geometric set of this postulation is given on Figure 2a.

Presented model is the planar framework which assumes that main structural parts are beams having uniform properties along their lengths. For other types of structures it can be applied with proper idealization of elements. The individual members of the frame, Figure 2, are assumed to be governed by the transverse vibration theory of an Euler-Bernoulli beam. Neglecting the axial and shear deformation and rotatory inertia effects can be done because of known structural behaviour of gantry cranes. Individual elements are made of same material (steel).

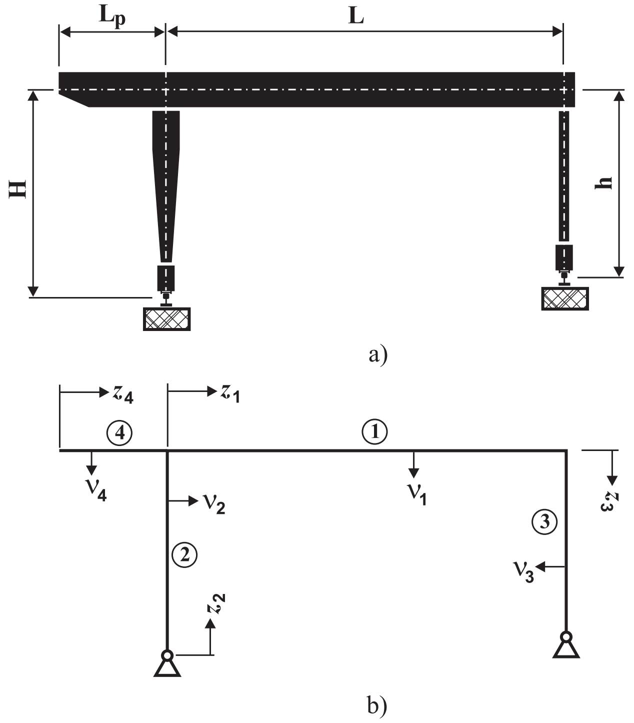


Figure 2: Mathematical model of gantry-like structure as distributed-parameter system

The partial differential equation for free transverse undamped vibrations of each element has the following form

$$\frac{\partial^2 v_i}{\partial t^2}+\frac{EI_i}{\rho A_i}\frac{\partial^4 v_i}{\partial z_i^4}=0 \quad (i=1,2,3,4) \qquad (1)$$

with postulation as

$$v_1=v_1(z_1,t)=Z_1(z_1)\cdot T(t) \quad 0\le z_1\le L \qquad (2)$$

$$v_2=v_2(z_2,t)=Z_2(z_2)\cdot T(t) \quad 0\le z_2\le H \qquad (3)$$

$$v_3=v_3(z_3,t)=Z_3(z_3)\cdot T(t) \quad 0\le z_3\le h \qquad (4)$$

$$v_4=v_4(z_4,t)=Z_4(z_4)\cdot T(t) \quad 0\le z_4\le L_p \qquad (5)$$

where A_i represent section area, I_i is moment of inertia, ρ is density and E is Young's modulus.

The mode shapes are presented throughout Krylov functions

$$Z_i(z_i)=G_i\mathbf{S}(k_iz_i)+B_i\mathbf{T}(k_iz_i)+C_i\mathbf{U}(k_iz_i)+D_i\mathbf{V}(k_iz_i)$$
$$(i=1,2,3,4) \qquad (6)$$

The time function is presented as

$$T(t)=X\cos(\omega t)+Y\sin(\omega t) \qquad (7)$$

where circular frequency is

$$\omega=k_i^2\sqrt{\frac{EI_i}{\rho A_i}} \quad (i=1,2,3,4) \qquad (8)$$

One can formulate the boundary conditions for the model under study as following.

The pinned joint at element 2 gives

$$Z_2(0)=0 \qquad (9a)$$

$$-EI_2Z_2''(0)=0 \qquad (9b)$$

The pinned joint at element 3 gives

$$Z_3(h)=0 \qquad (9c)$$

$$-EI_3Z_3''(h)=0 \qquad (9d)$$

Free end of element 4 gives

$$-EI_4Z_4''(0)=0 \qquad (9e)$$

$$-EI_4Z_4'''(0)=0 \qquad (9f)$$

The joints of elements 1 and 3, along with joints of element 1, 2 and 4 give following

$$Z_1(L)=0 \qquad (9g)$$

$$Z_1'(L)=Z_3'(0) \qquad (9h)$$

$$-EI_1Z_1''(L)=-EI_3Z_3''(0) \qquad (9j)$$

$$Z_1(0)=0 \qquad (9k)$$

$$Z_4(L_p)=0 \qquad (9l)$$

$$Z_2'(H)=Z_1'(0) \qquad (9m)$$

$$Z_4'(L_p)=Z_1'(0) \qquad (9n)$$

$$-EI_1Z_1''(0)=-EI_2Z_2''(H)-EI_4Z_4''(L_p) \qquad (9o)$$

$$Z_2(H)=-Z_3(0) \qquad (9p)$$

Finally, most important condition is equilibrium of shear forces at the top of the legs with inertial force developed in the main girders elements by sideway motion which provides the condition

$$-(\rho A_1L+\rho A_4L_p)\ddot{v}_3(0,t)=EI_2v_2'''(H,t)+EI_3v_3'''(0,t) \qquad (9r)$$

which becomes

$$(\rho A_1L+\rho A_4L_p)Z_3(0)\omega^2=EI_2Z_2'''(H)+EI_3Z_3'''(0) \qquad (10)$$

Afterwards, one may obtain a set of 16 homogenous system of equations with unknown guantities, G_i, B_i, C_i, D_i (i=1,2,3,4) and non-trivial solution of determinant of coefficients must be equal to zero.

From (8) one may express frequency parameters of every element with

$$k_i=k_1\sqrt[4]{\frac{A_iI_1}{A_1I_i}} \qquad (11)$$

and set up the postulation of frequency equation which is very complex because of combination of special functions and numerous parameters such as A_1, A_2, I_1, L, H ....

The form as

$$f(k_1)=0 \qquad (12)$$

don't allows analytical solution but only numerical one. Here, it is solved with state-of-the-art software Mathematica. Upon the finding the solutions for k_1 one may calculate the frequencies with (8) and

$$f=\frac{\omega}{2\pi} \qquad (13)$$

2.1. Numerical results

It is determined first 3 frequencies for the adopted cases of gantry-like structures. Main geometric parameters are set with L and H, while it is assumed that L_p=0,25 L and that H=h. Due to the fact that design of main girder is first step in design of a crane, the static characteristics of main girder are used as start point. They are determined with design recommendations for flexible and rigid case. The cantilever part is the same as span part of main girder.

The variation of parameters are given with following expressions

$$\alpha = \frac{I_1}{I_2}, \beta = \frac{I_1}{I_3}, \gamma = \frac{A_1}{A_2}, \delta = \frac{A_1}{A_3} \qquad (14)$$

The results are given in Table 2.

Table 2. First 3 frequencies of the adopted gantry-like structures

L = 30 m **H = 18 m**				I_1= 0.024 m^4 A_1= 0.07 m^2			I_1= 0.05 m^4 A_1= 0.09 m^2		
α	γ	β	δ	**f₁** [Hz]	**f₂** [Hz]	**f₃** [Hz]	**f₁** [Hz]	**f₂** [Hz]	**f₃** [Hz]
1	1	1	1	1.57	7.58	15.69	2.00	9.65	19.97
1	1	10	2	1.12	6.49	10.34	1.43	8.27	13.16
10	2	10	2	0.69	5.25	10.11	0.88	6.68	12.87
10	2	50	4	0.54	5.09	6.64	0.68	6.48	8.45

L = 40 m **H = 20 m**				I_1= 0.05 m^4 A_1= 0.09 m^2			I_1= 0.13 m^4 A_1= 0.11 m^2		
α	γ	β	δ	**f_1** [Hz]	**f_2** [Hz]	**f_3** [Hz]	**f_1**[Hz]	**f_2** [Hz]	**f_3** [Hz]
1	1	1	1	1.45	5.76	12.57	2.12	8.41	18.34
1	1	10	2	1.03	4.92	10.26	1.50	7.18	14.96
10	2	10	2	0.65	3.90	9.67	0.95	5.70	14.10
10	2	50	4	0.51	3.77	6.70	0.74	5.51	9.81

L = 45 m **H = 18 m**				I_1= 0.07 m^4 A_1= 0.1 m^2			I_1= 0.18 m^4 A_1= 0.14 m^2		
α	γ	β	δ	**f_1** [Hz]	**f_2** [Hz]	**f_3** [Hz]	**f_1** [Hz]	**f_2** [Hz]	**f_3**[Hz]
1	1	1	1	1.74	5.40	12.08	2.36	7.34	16.38
1	1	10	2	1.23	4.61	11.52	1.67	6.25	15.67
10	2	10	2	0.81	3.58	9.86	1.13	4.88	13.66
10	2	50	4	0.63	3.44	9.11	0.85	4.66	12.34

3. FINITE ELEMENT MODEL

The previous chapter offers free vibrations of the gantry-like structure as distributed-parameter system. Even that results are the closest to the *exact* solutions, one may respect the fact that discrete-parameter system analysis is widely accepted approach for this kind of problems [9]. It is especially needed for structures with complex form which can't be easily simplified to SDOF system or beam models. The gantry-like structures as in this paper are the end form for consideration with distributed-parameter system. Any additional structural parts would demand discrete-parameter analysis.

Here, the discrete model of the gantry-like structure is shown at Figure 3, with same geometric set as previous.

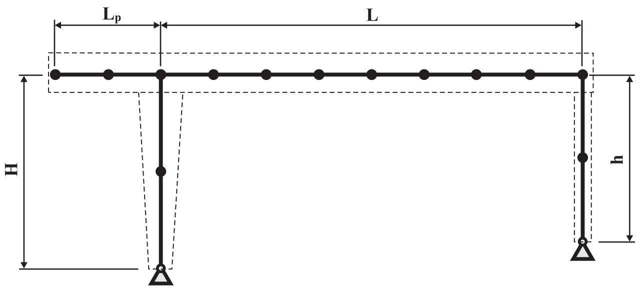


Figure 3: Discrete-parameter system of gantry-like structure

The main girder is divided into the 10 elements and legs into 2 elements, each. This can be described as enough level of discretization.

It is used the finite element method, in basic form, for postulation of the discrete-parameter system. The FE model is shown in Figure 4 and is consisted of 15 nodes and 14 elements. The length of elements (l_n) are the same for same construction part.

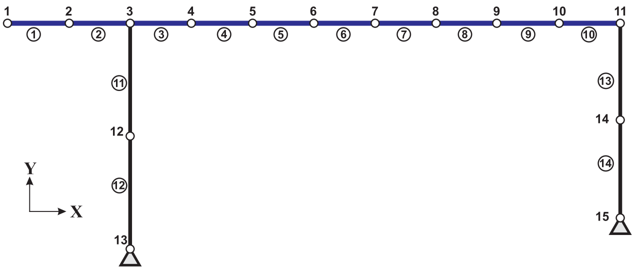


Figure 4: FE model of the gantry-like structure

Every node has 3 DOF's, horizontal displacement, vertical displacement and planar rotation. This is starting postulation for creating property matrices.

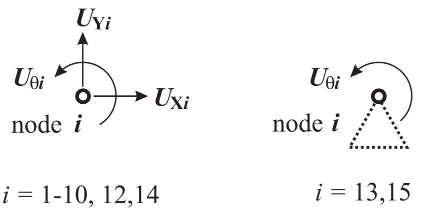


Figure 5: Nodal displacements

The restrained translations of pinned joints are not included in postulation of problems of any kind.

Thus, vector of structural displacements becomes

$$\mathbf{U} = \{U_{X1} \quad U_{Y1} \quad U_{\theta 1} ... U_{\theta 12} \quad U_{\theta 13} ... U_{\theta 14} \quad U_{\theta 15}\}^T \qquad (15)$$

The discretization of the framework (Fig. 4) is done by using FEM, with plane-frame elements [10].

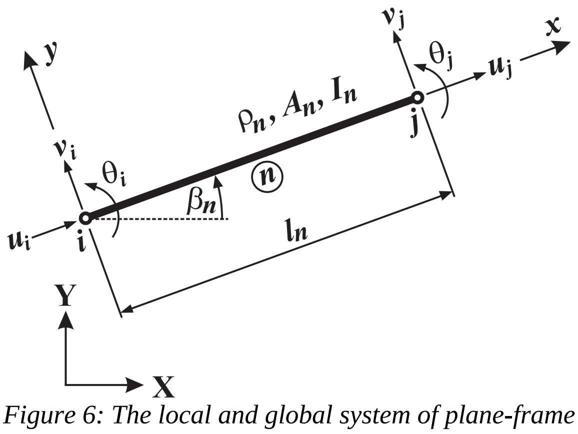


Figure 6: The local and global system of plane-frame element

This postulation gives following matrices, stiffness and mass matrix, respectively, in element local coordinate system.

The element stiffness matrix can be obtained by

$$\mathbf{k}_n = \begin{bmatrix} \frac{EA_n}{l_n} & 0 & 0 & -\frac{EA_n}{l_n} & 0 & 0 \\ 0 & \frac{12EI_n}{l_n^3} & \frac{6EI_n}{l_n^2} & 0 & -\frac{12EI_n}{l_n^3} & \frac{6EI_n}{l_n^2} \\ 0 & \frac{6EI_n}{l_n^2} & \frac{4EI_n}{l_n} & 0 & -\frac{6EI_n}{l_n^2} & \frac{2EI_n}{l_n} \\ -\frac{EA_n}{l_n} & 0 & 0 & \frac{EA_n}{l_n} & 0 & 0 \\ 0 & -\frac{12EI_n}{l_n^3} & -\frac{6EI_n}{l_n^2} & 0 & \frac{12EI_n}{l_n^3} & -\frac{6EI_n}{l_n^2} \\ 0 & \frac{6EI_n}{l_n^2} & \frac{2EI_n}{l_n} & 0 & \frac{6EI_n}{l_n^2} & \frac{4EI_n}{l_n} \end{bmatrix}$$

The element mass matrix is

$$\mathbf{m}_n = \frac{\rho_n A_n l_n}{420} \begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22l_n & 0 & 54 & -3l_n \\ 0 & 22l_n & 4l_n^2 & 0 & 13l_n & -3l_n^2 \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13l_n & 0 & 156 & -22l_n \\ 0 & -13l_n & -3l_n^2 & 0 & -22l_n & 4l_n^2 \end{bmatrix}$$

The transformation matrix to global coordinate system is

$$\mathbf{T}_n = \begin{bmatrix} \cos\beta_n & \sin\beta_n & 0 & 0 & 0 & 0 \\ -\sin\beta_n & \cos\beta_n & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\beta_n & \sin\beta_n & 0 \\ 0 & 0 & 0 & -\sin\beta_n & \cos\beta_n & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

3.1. Overall stiffness matrix

The global stiffness matrices for the main girder elements are the same as local stiffness matrices, while leg elements are obtained with transformation matrix for angle of $3\pi/2$, which give

$$\mathbf{K}_n = \mathbf{k}_n \text{, } n\text{=1-10} \qquad (16)$$

$$\mathbf{K}_n = \mathbf{T}_n{}^T \mathbf{k}_n \mathbf{T}_n \text{, } n\text{=11-14} \qquad (17)$$

Adjustment with all the DOF's and with combination of element stiffness matrices give [11]

$$[K_{st}]_{45x45} = \sum\nolimits_1^{14} \mathbf{K}_n \qquad (18)$$

Overall stiffness matrix of the structure, which includes only free structural displacements, is now obtained with

$$\mathbf{K}_{st} = [K_{st}]_{41x41} \qquad (19)$$

3.2. Overall mass matrix

Similarly, the global mass matrices of elements are obtained as

$$\mathbf{M}_n = \mathbf{m}_n \text{, } n\text{=1-10} \qquad (20)$$

$$\mathbf{M}_n = \mathbf{T}_n{}^T \mathbf{m}_n \mathbf{T}_n \text{, } n\text{=11-14} \qquad (21)$$

Adjustment with all the DOF's give

$$[\mathbf{M}_{st}]_{45x45} = \sum\nolimits_1^{14} \mathbf{M}_n \qquad (22)$$

Overall mass matrix of the structure becomes

$$\mathbf{M}_{st} = [\mathbf{M}_{st}]_{41x41} \qquad (23)$$

3.3. Free undamped vibrations

The governing equation of the free undamped vibration of the MDOF system is known as [12]

$$\mathbf{M}_{st}\ddot{\mathbf{U}}+\mathbf{K}_{st}\mathbf{U}=0 \tag{24}$$

where $\ddot{\mathbf{U}},\mathbf{U}$ are acceleration and displacement vectors of the system, respectively.

Frequency equation becomes

$$\left\|\mathbf{K}_{st}-\omega^2\mathbf{M}_{st}\right\|=0 \tag{25}$$

which gives a set of 41 circular frequencies for the system, while frequency is calculated as

$$f_i=\frac{\omega_i}{2\pi} \tag{26}$$

The given algorithm is also programmed in software Mathematica.

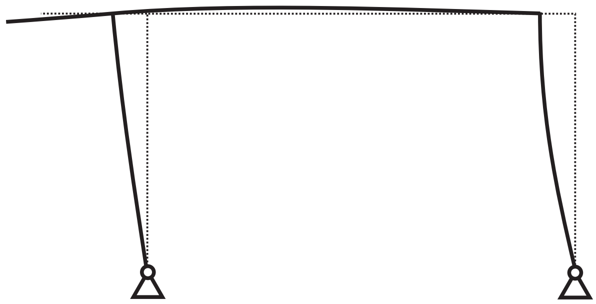
4. VERIFICATION

Verification of the given algorithm is done with finite element model in correlation with mathematical model with distributed-parameter system.

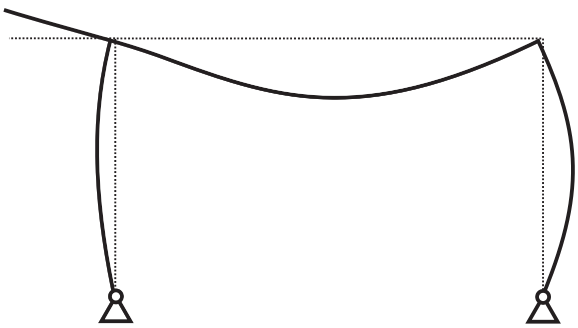
The adopted FE model has following characteristics: L=30 m, H=18 m, I_1=0,024 m^4, A_1=0,07 m^2 and other characteristics are done to comply with (14), following $\alpha=1$, $\beta=10$, $\gamma=1$ and $\delta=2$. It is obtained frequencies of f_1=1,122 Hz, f_2=6,45 Hz, f_3=10,386 Hz, f_4=15,43 Hz ...

For this case on can found only slight differences from the suitable case from Table 2.

Figure 7 shows the first 2 mode shapes of this structure.



a)



b)

Figure 7: a) 1st mode shape, f_1=1,12 Hz, b) 2nd mode shape, f_2=6,45 Hz

The character of mode shapes from Figure 7 describe the typical behaviour of gantry-like structure where 1st mode represent sideway motion of the structure and 2nd mode represent bending of the main girder and legs.

5. CONCLUSION

The paper is dealing with modal analysis of gantry-like structures. First part is devoted to modelling of these structures as distributed-parameter systems. It is obtained frequency equation for free vibrations and solution is obtained with numerical software for several cases.

The main advantages of this approach are:

- the results are closest to *exact* solutions of eigenfrequencies
- it is easy to track the influence of any structural parameter on natural and other frequencies of the structure

The biggest drawback is cumbersome mathematical expressions with special functions and graphically oriented tracking of solutions, but this should be the cost for gaining the *exact* solutions. Consideration of only transversal vibrations is quite suitable approximation for this kind of structures.

The second part deals with title problem with finite element method in basic form. With plane frame elements one can set the model of the framework as discrete-parameter MDOF system. The modal analysis is performed with basic matrix algebra.

The main advantages are:

- simple mathematical apparatus is needed to postulate the problem
- this approach is practically unavoidable when response of structures due to general dynamic loading is needed
- it is more flexible to the changes of framework element characteristics

The basic drawback is that the influence of some structural parameter is not so *visible* on eigenfrequencies analysis and require more detailed consideration.

Authors deliberately didn't show the modal analysis with commercial FEM software. The 2 mentioned concepts in this work serve as starting point of analysis of dynamic behaviour of gantry-like structure as well for validation of modal analysis results performed with commercial FEM.

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