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INFLUENCE OF THE TIGHTNER INCLANATION ANGLE OF THE BUCKET WHEEL EXCAVATOR BOOM TO ITS DYNAMIC BEHAVIOUR

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Abstract: The bucket wheel excavator boom is modeled as elastic body with infinite great number of degrees of freedom, and the analysis of its oscillatory behavior is considered. The process for solving the corresponding partial differential equations and for setting the frequent equation is explained. The non-linear transcendent equation is solved by symbolic "MATHEMATICA" , and the process for solving the problem itself, is exposed in details. The effect of changing the inclination angle, of the rope system for the boom suspension, on the eigenvalues, is analyzed.
Keywords: Bucket wheel excavator, oscillations, elastic body, frequencies, "MATHEMATICA" interpreter

INTRODUCTION

The analysis of the dynamic behavior of the bucket wheel excavator in the working conditions include the setting and the solving the small oscillations differential equations (Волков and Черкасов, 1969), but primary consider the determination of the eigenvalues and the corresponding mode shapes. The most sophisticated models and their solutions are given in dissertations (Petković, 1990. and Bošnjak, 1995.), with the analysis of the bucket wheel excavator linear oscillations. In that analysis the bucket wheel excavator is considered as mechanical system with finite number of degrees of freedom. This paper presents considered as the addition to that study, and refer to the boom oscillations in the vertical operating plane, whereas the boom is modeled as elastic body with infinite number of degrees of freedom. In the first part of this paper, the dynamic model of the bucket wheel excavator boom, taken from the monograph (Petković and Ostrić, 1998.), is explained. In the second part, the setting of mathematical model, as the system of differential equations of the multi-span beam support transversal oscillations and the modal equation from the boundary condition, is performed. The process for determination of the modal equation is totally defined, theoretically, (Бабаков, 1958., Rao, 1995.), but its solving for the special cases can be very difficult. The modal equation is transcendental, and the trigonometric and hyperbolic functions are present. This is not the case for the model with the finite number of degrees of freedom, where the modal equation is in polynomial shapepowered polinom. The new computer routines enable the determination of the eigenvalues of the elastic bodies. Such a routine is the symbolic interpreter "MATHEMATICA" (Volfram,1988.), and its application in solving the modal equation is shown in the third part of this paper. In the scope of considered problem, the effect of changing the inclination angle of the rope system for the boom suspension, on the eigenvalues, is analyzed.

1. DYNAMIC MODEL OF THE ROTOR EXCAVATOR BOOM

The dynamic model of the rotor excavator boom (Petković and Ostrić, 1998.), which oscillates merely in one plane, is given on Fig.1:

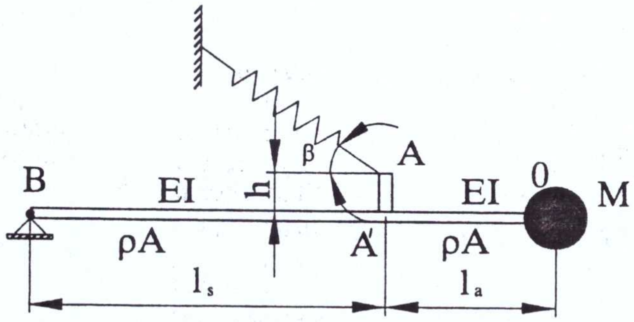


Fig.1

where the model parameters are defined as:

- ls = 52 m, span length of the boom,
- la = 12 m, the length of the cantilever part of the boom,
- h = 4.5 m, eccentric position of the boom connection,
- $EI = 8 \cdot 10^{10}\ Nm^2$, the flexural rigidity of the boom cross section of the span and the bracket parts,
- $c^* = 8 \cdot 10^6\ N/m$, reduced stiffness of the rope system for the boom suspension,
- $\rho A = 3000\ kg/m$, distributed boom mass (the product of the density and the cross sectional area),
- m = 200000 kg, mass of the rotor device,
- β, the inclination angle of the rope system for the boom suspension, which effects on the eigenvalues will be considered in this study.

2. MATHEMATICAL MODEL

Solving the partial differential equations for the transversal oscillations for the span of the boom:

$$\frac{\partial^2 y_s}{\partial t^2} = -c^2 \frac{\partial^4 y_s}{\partial z_s{}^4}, c = \sqrt{\frac{EI}{\rho A}} \tag{1}$$

and for the cantilever part:

$$\frac{\partial^2 y_a}{\partial t^2} = -c^2 \frac{\partial^4 y_a}{\partial z_a{}^4}, c = \sqrt{\frac{EI}{\rho A}} \tag{2}$$

the following solutions are obtained:

$$\begin{aligned} y_s(z_s,t) &= Z_s(z_s)T(t) \\ y_a(z_a,t) &= Z_a(z_a)T(t) \end{aligned} \tag{3}$$

where are:

$$\begin{aligned} &Z_s(z_s) = C_1Ch(kz_s) + C_2Sh(kz_s) + C_3\cos(kz_s) \\ &+ C_4\sin(kz_s), \\ &Z_a(z_a) = D_1Ch(kz_a) + D_2Sh(kz_a) + D_3\cos(kz_a) \\ &+ D_4\sin(kz_a), \\ &T(t) = A\cos(\omega t) + B\sin(\omega t), \\ &\omega = ck^2, \ddot{T}(t) = -\omega^2 T(t). \end{aligned} \tag{4}$$

Before the boundary conditions are formulated, it was necessary to determine the force in the rope system for the boom suspension, as:

$$F = c^*\Delta = c^*[\,y_a(0,t)\sin\beta + h y_a'(0,t)\cos\beta\,] \tag{5}$$

and, also, the corresponding transversal force and flexural momentum in the cross sections, as:

$$\begin{aligned} &F_{t1} = -EI\,y_s'''(l_s,t)), F_{t2} = -EI\,y_a'''(0,t)), F_{t3} = -EI\,y_a'''(l_a,t)) \\ &M_{f1} = -EI\,y_s''(l_s,t)), M_{f2} = -EI\,y_a''(0,t)) \end{aligned} \tag{6}$$

which are shown on Fig. 2 .

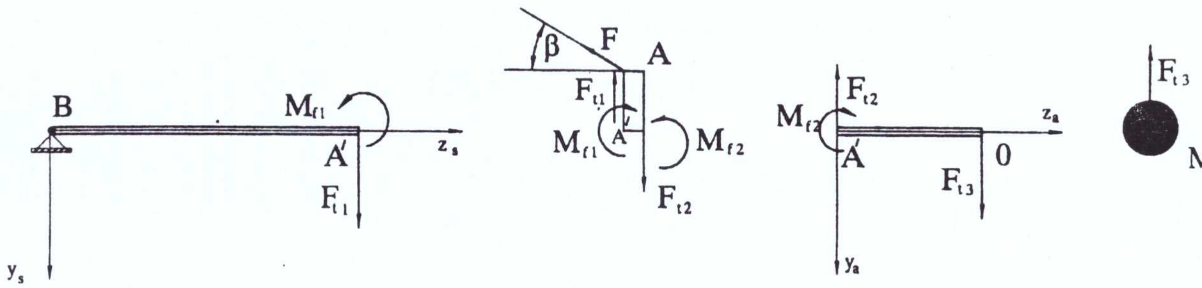


Fig. 2.

The deflection, in the suspension point B, is equal to the zero value:

$$y_s(0,t)=0 \tag{7}$$

as well as the flexural momentum:

$$-EI\,y_s''(0,t))=0. \tag{8}$$

The deflections, from the both sides of the point of connection of the span and the centilever parts of the boom, are equal:

$$y_s(l_s,t)=y_a(0,t) \tag{9}$$

as well as the inclinations:

$$y_s'(l_s,t)=y_a'(0,t) \tag{10}$$

From the equilibrium of the system of forces applied on the bracket, the following can be obtained:

$$F_{t1}+F\sin\beta=F_{t2} \tag{11}$$

as well as for the momentum :

$$M_{f1}=M_{f2}+hF\cos\beta. \tag{12}$$

The flexural momentum in the node O is equal to the zero value:

$$-EI\,y_a''(l_a,t))=0. \tag{13}$$

but when the rotor is in motion, the following is valid:

$$m\ddot{y}_a(l_a,t)=-F_{t3}\,. \tag{14}$$

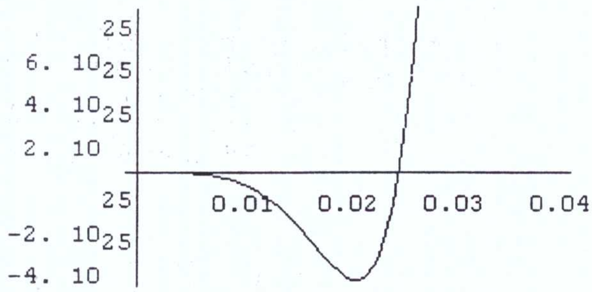
When (3)-(6) are input in the boundary conditions (7)-(14), the homogenous system of equations will be obtained, with unknown quantities $C_i, D_i, i=1,2,3,4$. Besides, it is determined that $C_1=0, C_3=0$, and that the values for $C_2, C_4, D_i, i=1,2,3,4$ have to be non-trivial (the determinant of the system has to be equal to the zero value):

$$\begin{vmatrix} Ch(kl_s) & cos(kl_s) & 0 & -1 & 0 & -1 \\ Sh(kl_s) & sin(kl_s) & -1 & 0 & -1 & 0 \\ -EIk^3Ch(kl_s) & EIk^3cos(kl_s) & c^*sin^2\beta & c^*hk\cos\beta\sin\beta+EIk^3 & c^*sin^2\beta & c^*hk\cos\beta\sin\beta-EIk^3 \\ -EIk^2Sh(kl_s) & EIk^2sin(kl_s) & -c^*h\cos\beta\sin\beta+EIk^2 & -c^*kh^2\cos^2\beta & -c^*h\cos\beta\sin\beta-EIk^2 & -c^*kh^2\cos^2\beta \\ 0 & 0 & Ch(kl_a) & Sh(kl_a) & -cos(kl_a) & -sin(kl_a) \\ 0 & 0 & mkc^2Ch(kl_a)+EI\,Sh(kl_a) & mkc^2Sh(kl_a)+EI\,Ch(kl_a) & mkc^2cos(kl_a)+EI\,sin(kl_a) & mkc^2sin(kl_a)-EI\,cos(kl_a) \end{vmatrix}=0 \tag{15}$$

3. SOLVING THE MODAL EQUATION BY USING THE "MATHEMATICA" INTERPRETER

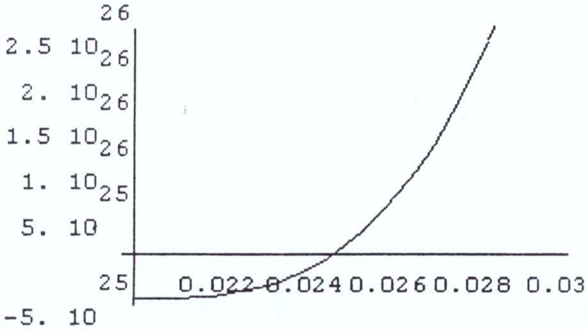
If the determinant (15) is developed, very complicated transcendent equation with unknown value k, will be obtained. If we bring the model parameters into the equation, the additional problem will appear, because the values of different exponents are present on the individual intervals. Since the equation is not a polinom, the roots can't be determined by using the simple commands (e. g. "SOLVE", Volfram, 1988.), thus, it is necessary to analyze the function on the small subintervals, to localize the roots, and to use some numerical methods for its solving. Since k is positive value, the analysis refer to the values greater than zero. Such an analysis can be obtained by plotting the modal (15), (i. e. by using the command "PLOT"). When the root is localized, the corresponding numerical method will be used. In this case, that will be the command "FINDROOT". This process is based on the tangent method, so the initial point have to be chosen for the case when the function and its second derivate have the same sign, which can be determined from the graphic. The process for determination of the first frequency for $\beta = 40^{0}$, with the basic commands, is given on Fig. 3.

Plot[DD,{k,0,0.04}]



Graphics

Plot[DD,{k,0.02, 0.03}]



Graphics

FindRoot[DD,{k,0.025,0.024,0.026}]
{k -> 0.0246831}
omega1 = 5164*(0.0246831)^2
3.1462

Fig. 3.

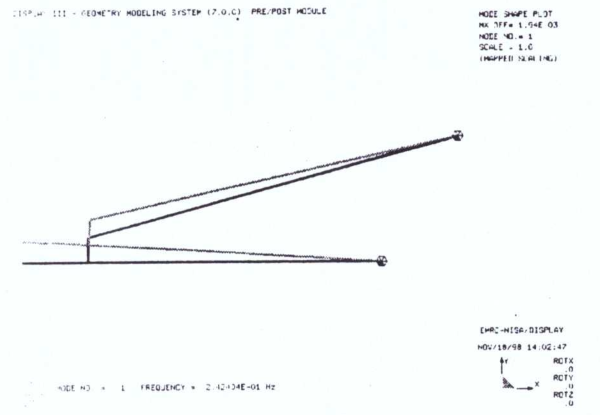
In the further analysis, from Eq.(1) and (4), the other roots and frequencies can be determined, as well. Only the first couple of frequencies are important for the study. This process can be repeated for different values for the incliunation angle of the rope system for the boom suspension, as well. In table 1, the values for the first three eigenvalues,. for different angles β, are given.

Table 1.

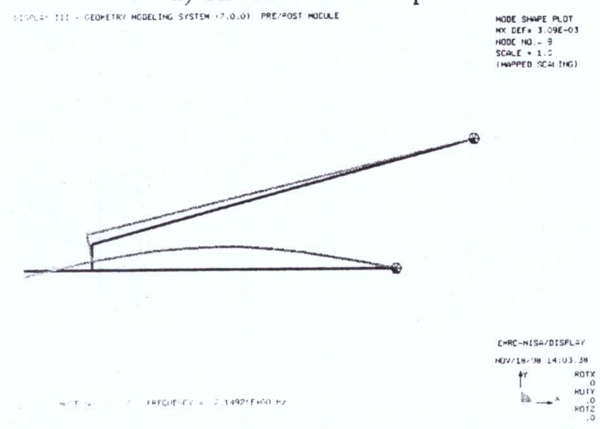
β [°]	ω1 [rad/s]	ω2 [rad/s]	ω3 [rad/s]
15	1.524	13.507	50.890
20	1.886	13.487	50.899
30	2.561	13.472	50.945
40	3.146	13.496	50.024
50	3.621	13.557	50.127
60	3.971	13.698	50.240
70	4.190	13.757	50.351
80	4.279	13.870	50.446

CONCLUSION

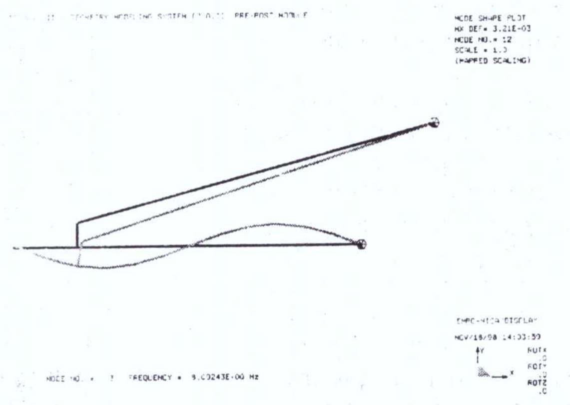
If the energy method was applied on the bucket wheel excavator boom model with finite number of degrees of freedom, it can be concluded as (Petković 1990.), that in the basic mode shape, in the excavator vertical operating plane (with its basic eigen value ω_1), the greatest part of the potential energy is accumulated in the system for the boom suspension. Also, the eigenvalues ω_2 and ω_3, correspond to the transversal oscillations of the boom itself. The analysis of the obtained results, from table 1, shows that the change in inclination angle of the system for the boom suspension, effects mostly on the system itself. Therefore, the less the angle for the boom suspension β is, the lower the eigenvalues ω_1 are, which mean that the system is more deformed, i. e. its stiffness is smaller in the corresponding direction. Analogous, the greater the inclination angle for the boom suspension corresponds to the greater stiffness. The small differences between the eigenvalues ω_2 and ω_3 for the different angles β, show that the transversal oscillations slightly depend on the system for the boom suspension. The first three basic mode shapes are shown on Fig. 4. a, b, c. Parallel with mentioned calculation, the results are confirmed by using the computer program for the finite elements analysis NISA II, and the equal results are obtained.



a) The first mode shape



b) The second mode shape



c) The third mode shape

Fig. 4

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