FUZZY OPTIMIZATION OF CANTILEVER BEAM

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Abstract. It happens very often that we want to design a cantilever beam, while all project requirements are not fully known. Namely, we know roughly what the structure should realize. In classical optimization standard procedure should be applied in order to satisfy all the pre-requirements. What happens if the requirements can only be described, but not explicitly set? This paper starts from the premise that some requirements are expressed linguistically. We want that the length and the largest deflection of a cantilever beam are suitable to satisfy the predetermined conditions. Also, the goals are that the bending stress and the largest deflection have to be less than the allowable maximum value. The objective of this paper is: on known constraints and known fuzzy goal functions we must execute fuzzy projecting of a beam. Constraints are: the length of cantilever beam and its deflection, while the goal functions are maximum bending stress and maximum deflection. On this basis, with known cross-sectional dimensions, we can determine the maximum cantilever beam load.

1. Introduction

Optimization in any field of science, in any problem, provides a solution that satisfies the criteria prescribed in advance. Optimization problems are very often present in design of machines and their elements, optimal control, in finding the optimal trajectory of the system... In defining the optimal criteria there is a situation that some parameters of the system get advantage over the other parameters. A particular problem is the multi criteria optimization. In such cases we are never sure whether that we choose the right criteria, and in particular, whether the criteria are defined in an appropriate manner. What happens in cases where the criterion of optimality does not have clear boundaries? Today, there are available several procedures being able to successfully do mentioned job for us.

In case of the optimization problems with an analytical solution without special restrictions, existing methods provide an exact optimal solution. When the mentioned situation occurs in problems with analytical solution, usually expressed like derivatives of functions, the final solution may depend on the numerical skills. However, in those cases, the well-known classical methods of optimization are present.

Unconventional methods of optimization, in the cases without precisely defined constraints and optimality criteria, began its development about 20 years ago. More of these methods are presented in [1]. A separate analysis of the process of optimization in engineering problems, using fuzzy logic is given in [2]. However, available methods have their disadvantages. The aim of this paper is to improve existing methods for optimization of fuzzy systems, so that the results are more realistic.

2. Theoretical postulation

Consider a function $y = f(x)$. It is necessary to determine the optimal solution x^*, for which the function $y = f(x)$ has a maximum value, whereas we need to be satisfied n constraints [2], for example

$$f_i(x) \le 0, \qquad i = 1,2,...,n \tag{1}$$

All constraints can be represented by a set A

$$A = A_1 \cap A_2 \cap ... \cap A_n = \{x \mid f_i(x) \le 0, \forall i = 1,2,...,n\}, \tag{2}$$

where is $A_i = \{x \mid f_i(x) \le 0\}$. That way we get to the optimal solution x^* defined as follows

$$f(x^*) = \max_{x \in A}\{f(x)\}. \tag{3}$$

In case of conflicting constraints and nonentity of analytical solutions, pre-defined problem can be expressed in a different way, using elements of fuzzy logic. Then, constraints presented with a set A, can be represented in an appropriate way, i.e. new set A, which is adopted in fuzzy set. This fuzzy set is the best way to set limits. It is now necessary to write the function $y = f(x)$ in the form of fuzzy. This can be done as follows [3], using the membership functions

$$\mu_B(x) = \frac{f(x) - m}{M - m}, \tag{4}$$

where are: $m = \inf_{x \in X} f(x)$ and $M = \sup_{x \in X} f(x)$. A set X denotes an area in which we look for the optimal solution, and a fuzzy set B is an appropriate goal function. Relation (4) is suitable for determining the maximum values, but in case you need to minimize a function, does not give proper results, and the membership functions of the goal function should be represented in the form of

$$\mu_B(x) = 1 - \frac{f(x) - m}{M - m}. \tag{5}$$

Obviously, the fuzzy solution obtained as $C = A \cap B$, i.e.

$$\mu_C(x) = \min\{\mu_A(x), \mu_B(x)\}, \tag{6}$$

and the optimal value x^* determined by the relation

$$\mu_C(x^*) \geq \mu_C(x), \qquad \forall x \in X . \tag{7}$$

If the goal functions and constraints are in conflict, fuzzy decision is taken in the form of [2]

$$\mu_C(x) = \alpha\mu_A(x) + (1-\alpha)\mu_B(x), \qquad \alpha \in [0,1]. \tag{8}$$

Applying the previous expression, in practice, does not yield to satisfactory solutions.

3. Problem of multicriteria optimization

Suppose the constraints given by fuzzy sets $A_i, i = 1,2,...,n$, and the goal functions with fuzzy sets $B_j, j = 1,2,...,m$. The corresponding membership functions are $\mu_{A_i}(x)$ and $\mu_{B_j}(x)$. Suppose that all constraints and all goal functions have not the same significance for the determination of the optimal solution.
Based on the above analysis it follows that the membership function of constraints

$$\mu_A(x) = \frac{1}{M_1}\sum_{i=1}^{n}\alpha_i\mu_{A_i}(x), \tag{9}$$

with $M_1 = \sup_{x\in X}\sum_{i=1}^{n}\alpha_i\mu_{A_i}(x)$, while α_i represent the weight coefficients for certain constraints. The same can be determined and the membership functions of goal, i.e.

$$\mu_B(x) = \frac{1}{M_2}\sum_{j=1}^{m}\beta_i\mu_{B_j}(x), \tag{10}$$

where is $M_2 = \sup_{x\in X}\sum_{j=1}^{m}\beta_i\mu_{B_j}(x)$, and β_j are the weight coefficients for certain goals.
In this case, fuzzy solution is obtained using (6), and the optimal solution is determined by relation (7).

4. Example

Suppose that the cantilever beam (Fig. 1.) of length l and square cross section with $a = 4cm$ is loaded with force $\vec{F}$ on its end. The cantilever beam is made of material whose elastic modulus is $E = 2{,}1 \cdot 10^4 \dfrac{kN}{cm^2}$, and allowed bending stress $200MPa$.

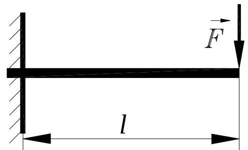

Fig.1.

It is well known, in this case, that the maximum value of bending stress and deflection can be determined by: $\sigma = \dfrac{Fl}{W_x}$ and $f = \dfrac{Fl^3}{3EI_x}$, where are $W_x = \dfrac{a^3}{6}$ and $I_x = \dfrac{a^4}{12}$.

Constraints are: $l \le 2m$ and $f \le 4mm$. Also, we ask that deflection at the end of cantilever beam of the length $1m$ is as close as possible to $0mm$. The goals are: to be sure the deflection does not exceed $4mm$, and that allowed bending stress does not exceed $200MPa$. All constraints and all goals are equally significant to us. On this basis, it is necessary to determine the intensity of force $\vec{F}$.

Using the given constraints, we can introduce them in the form of fuzzy sets

$$l = \begin{cases} d_1 & 0 \le d_1 \le 1 \\ 2 - d_1 & 1 \le d_1 \le 2 \end{cases}, \qquad f = \begin{cases} 4 - c_1 & 0 \le c_1 \le 4 \\ 0 & c_1 \ge 4 \end{cases}. \tag{11}$$

Previously shown functions express our demand that the length of a cantilever beam should be approximately $1m$, and the deflection is as small as possible and never exceeds $4mm$. Functions (11) remind us to the membership function expressions, which are interconnected [4-5]. Transformation of the previous function, for $x \in [0,1]$, we get

$$\mu_{A_1}(x = \frac{l}{2}) = \begin{cases} 2x & 0 \le x \le 0{,}5 \\ 2 - 2x & 0{,}5 \le x \le 1 \end{cases}, \qquad \mu_{A_2}(x = \frac{f}{4}) = \begin{cases} 1 - x & 0 \le x \le 1 \\ 0 & x \ge 4 \end{cases}. \tag{12}$$

Using (2) we obtain the membership function of constraints (Fig.2.)

$$\mu_A(x) = \begin{cases} 2x & 0 \le x \le \frac{1}{3} \\ 1 - x & \frac{1}{3} \le x \le 1 \end{cases}. \tag{13}$$

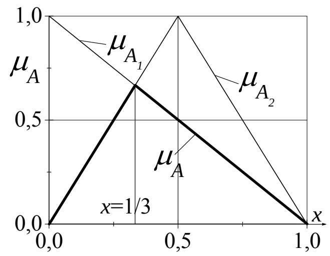

Fig.2.

As the goal functions of the linear functions of force $\vec{F}$, membership functions of goal functions can be expressed as

$$\mu_{B_1}(x = \frac{\sigma}{200}) = x, \qquad \mu_{B_2}(x = \frac{f}{4}) = x, \qquad x \in [0,1], \tag{14}$$

so $\mu_B(x) = x$. Then, based on (6) (Fig.3.)

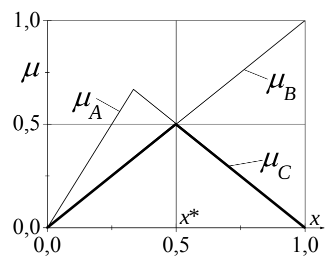

Fig.3.

$$\mu_C(x) = \begin{cases} x & 0 \le x \le 0{,}5 \\ 1 - x & 0{,}5 \le x \le 1 \end{cases}, \tag{15}$$

whence, using (7), it follows that the optimal solution $x^* = 0{,}5$. Therefore, obtained solutions are: $l = 1m$, $f = 2mm$, $\sigma = 100MPa$, and the required intensity of the force is

$$F = \min\left\{ \frac{W_x}{l}\sigma, \quad \frac{3EI_x}{l^3} f \right\} = \min\{1.07, \quad 0.27\} = 0{,}27kN$$

Applying relation (9) and (10), we get

$$\mu_A^*(x) = \frac{1}{M_1}\mu_A(x) = \begin{cases} 3x & 0 \le x \le \frac{1}{3} \\ \frac{3}{2}(1 - x) & \frac{1}{3} \le x \le 1 \end{cases}, \quad \mu_B^*(x) = \frac{1}{M_2}\mu_B(x) = x, x \in [0,1], \tag{16}$$

as it is shown in Fig.4.

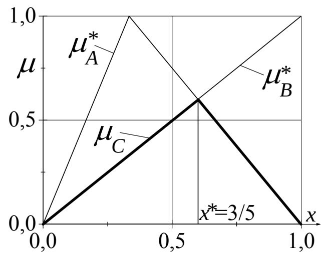

Fig.4.

Then $\frac{3}{2}\left(1-x^*\right)=x^*$, ie. $x^*=0{,}6$. Now the solutions are: $l=1{,}2m$, $f=2{,}4mm$, $\sigma=120MPa$, and the required intensity of the force is

$$F=\min\left\{\frac{W_x}{l}\sigma,\quad \frac{3EI_x}{l^3}f\right\}=\min\left\{1.28,\quad 0.32\right\}=0{,}32kN$$

5. Conclusion

This paper discusses the process of optimization using fuzzy sets. Theoretically, an optimization problem with the presence of more than one constraint and one goal function is considered. This procedure is generalized for the case of more constraints and more goal functions. It is especially considered the case when all the fuzzy constraints and fuzzy objective functions have the same practical significance. Improvement of existing methods, in order to obtain more realistic solutions, is done by a procedure of bringing the membership function to their maximum value. Theoretical studies of this optimization process are shown in the example of a cantilever beam.

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