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ANALYSIS OF DYNAMIC BEHAVIOR OF THE BUCKET WHEEL EXCAVATOR BOOM MODELED AS AN ELASTIC BODY

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Abstract. This paper discusses the effects of structural parameters of the bucket wheel excavator boom on its dynamic behavior in the vertical plane. The boom is modeled as an elastic body with infinite degrees-of-freedom, consisting of several structural parts with constant cross section. The traditional approach in the literature by the discretization of the continuum is avoided by solving partial differential equations of transverse vibrations of the elastic body. The advances in modern software for symbolic programming have enabled the possibility of solving the above mentioned equations. The paper presents the procedure for symbolic set up of characteristic transcendent equation in analytical form and its numerical solving, i.e., defining eigenfrequencies. The original procedure for defining and graphical representation of the eigenfunctions is also given. The results obtained in this paper enable the possibility of the simple analysis of the effects of various parameters on the dynamic behavior of the boom. As an example the influence of the boom inclination angles is analyzed.

1. Introduction

The analysis of dynamic behavior of the bucket wheel excavator (BWE) in the operating conditions includes set up and solving differential equations for small oscillations of the system, but primary considers the determination of eigenvalues and corresponding mode shapes. In some previous references BWE is considered as a mechanical system with finite degrees-of-freedom (DOF), and the corresponding list of references is given in [1]. The analysis of the bucket wheel boom (BWB) oscillations in vertical plane, whereas the boom is modeled as elastic body with infinite DOF is shown in [1]. Also, the procedure of modeling the waterside boom of large container cranes with infinite DOF is given in [2]. Basic principles of modeling of a slewing flexible beam under the moving payload are given in [3]. In this paper the method for solving the system of partial differential equations of transverse oscillations of the beam-type system with arbitrary number of structural parts with constant cross section is developed. The advantage of the developed method is in using the maximum rank of the matrix 4x4 instead the usual rank of 4nx4n for the beam-like structure consisting of n structural elements with constant cross section. The procedure can be fully automated by using modern software for analytical manipulation of mathematical expressions. The possibility for obtaining the complex analytical shape of transcendent characteristic equation enables considering the effects of various structural parameters on the natural frequencies and mode shapes. The method developed in this paper enables also to give the numerical and visual presentation of the characteristic equation solution depending on one or several structural parameters of the system under consideration. That makes possible to define the qualitative effects of some parameters. The proposed method, after analyzing the effects of parameters and estimation of interval of each characteristic value, enables numerical solving of characteristic equation, as well as obtaining the eigenfunctions in analytical form and their graphical representation. In the case of applying any kind of software for structural analysis it is necessary to repeat the computation for each set of structural influence parameters that is avoided for the procedure given in this paper. The adopted procedure leads to the fundamental approach for solving partial differential equations against the usual methods for system discretization by introducing lumped masses. Finally, the developed method is implemented for analyzing dynamic behavior of BWB in the vertical plane. As an example of structural influence parameter the slope angle of BWB is assumed, as well as its influence on the fundamental frequency and higher harmonics and mode shapes.

2. Transverse oscillations of the beam-like structure

The oscillations around the equilibrium position of the beam-like structure consisting of n structural parts with constant cross section are considered, Figure 1.

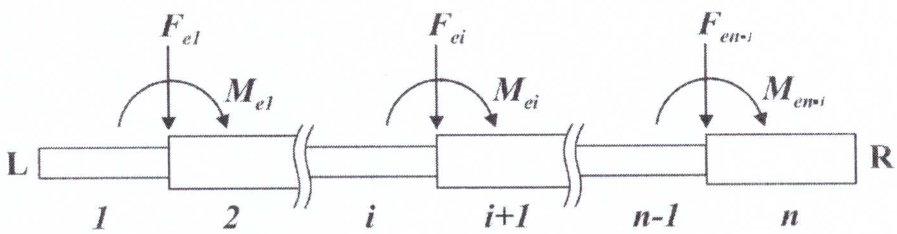

Figure 1. Beam-like structure consisting of n structural parts with constant cross section

In each section we have the following differential equation [4]

$$\frac{\partial^2 y_i}{\partial t^2}+c_i^2\frac{\partial^4 y_i}{\partial z_i^4}=0, c_i=\sqrt{\frac{E_iI_i}{\rho_iA_i}}, i=1,...,n, \tag{1}$$

where E_iI_i is the flexural stiffness, while ρ_i is the density and A_i is the cross section area.

By solving the system of differential equations (1) by using the method of separation of variables

$$y_i(z_i,t)=Z_i(z_i)T(t) \tag{2}$$

the well-known solutions are obtained

$$Z_i(z_i)=C1_iCosh(k_iz_i)+C2_iSinh(k_iz_i)+C3_iCos(k_iz_i)+C4_iSin(k_iz_i)$$
$$T(t)=ACos(\omega t)+BSin(\omega t), \tag{3}$$

where

$$k_1 = k, \quad c = c_1 = \sqrt{\frac{E_1 I_1}{\rho_1 A_1}}, \quad k_i = k\sqrt{\frac{c}{c_i}}, \qquad \omega = c\ k^2. \tag{4}$$

The characteristic value k is defined from 4n boundary conditions, whereof we have two of them at each end, while we have four of them at each connection between the elements. By introducing the matrices

$$\widetilde{C}_i = \left[C1_i\, C2_i\, C3_i\, C4_i\right]^T \tag{5}$$

and by using the fact that the vectors which corresponds to the contiguous parts can be expressed one over the other, the following recurrent relation can be introduced

$$\widetilde{C}_{i+1} = \widetilde{M}_i \widetilde{C}_i \tag{6}$$

where

$$\widetilde{M}_i = \widetilde{M}_{iR}^{-1} \widetilde{M}_{iL} \tag{7}$$

whereby

$$\widetilde{M}_{iL} = \begin{bmatrix} Cosh(k_i l_i) & Sinh(k_i l_i) & Cos(k_i l_i) & Sin(k_i l_i) \\ k_i Sinh(k_i l_i) & k_i Cosh(k_i l_i) & -k_i Sin(k_i l_i) & k_i Cos(k_i l_i) \\ k_i^2 E_i I_i Cosh(k_i l_i) & k_i^2 E_i I_i Sinh(k_i l_i) & -k_i^2 E_i I_i Cos(k_i l_i) & -k_i^2 E_i I_i Sin(k_i l_i) \\ k_i^3 E_i I_i Sinh(k_i l_i) & k_i^3 E_i I_i Cosh(k_i l_i) & k_i^3 E_i I_i Sin(k_i l_i) & -k_i^3 E_i I_i Cos(k_i l_i) \end{bmatrix}$$

$$\widetilde{M}_{iR} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & k_{i+1} & 0 & k_{i+1} \\ k_{i+1}^2 E_{i+1} I_{i+1} + M1_i & M2_i & -k_{i+1}^2 E_{i+1} I_{i+1} + M3_i & M4_i \\ F1_i & k_{i+1}^3 E_{i+1} I_{i+1} + F2_i & F3_i & -k_{i+1}^3 E_{i+1} I_{i+1} + F4_i \end{bmatrix} \tag{8}$$

Equivalent forces between the sections depend on the character of connection. The connection type can be just a simple change of cross section, where the equivalent force and the moment equal zero. In the case of inserted masses, connection by elastic elements or similar, the equivalent force and the moment can be expressed by $\widetilde{C}_{i+1}$

$$F_{ei} = \widetilde{F}_i \widetilde{C}_{i+1} T(t), \qquad M_{ei} = \widetilde{N}_i \widetilde{C}_{i+1} T(t) \tag{9}$$

where

$$\widetilde{F}_i = \left[F1_i\, F2_i\, F3_i\, F4_i\right]$$
$$\widetilde{N}_i = \left[M1_i\, M2_i\, M3_i\, M4_i\right] \tag{10}$$

On the left and the right end of the complex structure, depending on the type of connection between the end sections, we always have two boundary conditions which can be expressed in the matrix form

$$\widetilde{L}\widetilde{C}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \widetilde{R}\widetilde{C}_n = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{11}$$

Having in mind the recurrent relation (6), the boundary conditions (11) can be together written as

$$\widetilde{D}\widetilde{C}_1 = \widetilde{0} , \tag{12}$$

where

$$\widetilde{D} = \begin{bmatrix} \widetilde{L} \\ \widetilde{R}' \end{bmatrix}, \quad \widetilde{R}' = \widetilde{R}\widetilde{M}_n\widetilde{M}_{n-1}...\widetilde{M}_2\widetilde{M}_1, \quad \widetilde{0} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T . \tag{13}$$

Characteristic values $k_j, j = 1,...,\infty$, are defined by solving characteristic equation

$$\det(\widetilde{D}) = 0 , \tag{14}$$

with a note that is the determinant of the forth order independently on the number sections and this is the main advantage of the mentioned way of solving such problem. For each characteristic value can be defined from (12) $\widetilde{C}_1(k_j)$, with a note that one non-zero constant can be equalized to one. By using the recurrent relations (6) all other remaining constants are defined $\widetilde{C}_i(k_j), i = 2,...,n$ in all modes, so we have in each section (i) for the (j-th) shape of oscillations

$$\begin{aligned} Z_{ij}(z_i) &= C1_{ij}Cosh(k_{ij}z_i) + C2_{ij}Sinh(k_{ij}z_i) + C3_{ij}Cos(k_{ij}z_i) + C4_{ij}Sin(k_{ij}z_i) \\ T_j(t) &= A_jCos(\omega_j t) + B_jSin(\omega_j t), \end{aligned} \tag{15}$$

and finally in each section,

$$y_i(z_i,t) = \sum_{j=1}^{\infty} Z_{ij}(z_i)T_j(t), \; i = 1,...,n. \tag{16}$$

In addition to the fact that the rank of the matrices doesn't exceed 4, and the fact that the exposed procedure can be algorithmically solved for numerous various boundary conditions, here is mentioned the possibility of obtaining analytical shape of characteristic equation (14) on the basis of which can be analyzed the influence of any structural parameter on the characteristic values and mode shapes. The application of this method will be expressed on the example of transverse oscillations of BWB in the vertical plane.

By introducing into consideration differential equations of longitudinal oscillations for the in-plane frame structures with arbitrary orientation this method can be generalized, whereby the maximum rank of the corresponding matrix will be 6.

3. Analysis of dynamic behavior of the BWB

The dynamic model of the BWB is given in [1], whereby the BWB primary oscillates in the vertical plane, Figure 2. The model parameters are defined as:

$l_1 = l_s = 52\,m$, span between the boom hinge and the stay connection;

$l_2 = l_a = 12\,m$, span between the stay connection and the free end of the boom - overhanging;

$h = 4.5\,m$, eccentricity of the boom connection;

$b = E_1 I_1 = E_2 I_2 = 8 \cdot 10^{10}\ Nm^2$, flexural stiffness of the BWB;

$co = 8 \cdot 10^{10}\ N/m$, reduced stiffness of the stay (system of ropes for BWB hanging);

$\rho_1 A_1 = \rho_2 A_2 = 3 \cdot 10^3\ kg/m$, distributed mass of the BWB;

$m = 2 \cdot 10^5\ kg$, mass of the bucket wheel (BW);

β, inclination angle of the BWB stay (rope system for BWB hanging).

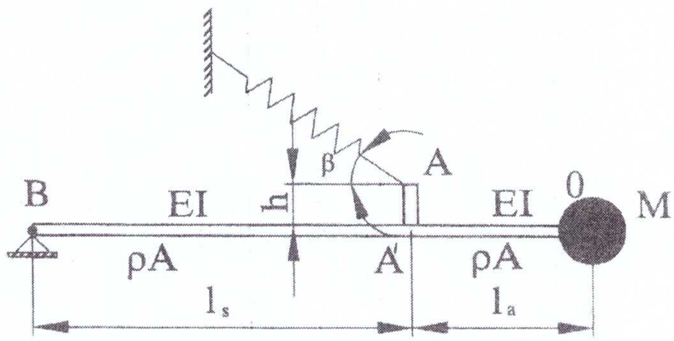

Figure 2. Dynamic model of the BWB in the vertical plane.

The boundary conditions in the left and the right end are defined by matrices

$$
\widetilde{L} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \end{bmatrix},
$$

$$
\widetilde{R} = \begin{bmatrix} Cosh(kl_a) & Sinh(kl_a) & -Cos(kl_a) & -Sin(kl_a) \\ R_{21} & R_{22} & R_{23} & R_{24} \end{bmatrix},
$$

$$
\begin{aligned}
R_{21} &= mkc^2\, Cosh(kl_a) + b\, Sinh(kl_a), \\
R_{22} &= b\, Cosh(kl_a) + mkc^2\, Sinh(kl_a), \\
R_{23} &= mkc^2\, Cos(kl_a) + b\, Sin(kl_a), \\
R_{24} &= -b\, Cos(kl_a) + mkc^2\, Sin(kl_a).
\end{aligned}
\tag{17}
$$

Equivalent force and the moment between the sections are:

$$
\begin{aligned}
\widetilde{F}_1 &= co\, Sin\beta\, \left[Sin\beta \quad h\,k\, Cos\beta \quad Sin\beta \quad h\,k\, Cos\beta \right] \\
\widetilde{N}_1 &= -co\, h\, Cos\beta\, \left[Sin\beta \quad h\,k\, Cos\beta \quad Sin\beta \quad h\,k\, Cos\beta \right]
\end{aligned}
\quad . \tag{18}
$$

By applying the procedure developed in the previous section of this paper it is possible to write the analytical form of the characteristic equation (14)

$$
-2\ b\ k^{2}\ (Cosh[k\ ls]\ (b\ co\ h\ k\ Cos[\beta]\ (h\ k\ Cos[k\ ls]\ Cos[\beta]+Sin[k\ ls]\ Sin[\beta])+b\ co\ h\ k\ Cos[\beta]\ Cosh[k\ la]^{2}\ (h\ k\ Cos[k\ ls]\ Cos[\beta]+Sin[k\ ls]\ Sin[\beta])-Cosh[k\ la]\ (Sin[k\ la]\ (2\ k^{3}\ Cos[k\ ls]\ (b^{2}+c^{2}\ co\ h^{2}\ m\ Cos[\beta]^{2})+b\ co\ Sin[k\ ls]\ (h^{2}\ k^{2}\ Cos[\beta]^{2}+Sin[\beta]^{2}))+k\ Cos[k\ la]\ (-2\ b\ co\ h^{2}\ k\ Cos[k\ ls]\ Cos[\beta]^{2}+Sin[k\ ls]\ (2\ b^{2}\ k^{2}+2\ c^{2}\ co\ h^{2}\ k^{2}\ m\ Cos[\beta]^{2}-b\ co\ h\ Sin[2\ \beta])))+(Cos[k\ la]\ (2\ k^{3}\ Cos[k\ ls]\ (b^{2}+c^{2}\ co\ h^{2}\ m\ Cos[\beta]^{2})+Sin[k\ ls]\ (-4\ b\ c^{2}\ k^{4}\ m-b\ co\ h^{2}\ k^{2}\ Cos[\beta]^{2}+4\ c^{2}\ co\ h\ k^{2}\ m\ Cos[\beta]\ Sin[\beta]+b\ co\ Sin[\beta]^{2}))+k\ Sin[k\ la]\ (-4\ b\ c^{2}\ k^{3}\ m\ Cos[k\ ls]+Sin[k\ ls]\ (-2\ b^{2}\ k^{2}-2\ c^{2}\ co\ m\ Sin[\beta]^{2}+b\ co\ h\ Sin[2\ \beta])))\ Sinh[k\ la]-b\ co\ h\ k\ Cos[\beta]\ (h\ k\ Cos[k\ ls]\ Cos[\beta]+Sin[k\ ls]\ Sin[\beta])\ Sinh[k\ la]^{2})+(b\ co\ Sin[\beta]\ (h\ k\ Cos[k\ ls]\ Cos[\beta]+Sin[k\ ls]\ Sin[\beta])+b\ co\ Cosh[k\ la]^{2}\ Sin[\beta]\ (h\ k\ Cos[k\ ls]\ Cos[\beta]+Sin[k\ ls]\ Sin[\beta])+Cosh[k\ la]\ (2\ Cos[k\ la]\ (Cos[k\ ls]\ (b^{2}\ k^{3}+c^{2}\ co\ h^{2}\ k^{3}\ m\ Cos[\beta]^{2}+b\ co\ h\ k\ Cos[\beta]\ Sin[\beta])+b\ Sin[k\ ls]\ (-2\ c^{2}\ k^{4}\ m+co\ Sin[\beta]^{2}))+Sin[k\ la]\ (Cos[k\ ls]\ (-4\ b\ c^{2}\ k^{4}\ m+b\ co\ h^{2}\ k^{2}\ Cos[\beta]^{2}-4\ c^{2}\ co\ h\ k^{2}\ m\ Cos[\beta]\ Sin[\beta]+b\ co\ Sin[\beta]^{2})-2\ k\ Sin[k\ ls]\ (b^{2}\ k^{2}+c^{2}\ co\ m\ Sin[\beta]^{2})))+(-2\ k\ Cos[k\ ls]\ Sin[k\ la]\ (b^{2}\ k^{2}+b\ co\ h\ Cos[\beta]\ Sin[\beta]-c^{2}\ co\ m\ Sin[\beta]^{2})+Cos[k\ la]\ (b\ co\ Cos[k\ ls]\ (h^{2}\ k^{2}\ Cos[\beta]^{2}-Sin[\beta]^{2})+2\ k\ Sin[k\ ls]\ (-b^{2}\ k^{2}+c^{2}\ co\ m\ Sin[\beta]^{2})))\ Sinh[k\ la]-b\ co\ Sin[\beta]\ (h\ k\ Cos[k\ ls]\ Cos[\beta]+Sin[k\ ls]\ Sin[\beta])\ Sinh[k\ la]^{2})\ Sinh[k\ ls]) = 0
$$

(19)

Analytical form of the characteristic equation enables to analyze the effects of any kind of structural parameters on the mode shapes. Structural parameters are various and may depending on the geometry, cross section area, boom overall dimensions, performances of used materials and stiffness of ropes in the system for BWB hanging or similar. In this paper as an example is shown the analysis of the BWB inclination angle effects. Modern software for manipulating symbolic expressions gives the possibility for immediately visual representation of such effects [5]. In Figure 3 is shown the graphical representation of finding the lowest root of the characteristic equation for various values of BWB inclination. It is easy to observe the increase of the fundamental frequency on the increase of the slope angle (inclination). The cause of this dependency originates from the fact that the fundamental mode shape approximately corresponds to the BWB oscillations as the rigid body. By similar analysis it is conclusive the negligible effect of the inclination to the 2nd and the 3rd mode shape.

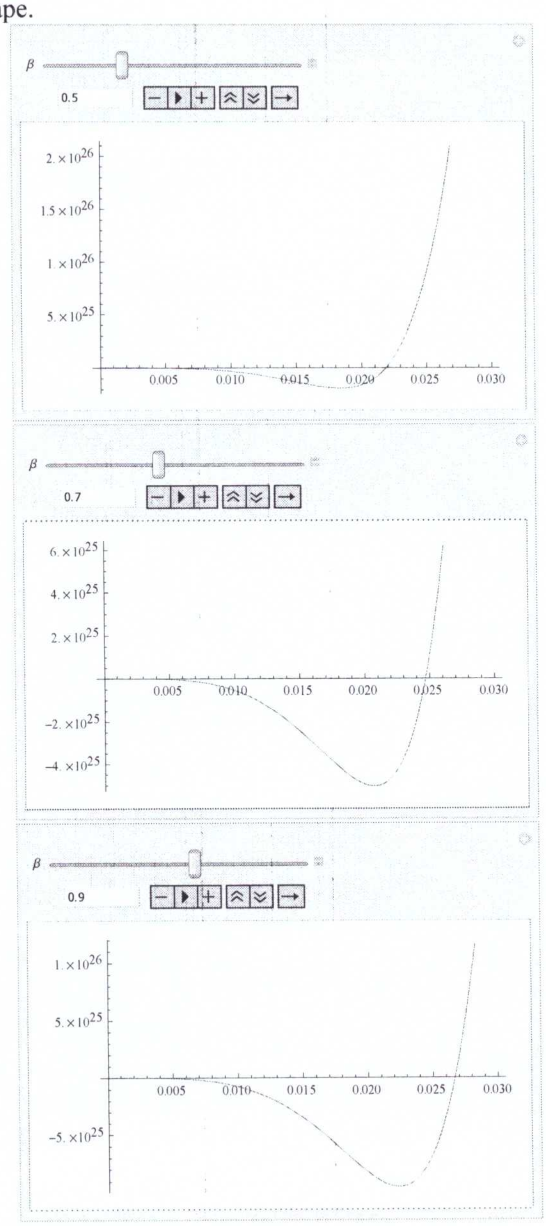

Figure 3. Influence of the inclination angle on the characteristic values of fundamental mode shape.

Figure 4 shows the dependency of characteristic values for the three mode shapes on the BWB inclination angle. The identical procedure can be applied to consider the effects of any other parameter.

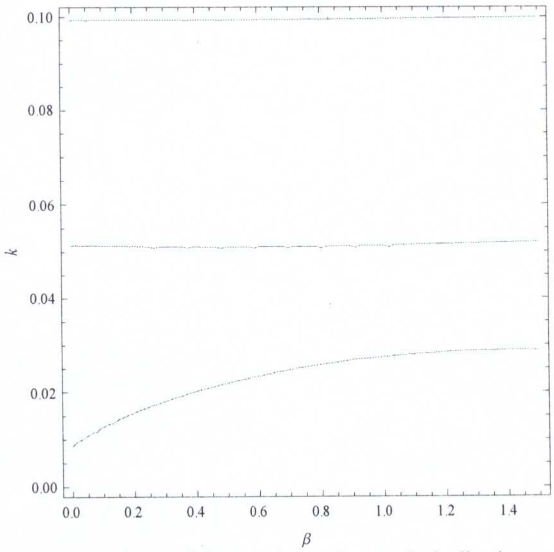

Figure 4. Dependence of characteristic values on the inclination angle.

For $\beta = 40^0 = 2\pi / 9$ the following characteristic values are obtained:

$$\begin{aligned} k1 &= 0.024683108\,\mathrm{m}^{-1} \\ k2 &= 0.051122300\,\mathrm{m}^{-1} \\ k3 &= 0.099402091\,\mathrm{m}^{-1} \end{aligned} \quad (20)$$

which correspond to the constants

$$\begin{aligned} &\tilde{C}_1^T(k_1) = [0 \quad 1 \quad 0 \quad 1.0067317]\,, \\ &\tilde{C}_2^T(k_1) = [2.1983242 \quad -2.077194 \quad 0.43328531 \quad 4.3056356], \\ &\tilde{C}_1^T(k_2) = [0 \quad 1 \quad 0 \quad -114.50673], \\ &\tilde{C}_2^T(k_2) = [6.4087776 \quad 9.6977613 \quad -52.512300 \quad 98.869108], \\ &\tilde{C}_1^T(k_3) = [0 \quad 1 \quad 0 \quad 4706.5647], \\ &\tilde{C}_2^T(k_3) = [54.401435 \quad 150.62610 \quad -4191.1181 \quad 2012.0188]. \end{aligned} \quad (21)$$

on the basis of which it is possible to define the eigenfunctions of oscillations (3). In Figure 5 are shown the first three mode shapes.

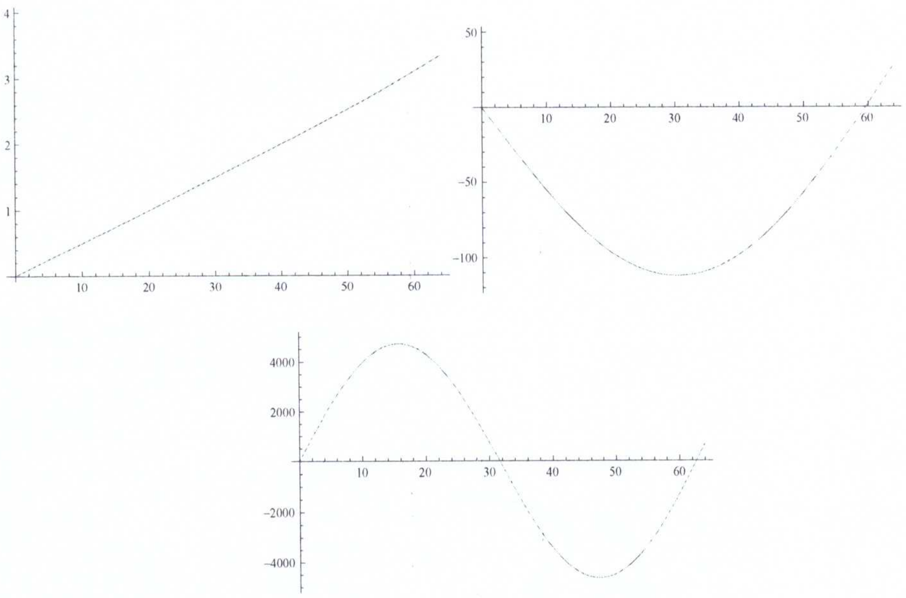

Figure 5. First three mode shapes of BWB oscillations.

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