PAMM · Proc. Appl. Math. Mech. **13**, 591 – 592 (2013) / **DOI** 10.1002/pamm.201310276

Numerical Studies of Viscoelastic Flow Using the Software OpenFOAM

Iva Guranov[1,*], **Aleksandar Ćoćić**[1], and **Milan Lečić**[1]

[1] Faculty of Mechanical Engineering, University of Belgrade, Kraljice Marije 16, 11000 Belgrade

Viscoelastic fluids are a special class of non-Newtonian fluids. There are several types of viscoelastic fluid models, and all of them have a complex rheological response in comparison to Newtonian fluids. This response can be viewed as a combination of viscous and elastic effects and non-linear phenomena. This complex physics makes a numerical simulation a rather challenging task, even in simple test-cases. Studies presented in this paper are numerical studies of the viscoelastic fluid flow in several test cases. These studies have been done in OpenFOAM, an open-source CFD package. Implementation of viscoelastic models and a solver is only available in a community driven version of software (OpenFOAM-ext). One of the goals of research in this paper was to test the solver and models on some simple test cases. We considered start-up and pulsating flows of viscoelastic fluid in a channel and a circular pipe. The important thing is that an analytical solution can be found in these cases, making in possible to test all aspects of numerical simulation in OpenFOAM. Obtained results showed an excellent agreement with the analytical solution for both velocity and stress components. These results encouraged authors' motivation and a choice to use OpenFOAM for simulation of viscoelastic flows. We hope that our research will make a contribution to the OpenFOAM community. Our plan for the further research is a simulation of blood flow in arteries with the viscoelastic solver.

1 Introduction

Viscoelastic fluids are a class of non-Newtonian fluids. The main characteristic of viscoelastic fluids is a simultaneous existence of viscous and elastic effects of the fluid. Model of flow which will be discussed in this work is an isothermal flow of an incompressible viscous non-Newtonian (mean viscoelastic) fluid. This isothermal flow of the incompressible fluid is described by the system of equations, which consists of mass conservation (continuity equation):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{U}) = 0 \tag{1}$$

and the momentum equation:

$$\rho \frac{\mathrm{D}\underline{U}}{\mathrm{D}t} = \rho \underline{f} - \nabla p + \nabla \cdot \underline{\underline{\tau}} \tag{2}$$

Additional equations, which are necessary for closing the system of equations, are constitutive relations or rheological laws. For specific problems considered in this work authors assumed a fully developed planar flow, $\underline{U} = [u(y), 0, 0]$. With this assumption, the system of equations is reduced to a single scalar equation - projection of the momentum equation on x-direction. The analytical solution of velocity for this type of flow is a solution of the mentioned equation with appropriate initial and boundary conditions, and a constitutive relation, which depends on the considered fluid model. Three cases will be examined in the further text: start-up flow in the channel, pulsating flow in the channel, and start-up flow in the circular pipe; every with two constitutive models: the upper convected Maxwell (UCM) and the Oldroyd-B model.

2 Constitutive models and analytical solutions

Firstly two constitutive models will be considered. The UCM model is a linear equation, which is a combination of elastic and viscous behavior of material. Physically it can be represented by a purely viscous damper and a purely elastic spring connected in series. It is the simplest differential model, however it is the most difficult for numerical calculations. Hence, the UCM model is expressed by:

$$\underline{\underline{\tau}}_p + \lambda \overset{\triangledown}{\underline{\underline{\tau}}}_p = 2\eta_p \underline{\underline{S}}. \tag{3}$$

The Oldroyd-B model is an extension of the UCM model. It is a quasilinear model and a combination of the Newtonian model and the UCM model.

$$\underline{\underline{\tau}}_p + \lambda \overset{\triangledown}{\underline{\underline{\tau}}}_p = 2\eta_0 \left(\underline{\underline{S}} + \lambda_r \overset{\triangledown}{\underline{\underline{S}}} \right). \tag{4}$$

Start-up flows generally occur when a constant pressure gradient is applied to a liquid initially at rest. Geometry of the problems is shown in literature [1] (valid distance depends on the considered case - planar channel (H) or circular pipe (R)). Analytical solutions for the start-up flow in the planar channel and in the circular pipe are available in the literature [1], the solution for pulsating flow is published in [2].

---

[*] Corresponding author: Email iguranov@mas.bg.ac.rs,

3 Numerical results and conclusion

OpenFOAM (*Open Source Field Operation Manipulation*) is a free, open-source CFD software package. Solver for flow of incompressible viscoelastic fluids is viscoelasticFluidFoam, implemented by Favero et al. [3]. It is only present in community driven version of the software - OpenFOAM-1.6-ext and it allows analyzing complex geometries using a wide range of meshes and a large variety of interpolation schemes. Numerical results obtained with OpenFOAM were evaluated by comparing with analytical solutions.

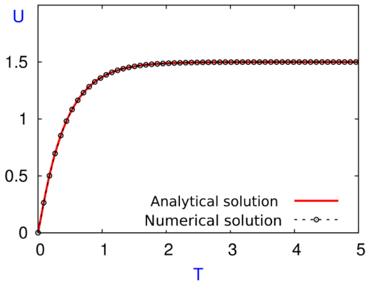

Fig. 1: Evolution of centerline velocity with time during the start-up flow of Newtonian fluid.

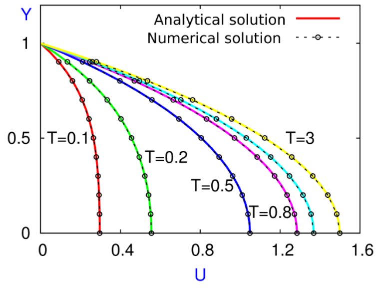

Fig. 2: Velocity profiles at different non-dimensional times for start-up flow of Newtonian fluid.

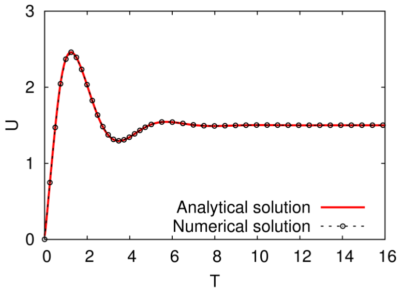

Fig. 3: Evolution of centerline velocity with time during start-up flow of Oldroyd-B fluid.

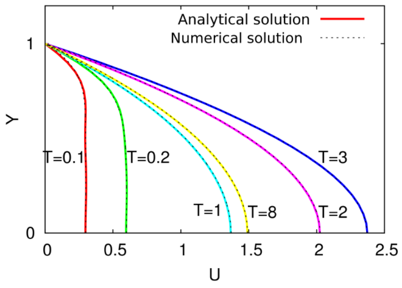

Fig. 4: Velocity profiles at different non-dimensional times for start-up flow of Oldroyd-B fluid.

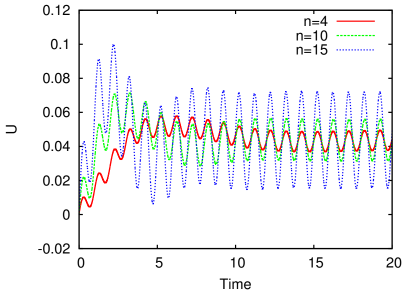

Fig. 5: Evolution of centerline velocity with time during pulsating flow with 100 cells in y direction of Oldroyd-B fluid.

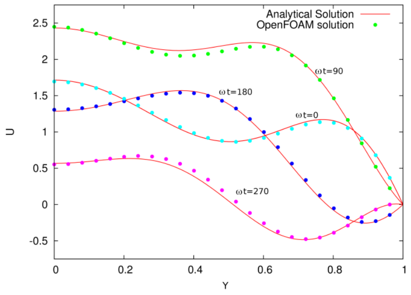

Fig. 6: Comparison between analytical and numerical solution for pulsating flow of Oldroyd-B fluid (4 profiles during one oscillation period).

In figures 1 and 2 comparison between analytical and numerical results for the Newtonian fluid is presented. For start-up flows of UCM and Oldroyd-B fluid velocity grows with time, and the final steady profile is still parabolic due to constant shear viscosity of these fluid models (figure 3, 4). Comparison between Newtonian and viscoelastic fluids for the start-up flow in the planar channel demonstrated a different behaviour during the transient regime before reaching the steady state. UCM fluid took longer to attain a steady regime and Oldroyd-B model showed a smoother development of the flow field. For the start-up flow in the planar channel, period of time required for the flow to be established decreases as the fluid elasticity decreases. For the pulsating flow in channel, which is periodic, the Oldroyd-B model showed good agreement between theoretical and numerical solutions. Excellent agreement between numerical and analytical solution was also achieved for stress components and for start-up flow in the circular pipe.

Acknowledgements This work is supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia, project TR 35046, which authors gratefully acknowledge.

References

[1] A.I.P. Miranda, P.J. Oliveira, Start-up times in viscoelastic channel and pipe flows, Korea-Australia Rheology Journal **22**, 65-73 (2010).
[2] A.S.R. Duarte, A.I.P. Miranda, and P.J. Oliveira, Numerical and analytical modeling of unsteady viscoelastic flows: The start-up and pulsating test case problems, J. Non-Newtonian Fluid Mech **154**, 153-169 (2008).
[3] J.L. Favero, A.R. Secchi, N.S.M. Cardozo and H. Jasak, Viscoelastic flow analysis using the software OpenFOAM and differential constitutive equations, J. Non-Newtonian Fluid Mech **165**, 1625-1636 2010.