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THE INFLUENCE OF THE NONLINEAR TEMPERATURE FIELD ON THE BEHAVIOR OF THE METALLIC PLATE INDUCED BY TWO ELECTROMAGNETIC WAVES OBTAINED BY GSP

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Abstract. *In the paper the behavior of the elastic metallic plate produced by two harmonic electromagnetic plane waves (one at the upper and the other at the lower surface) is considered. Direction of the propagation of the both waves is normal on the surfaces of the plate (axe x_3). As a result of time-changing electromagnetic field conducting currents appear. Distribution of the power of the eddy-current losses across the plate thickness is obtained using complex calculation. It is of exponential type and depends of the plate thickness, wave frequency, electric conductivity, magnetic permeability and magnetic intensity. In further consideration that power is treated as a volume heat source. So, temperature field across the plate thickness is assumed in nonlinear form as $\theta(x_1,x_2,t)=\tau_0(x_1,x_2,t)+\tau_1(x_1,x_2,t)x_3+\tau_2(x_1,x_2,t)x_3^2$ and the system of three coupled differential equations governing temperature field is formed. Equations are solved in analytical form for the stationary state using by integral transform technique (Double Finite Fourier transform). Strain and stress fields are obtained using by FEM for several cases of support position. As the considered structure has symmetry properties, the problem is analyzed using by group supermatrix procedure in the direct stiffness method (GSP), developed by Zlokovic. Performances of GSP are compared with those of the standard finite element analysis (FEM).*

Key words: *electromagnetic field, temperature, plate, induction, heat, vibration, finite element*

1. INTRODUCTION

Eelectro-magneto-thermoelastic problem considered in paper shows one type of interaction between electromagnetic, temperature and strain field in a solid plate. It is assumed that the material of the plates is elastic, isotropic, soft ferromagnetic, which possesses a good electric conductivity. Many nickel-iron alloys used for motors, generators, inductors, transformers are of this type.

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As a result of time changing electromagnetic field conducting currents appear in electric conductors. This problem is mathematically described by the system of Maxwell's equations with the relations for slowly moving media and modified Ohm's low [6]:

$$rot\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}, \quad rot\vec{K} = -\frac{\partial \vec{B}}{\partial t}, \quad div\vec{D} = 0, \quad div\vec{B} = 0, \tag{1}$$
$$\vec{D} = \varepsilon_0(\vec{K} + \dot{\vec{u}} \times \vec{B}), \quad \vec{B} = \mu(\vec{H} - \dot{\vec{u}} \times \vec{D}), \quad \vec{J} = \sigma(\vec{K} + \dot{\vec{u}} \times \vec{B}).$$

The following notations are applied: H – magnetic intensity, K – electric intensity, B – magnetic flux density (magnetic induction), D – electric induction, J – current density, u – deflection, μ_0 – permeability of free space, σ - electric conductivity, ε_0 – dielectric constant of free space.

The power of the conducting currents is represented one type of volume heat source in the plate. So, system of equations describes temperature field in a plate is [4]

$$\left(\nabla^2 - \frac{1}{\kappa}\frac{\partial}{\partial t}\right)\theta - \eta\,\dot{u}_{j,j} = -\frac{Q}{\kappa}, \quad Q = \frac{W + W_H + \dfrac{J^2}{\sigma}}{C_\varepsilon}, \quad (j=1,2,3) \tag{2}$$

where $\kappa=\lambda_o/C_\varepsilon$ is coefficient of thermal intensity, η is representing the coupling between the temperature and the deformation fields and ∇^2 is Laplace operator. Losses in a plate $Q(x_1, x_2, x_3, t)$ are consisted of three factors: external volume heat source intensity, hysterisis losses and Joule's heat (eddy-current losses).

Of course, presented systems of equations have to be added with an appropriate boundary and initial conditions.

2. ELECTROMAGNETIC WAVE

In this section an analytical solution for the power of the eddy-current losses is given for one plane wave with E_{xo} and H_{yo} components on the surface of the plate. It is assumed that all field quantities vary with time as exp(jωt) and are represented in the complex form.

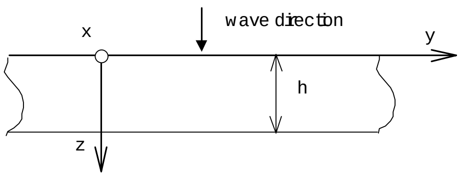


Fig. 1 Primordial coordinate system

In the case of high electric conductivity dielectric current is negligible compared with the conducting current and free electric charges are zero. For homogeneous and isotropic medium which is linear magnetic, using by symbolic-complex method ($\vec{A} = \underline{\vec{A}}e^{j\omega t}$), system of Maxwell's equations (1) can be presented in the next form:

$$rot\underline{\vec{H}} = \sigma\underline{\vec{K}}, \quad div\underline{\vec{K}} = 0, \quad rot\underline{\vec{K}} = -\mu(j\omega)\underline{\vec{H}}, \quad div\underline{\vec{H}} = 0. \tag{3}$$

For the simple plane electromagnetic wave, only normal components of the electric and magnetic field depend of each other [7]. Next analysis is given for one wave with components $\underline{K}_x$ and $\underline{H}_y$. Let in the plane $z = 0$ they have next values

$$\vec{K} = K_x\vec{i} = K_0 \cos\omega t\vec{i}\ ,\ \vec{H} = H_y\vec{j} = H_0 \cos\omega t\vec{j}\ ,\ H_0 = \sqrt{\frac{\varepsilon}{\mu}}K_0\,. \tag{4}$$

Appropriate solution for the electromagnetic field can be presented in the form [4][5]

$$H_y = \mathrm{Re}[\underline{H}_y e^{j\omega t}] = H_0 e^{-\alpha z}\cos(\omega t - \beta z)\,, \tag{5}$$

$$K_x = \mathrm{Re}[\underline{K}_x e^{j\omega t}] = H_0\sqrt{\frac{\omega\mu}{\sigma}}e^{-\alpha z}\cos\left(\omega t - \beta z + \frac{\pi}{4}\right).$$

where $\gamma^2 = j\sigma\mu\omega$, $\gamma = \alpha + j\beta$, $\alpha = \beta = \sqrt{\frac{\sigma\mu\omega}{2}}$.

Electromagnetic wave is followed with the conducting currents, density

$$\underline{J}_x = \sigma\underline{K}_x = H_0\sqrt{\omega\mu\sigma}\,e^{-\alpha z}e^{-j\beta z}e^{j\frac{\pi}{4}}\,. \tag{6}$$

Field amplitudes and current amplitudes are decreased on the exponential law $e^{-\alpha z}$ along the trajectory of the wave propagation. The constant of the penetration is proper to the decay of one Neper (0.368) and its value is

$$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\sigma\mu\pi f}}\ ,\quad \left(\omega = \frac{2\pi}{T} = 2\pi f\right). \tag{7}$$

Skin depth is decided with increasing of frequency, conductivity and permeability. The reason for that phenomenon is heat losses in the metal. The power of the eddy-current losses across the plate thickness is

$$P(z) = \frac{1}{2}H_0^2\omega\mu e^{-\sqrt{2\sigma\mu\omega}z}\,. \tag{8}$$

Express (8) shows that heat source intensity decreasing on exponential type throw the plate thickness. Gradient of the exponential curve increases with the increasing of the wave frequency, permeability and electric conductivity of the material.

The phenomenon of the conducting current concentration on the surface, valid for conductors with very high electric conductivity and magnetic permeability subjected to high frequency wave is known as skin effect.

3. TEMPERATURE FIELD

Let the plate is isolated on the upper and the lower surface and the temperature along the lateral sides is equal to initial temperature T_0 (θ=0). The initial and the boundary conditions have the form

$$\theta\big|_{t=0} = 0\,,\ \ \theta\big|_{x_1=0,a} = 0\,,\ \ \theta\big|_{x_2=0,b} = 0\,,\ \ \frac{\partial\theta}{\partial x_3}\bigg|_{x_3=\pm\frac{h}{2}} = 0\,. \tag{9}$$

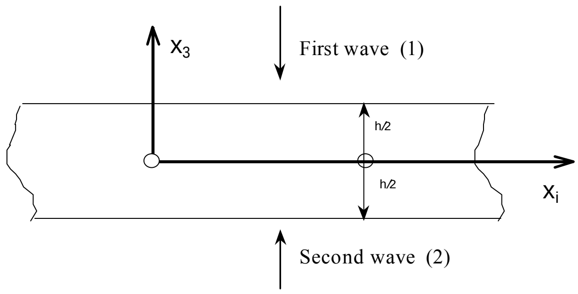


Fig. 2. Coordinate system (middle surface of the plate), waves directions

Using coordinate system shown on Figure 2 and express (8), powers of the eddy-current losses can be presented as

$$P_1(x_3)=\frac{1}{2}H_{01}^2\omega_1\mu e^{-h\sqrt{\frac{\sigma\mu\omega_1}{2}}}e^{x_3\sqrt{2\sigma\mu\omega_1}}=P_1 e^{2\alpha_1 x_3}\,, \tag{10}$$

$$P_2(x_3)=\frac{1}{2}H_{02}^2\omega_2\mu e^{-h\sqrt{\frac{\sigma\mu\omega_2}{2}}}e^{-x_3\sqrt{2\sigma\mu\omega_2}}=P_2 e^{-2\alpha_2 x_3}\,.$$

In further calculation that power was treated as a volume heat source with intensity $P(x_3) = P_1(x_3) + P_2(x_3)$.

Distribution of the heat power can be described with two exponential lines. So, we can take the assumption that the temperature changes squarly across the thickness of the plate. Temperature field $\theta(x_1,x_2,x_3,t)$ can be described using three values, τ_0, τ_1 and τ_2

$$\theta(x_1,x_2,x_3,t)=\tau_0(x_1,x_2,t)+x_3\tau_1(x_1,x_2,t)+x_3^2\tau_2(x_1,x_2,t)\,. \tag{11}$$

If we multiply equation (2) with x_3^k ($k = 0,1,2$) and make integration from the plate thickness, we arrive to three partial differential equations described temperature field in a plate:

$$\left(\nabla_1^2-\frac{1}{\kappa}\frac{\partial}{\partial t}\right)\left(\tau_0+\frac{h^2}{12}\tau_2\right)+\frac{1}{h}\left[\frac{\partial\theta}{\partial x_3}\right]_{-\frac{h}{2}}^{\frac{h}{2}}=-\frac{W_0}{h\,\lambda_0}\,,$$

$$\left(\nabla_1^2-\frac{12}{h^2}-\frac{1}{\kappa}\frac{\partial}{\partial t}\right)\tau_1+\frac{12}{h^3}\left[x_3\frac{\partial\theta}{\partial x_3}\right]_{-\frac{h}{2}}^{\frac{h}{2}}=-\frac{12W_1}{h^3\lambda_0}\,, \tag{12}$$

$$\left(\nabla_1^2-\frac{1}{\kappa}\frac{\partial}{\partial t}\right)\left(\tau_0+\frac{3h^2}{20}\tau_2\right)-4\tau_2+\frac{12}{h^3}\left[x_3^2\frac{\partial\theta}{\partial x_3}\right]_{-\frac{h}{2}}^{\frac{h}{2}}=-\frac{12W_2}{h^3\lambda_0}\,,$$

$$W_k=\int_{-\frac{h}{2}}^{\frac{h}{2}}W(x_1,x_2,x_3,t)x_3^k dx_3\,,\quad (i{=}0,1,2).$$

where κ is coefficient of thermal intensity, h is the plate thickness and ∇_1^2 is Laplace operator.

Subjected to the presented boundary conditions and relations (10) equations (12) can be represented in the next form

$$\left(\nabla_1^2 - \frac{12}{h^2} - \frac{1}{\kappa}\frac{\partial}{\partial t}\right)\tau_1 = -\frac{12W_1}{h^3\lambda_0} = C_1 \quad ,$$

$$\left(\nabla_1^2 - \frac{60}{h^2} - \frac{1}{\kappa}\frac{\partial}{\partial t}\right)\tau_2 = \frac{15}{h^3\lambda_0}\left(W_0 - \frac{12}{h^2}W_2\right) = C_2 , \qquad (13)$$

$$\left(\nabla_1^2 - \frac{1}{\kappa}\frac{\partial}{\partial t}\right)\tau_0 + 5\tau_2 = \frac{15}{h^3\lambda_0}\left(W_2 - \frac{3h^2}{20}W_0\right) = C_0 ,$$

where

$$W_k = \begin{cases} \frac{P_1}{\alpha_1}Sh(\alpha_1 h) + \frac{P_2}{\alpha_2}Sh(\alpha_2 h), \quad k=0 \\ \frac{P_1}{2\alpha_1^2}[\alpha_1 hCh(\alpha_1 h) - Sh(\alpha_1 h)] - \frac{P_2}{2\alpha_2^2}[\alpha_2 hCh(\alpha_2 h) - Sh(\alpha_2 h)], \quad k=1 \\ \frac{P_1}{4\alpha_1^3}[(\alpha_1^2 h^2 + 2)Sh(\alpha_1 h) - 2\alpha_1 hCh(\alpha_1 h)] + \frac{P_2}{4\alpha_2^3}[(\alpha_2^2 h^2 + 2)Sh(\alpha_2 h) - 2\alpha_2 hCh(\alpha_2 h)], \quad k=2 \end{cases} . \qquad (14)$$

Using integral transform technique, double finite Fourier transform, we arrive to the solution for the temperature field for the stationary state in the form

$$\tau_0 = \frac{4}{ab}\sum_{m=1,3,\dots}^{\infty}\sum_{n=1,3,\dots}^{\infty}\left[-\frac{C_0}{\alpha_n\alpha_m\Delta_{mn}} - \frac{5C_2}{\alpha_n\alpha_m\Delta_{mn}\left(\Delta_{mn} + \frac{60}{h^2}\right)}\right]\sin\alpha_n x_1 \sin\alpha_m x_2 \quad ,$$

(15)

$$\tau_1 = \frac{4}{ab}\sum_{m=1,3,\dots}^{\infty}\sum_{n=1,3,\dots}^{\infty}\left[-\frac{C_1}{\alpha_n\alpha_m\left(\Delta_{mn} + \frac{12}{h^2}\right)}\right]\sin\alpha_n x_1 \sin\alpha_m x_2 ,$$

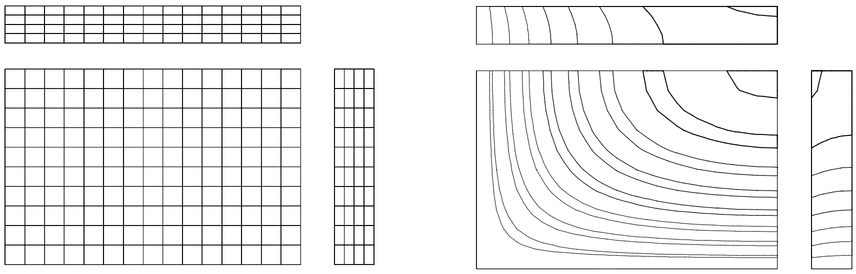
$$\tau_2 = \frac{4}{ab}\sum_{m=1,3,\dots}^{\infty}\sum_{n=1,3,\dots}^{\infty}\left[-\frac{C_2}{\alpha_n\alpha_m\left(\Delta_{mn} + \frac{60}{h^2}\right)}\right]\sin\alpha_n x_1 \sin\alpha_m x_2 ,$$

$$\Delta_{mn} = \alpha_n^2 + \alpha_m^2 = \left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2 .$$

Numerical example is given for the steel rectangular plate dimensions a=30cm, b=20cm, h=2cm. Material constants are: λ_o=0.5W/cmK, σ=7.710^{6}S/m and μ_r=*1000*. Electromagnetic field quantities are: H_{01}=20000A/m, H_{02}=5000A/m, f_1 =50Hz and f_2 =25Hz. Appropriate skin depths are 0.8mm and 1.15mm.

For FEM and GSP analysis 3D volume finite element model with 3255 nodes and 2400 elements is formed. Based on the analytical solution (15) and relation (11)

FORTRAN program obtaining temperature in nodes is made. Finite element model and temperature field for one quarter of the plate is presented on Figure 3.

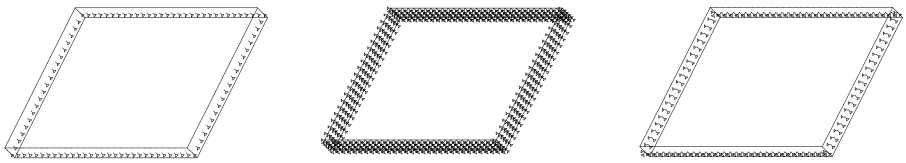


Volume finite element model-one group Isotherm lines, θ=0÷34.68/4°C

Fig. 3 Model for FEM and GSP analysis

4. DISPLACEMENT. STRESS FIELD

Displacement and stress field is obtained using standard FEM calculation and GSP calculation developed by Zlokovic [1]. In GSP calculation structures with symmetry properties described by groups G are analysed by the group supermatrix procedure in the direct stiffness method (GSP). By GSP, the global system stiffness equation of the structure was obtained with the stiffness group supermatrix in diagonal form, where its submatrices pertained to individual G-invariant subspaces of the G-vector space of the problem [2].



First case Second case Third case

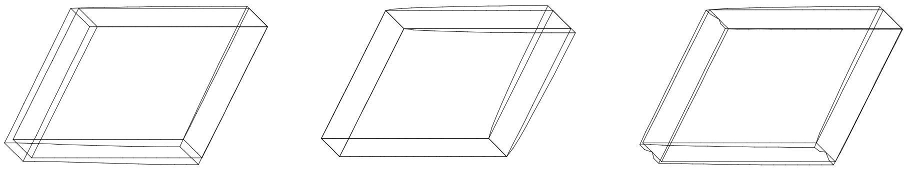
Fig. 4. Boundary conditions

FEM and GSP analyses were done using Program package KOMIPS [3]. Three cases of boundary conditions shown on Figure 4 were considered: plate was simply supported along the entire edge, fixed along the entire edge and fixed only in the middle surface. Appropriate deformation fields are presented on Figure 5 for one quarter of the plate.

Maximal stress obtained in first case was 5.112kN/cm^2, in second case 7.9kN/cm^2 and in third 11.091kN/cm^2. Great value of stress in third case is appearing on the edges because of the applied boundary conditions (see deformation field). Stress fields are presented on Figure 6.

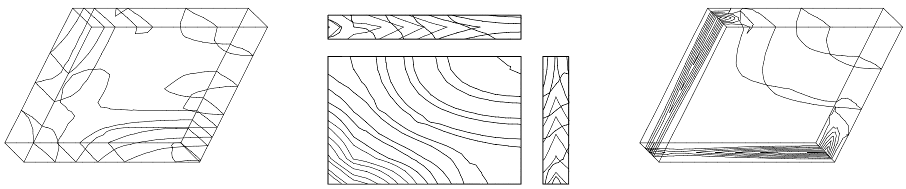
Performances of GSP are compared with those of standard finite element method (FEM). Drastic reduction in the amount of computation and memory space is realized

(GSP/FEM=4/33) and drastic reduction in working time for solving equations is noticed (GSP/FEM=102/1280).



I f_{max}=0.0421mm II f_{max}=0.0114mm III f_{max}=0.0158mm

Fig. 5. Deformation of the plate



I σ=0÷5.112/1 kN/cm^2 II σ=0÷7.902/0.5 kN/cm^2 III σ=5÷11.091/1 kN/cm^2

Fig. 6 Stress field

For presented type of problem, finite element method can be involved in calculation and using by analogy with the finite element of composite plate as it is explained in [8].

5. Conclusion

The problem of the metallic plate subjected transversal to the directions of propagation of the simple harmonic electromagnetic waves can be described throw three systems of differential equations: Maxwell's equations, equations governing temperature field and equations describing deformation and stress field. In considered case of loading the most influence on the stress field has increasing the temperature. That is a result of the time changing electromagnetic field. Intensity of the eddy-current losses is of exponential type across the thickness of the plate, so distribution of the temperature across the plate thickness has to be obtained in nonlinear form. Very suitable method for solving equations governing temperature field in analytical form is integral transform technique. But for problems with non-homogeneous boundary conditions it is very difficult to find displacement and stress fields in analytical form. So, finite element method has been involved in calculation. Because of the drastic reduction of computational memory and working time, finite element analysis was done using Program package KOMIPS with group supermatrix procedure (GSP).

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UTICAJ NELINEARNOG TEMPERATURNOG POLJA NA PONAŠANJE METALNE PLOČE INDUKOVANO POMOĆU DVA ELEKTROMAGNETNA TALASA DOBIJENO POMOĆU GSP

Vesna Milošević-Mitić, Taško Maneski

U radu su prikazani rezultati istraživanja ponašanja elastične metalne ploče proizvedenog pomoću dva harmonijska elektromagnetna ravna talasa (jedan sa donje, a drugi sa gornje površi ploče) . Pravci prostiranja oba talasa su upravni na ravan ploče (osa x_3*). U rezultatu vremenske promene elektromagnetnog polja javlja se struja. Temperaturno polje je pretpostavljeno u vidu nelinearne funkcije po koordinati u pravcu debljine ploče. Ponašanje sistema je opisano pomoću sistema od tri spregnute jednačine, koje su rešene za stacionarno stanje korišćenjem integralnih transformacija (dvostrukih konačnih Fourier-ovih transformacija). Polje napona i deformacija je dobijeno korišćenjem MKE za neke slučajeve. Razmotrene su strukture koje imaju simetrična svojstva i problem je analiziran korišćenjem grupne supermatrične procedure korišćenjem GSP metode koju je razvio Zloković. Uporedjivani su rezultati dobijeni pomoću GSP kao i MKE analize.*