BUCKLING AND POST-BUCKLING BEHAVIOR OF SHELL TYPE STRUCTURES UNDER THERMO MECHANICAL LOADS

by

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The thermo mechanical buckling and post-buckling behavior of layered composite shell type structure are considered with the finite element method under the combination of temperature load and applied mechanical loads. To account for through-thickness shear deformation effects, the thermal elastic, and higher-order shear deformation theory is used in this study. The refined higher order theories, that takes into account the effect of transverse normal deformation, is used to develop discrete finite element models for the thermal buckling analysis of composite laminates. Attention in this study is focused on analyzing the temperature effects on buckling and post-buckling behavior of thin shell structural components. Special attention in this paper is focused on studying of values of the hole in curved panel on thermal buckling behavior and consequently to expend and upgrade previously conducted investigation. Using finite element method, a broader observation of the critical temperature of loss of stability depending on the size of the hole was conducted. The presented numerical results based on higher-order shear deformation theory can be used as versatile and accurate method for buckling and post-buckling analyzes of thin-walled laminated plates under thermo mechanical loads.

Key words: *geometric non-linearity, buckling, post-buckling, thermal loads, finite element method, shells*

Introduction

Thin-walled layered composite structures are increasingly used in any engineering branch where structural weight is one of the major aspects in the design process. When composite materials are being employed computation methods by using numerical simulation (FEM) are required. These methods adequately take effects such as material anisotropy, coupling effects, and shear deformations into account which are inherent in this class of materials.

The composite laminates due to their high specific strength and stiffness are increasingly used in weight-sensitive applications such as aircraft and space vehicles [1-3]. Most of

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these vehicles have to operate in hostile thermal environments. As a result, the structural components of these vehicles are subjected to thermal loads [4-6].

Their components are often subjected to combinations of mechanical and thermal loading. In fact, many structures are subjected to high load levels and consequently may results in non-linear load-deflection relationships due to large deformations of the plate. One of the important problems deserving special attention is the study of their non-linear response to large deflections and post-buckling.

Many studies according to classical plate theory for the large deflection of multilayered composite plates subjected to mechanical or thermal loading are presented in [7, 8]. Numerous studies involving the application of the shear deformation plate theory to non-linear bending analysis can be found in [9-11]. In contrast, there have been fewer investigations for the thermal post-buckling of composite laminated plates [12, 13]. The problem of buckling under thermo mechanical loading has been considered by relatively few investigators [14-16]. The analysis of the buckling and post-buckling behavior of isotropic or composite laminated shells is a topic of considerable technical importance in number of branches of engineering. Such behavior may results from mechanical loading or from thermal loading or from a combination of the two, *i.e.* from thermo mechanical loading. Tauchert [17] contains much information on available methods, particularly as related to flat, rectangular plates, and includes details of several hundred pertinent references dating up until the mid-1990's. A large part of this literature, however, is naturally concerned with buckling under mechanical loading but less information exists on the buckling of shells under thermal loading.

In the present paper the particular concern is with the analysis of the buckling and post-buckling behavior under thermo mechanical loading of isotropic and composite shell type structures. Here the use of FEM in predicting buckling and post-buckling responses of isotropic and composite shells subjected to thermal or mechanical loading or combined thermo mechanical loading is observed.

It is useful to mention that various geometric and material non-linearity problems are solved by using FEM based on the first order shear deformation theory (FOST) [18-23] and here the particular adopted approach presents the FEM in the context of higher-order shear deformation theory (HOST).

Non-linear analysis

In the present work, thermal buckling analyses of multilayered composite panel using discrete finite element model is presented. The finite element model is based on the refined higher order theories [24-28] that considers the effect of transverse normal deformation.

The formulation of the presented shell finite element is based on the single-layer 2-D theory because of its ability of an adequate representation of the global behavior (deflections, stresses, and buckling loads) of thin composites. The HOST used here, assumes the parabolic distribution of the transverse shear stresses across the laminate thickness. The displacement field for the parabolic transverse shear deformation through the shell thickness is given:

$$\begin{aligned} u_1(x,y,z) &= u + z\left[-a\frac{\partial w}{\partial x} + b\Psi_1 - c\frac{4}{3}\left(\frac{z}{h}\right)^2\left(\Psi_1 + \frac{\partial w}{\partial x}\right)\right] \\ u_2(x,y,z) &= v + z\left[-a\frac{\partial w}{\partial y} + b\Psi_2 - c\frac{4}{3}\left(\frac{z}{h}\right)^2\left(\Psi_2 + \frac{\partial w}{\partial y}\right)\right] \\ u_3 &= (x,y,z) = w \end{aligned} \tag{1}$$

The relations (1) are obtained assuming that the transverse shear stresses σ_4 and σ_5 are zero on the shell surfaces:

$$\sigma_4\left(x,y,\pm\frac{h}{2}\right)=0,\quad \sigma_5\left(x,y,\pm\frac{h}{2}\right)=0 \tag{2}$$

The parameters *a*, *b,* and *c* introduced in eq. (1) can have the values zero and one. By combining their values, the displacement field given by eq. (1) can very simply describe the third order shear deformation theory, the first order shear deformation theory and the classical Kirchhoff's plate theory. In that way it is possible to say that eq. (1) represent the general expressions for the displacements of an arbitrary point of a multi-layered shell for the third order theory. Such a way of presentation of displacements is suitable for subsequent considerations of the formulation of a general shell finite element. This is particularly suitable for computer programme realization, since by combining parameters *a*, *b*, and *c*, it is possible to obtain the desired type of the shell finite element able to describe the thin and thick multi-layered composite shells.

The next governing equation can be used to study the linear/non-linear static and eigenvalue buckling analysis and can be written [10, 15, 23]:

$$\left[[K]-[K_T]+[K_G]+\frac{1}{2}\left[N_1(\delta)\right]+\frac{1}{3}\left[N_2(\delta)\right]\right]\{\delta\}=\{F_M\}+\{F_T\} \tag{3}$$

The governing eq. (3) can be used to study the linear/non-linear static and eigenvalue buckling analysis by neglecting the appropriate terms [15]:

– Linear static analysis:

$$[K]\{\delta\}=\{F_M\}+\{F_T\} \tag{4}$$

– Non-linear static analysis:

$$\left[[K]-[K_T]+\frac{1}{2}\left[N_1(\delta)\right]+\frac{1}{3}\left[N_2(\delta)\right]\right]\{\delta\}=\{F_M\}+\{F_T\} \tag{5}$$

– Eigenvalue buckling analysis:

$$[K]\{\delta\}=\Delta T\left[K_G^*\right] \tag{6}$$

where determining matrix $[K_G^*]$ means that the linear static analysis of the shell using eq. (4) have to be carried out. In this analysis the resulting deformation filed is used to determine the initial state of stress resultants. For that purpose Mindlin formulation is used [18, 21]. Mentioned formulation is also used for matrix $[K_G^*]$ determination. Solution of eq. (5) can be obtained using Newton-Raphson iteration procedure coupled with displacement control method [18]. To achieve equilibrium for each load/displacement step Bergan and Clough [19] proposed convergence criteria within the specific tolerance limit of less than 1%.

Following the usual procedure for assembling element stiffness matrices, the equilibrium and stability conditions are expressed:

$$\left[K+\lambda K_G\right]\delta u=[F] \tag{7}$$

Thermal buckling analysis

Calculating the critical temperature of buckling due to thermal load is a two-stage process. For a specified rise, ΔT, in temperature the thermal loads are computed and a linear static analysis is carried out to determine the thermal stress resultants. These stress resultants are then used to compute the geometric stiffness matrix, which is subsequently used in eq. (7), to determine the least eigen value, λ, and the associated mode shape δu. The critical temperature T_{CR} of the plate is calculated:

$$T_{\mathrm{CR}} = \lambda \Delta T \tag{8}$$

In the present analysis, a four-node quadrilateral from the *serendipity* family of 2-D C^0 continuous isoparametric element with 8 DoF per node [28] is used. The formulation of a four-nodes shell finite element that can be good enough also if applied to the thin multilayered plates or shells is by no means an easy matter. The authors' experience has shown that a good approach to the formulation of a four-node shell finite element can be based on the application of the discrete Kirchhoff's theory (DKT) for bending behavior. The DKT ensures C^1 continuity at discrete points on inter-element boundaries. The improved four-nodes layered shell element is derived combining HOST and DKT, fig. 1. More details about that element can be found in [27, 28].

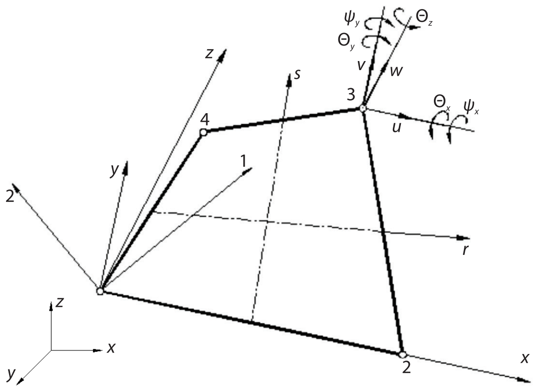


Figure 1. Improved four-nodes shell finite element

In the C^0 finite element theory the continuum displacement vector within the element is defined:

$$a = \sum_{i=1}^{M} N_i\left(r,s\right) a_i \tag{9}$$

In the case of the negligible mid-surface normal stress, σ_z, the stress-displacement relationships, stress resultants and the constitutive equations associated with HOST are given in [27, 28].

The total stiffness matrix of the element is obtained by the linear superposition of the following three independent parts:
- Part I: Membrane stiffness matrix K_M,
- Part II: Bending stiffness matrix K_B, and
- Part III: Rotational stiffness matrix K_{Θ_z}

In order to avoid irregular systems of equations in the case of completely plane systems, a very small rotational stiffness is adjoined to the variable, Θ_z, defining the rotation about the z-axis and it causes a larger stiffness of the system. The displacements (u, v) for the membrane element behavior are approximated by six-term quadratic polynomials as shown in [11] and are defined:

$$\begin{aligned} u &= \sum_{i=1}^{4} N_i\left(r,s\right)U_i + \left(1-r^2\right)\alpha_1 + \left(1-s^2\right)\alpha_2 \\ v &= \sum_{i=1}^{4} N_i\left(r,s\right)V_i + \left(1-r^2\right)\beta_1 + \left(1-s^2\right)\beta_2 \end{aligned} \tag{10}$$

The displacements $b = [\alpha_i, \beta_i]^T$ can be taken as some internal displacements having a quadratic effect on actual displacement. The membrane element equilibrium relations are organized in a matrix form:

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} d_\text{n} \\ b \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \tag{11}$$

By static condensation of the internal variables b, one obtains:

$$\begin{gathered} K_M d_\text{n} = F_\text{n} \\ K_M = K_{11} - K_{12} K_{22}^{-1} K_{21} \\ F_n = F_1 - K_{12} K_{22}^{-1} F_2 \\ b = -K_{22}^{-1} K_{21} d_n + K_{22}^{-1} F_2 \end{gathered} \tag{12}$$

For the element properties the Gauss Quadratic formulae with 2×2 points are used. By static condensation the internal variables α_i, β_i are eliminated on element level and the total number of membrane DoF per element is not changed.

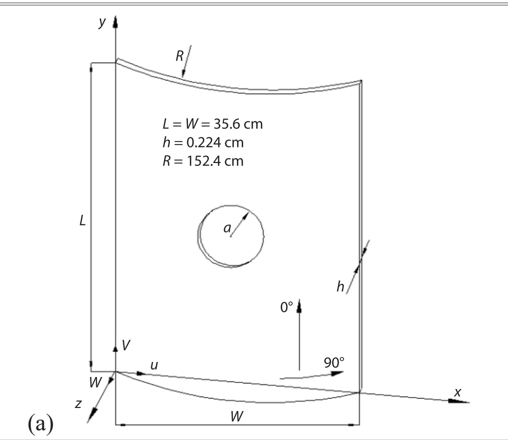
Numerical examples

In this study three software packages were used. The FEM-based commercial software was used for individual validation and comparison of the results, in-house FEM-based software SAMKE [29] was used for structural analyses, in which HOST is embedded, and also analytical in-house software was used, primary for the purpose of assessment of the service life of the structures.

This section presents some numerical examples of non-linear behavior of isotropic and composite structure affected by temperature.

Thermal buckling and post-buckling behavior of a curved laminated panel with a hole

In this section results of buckling and post-buckling behavior of a curved laminated composite panel with circular hole subjected to thermal loads are shown. The primary goal is to verify the computation analyzes of the loss of stability of the considered panel under the action of thermo mechanical loads. When considering the stability of a curved composite panel with circular hole the effects of variations in laminate stacking sequence, fiber orientation, number of layers and aspect ratio of the panels are important parameters to their buckling and post-buckling behavior.



Material properties:

Material 1: Isotropic
$E = 200$ GPa
$\nu = 0.33$
$\alpha = 9.0 \cdot 10^{-6}$ °C^{-1}
Material 2: Composite
$E_L = 130.3$ GPa
$E_T = 9.37$ GPa
$G_{LT} = 4.502$ GPa
$\nu_{LT} = 0.33$
$\alpha_L = 0.139 \cdot 10^{-6}$ °C^{-1}
$\alpha_T = 9.0 \cdot 10^{-6}$ °C^{-1}
$[\pm 45°/0°/90°]_{2S}$

(b)

Figure 2. Geometry presentation (a) and material properties (b) of a curved panel with a hole [15]

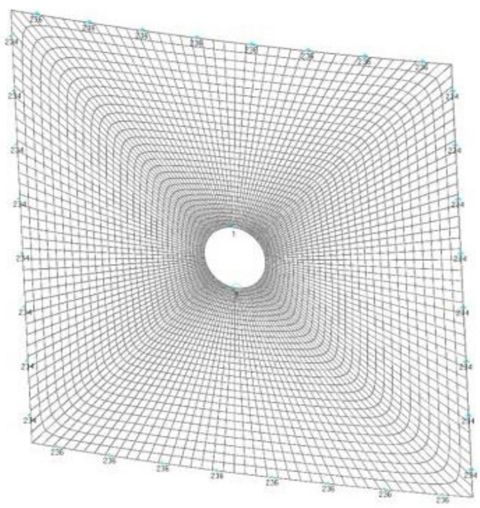


Figure 3. Established finite element mesh

For the purpose of analyzing the effect of a hole size and radius of curvature on panel stability the curved panel subjected to uniform temperature change is considered. For numerical validation geometry and material properties (Material 2) associated with each lamina are given in fig. 2. Boundary conditions of the model in numerical simulations are: $x = 0$, $x = W$, $y = 0$, $y = L$, $v = w = w_x = 0$, $w = w_y = 0$ (without edge restraints). For this problem, several finite element meshes were tested and practically the same results were obtained but the finest finite element mesh was selected. Established finite element mesh is modeled by 4800 elements, *i.e.* 4960 nodes and it is presented in fig. 3.

Additional information is in detail elaborated and depicted in [15], such as elastic-plastic material behavior and stress distribution as a function of linearity and non-linearity.

The results of buckling and post-buckling behavior of a curved layered composite panel with a circular hole subjected to thermal loads are presented in figs. 4-7.

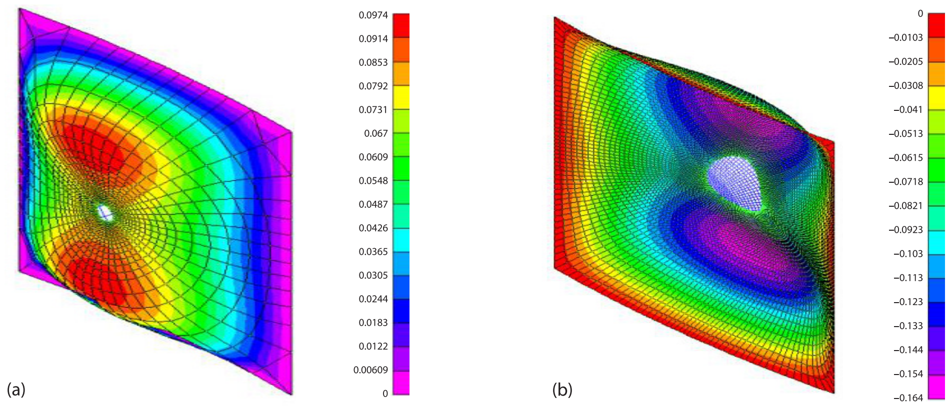


Figure 4. The first buckling mode using linear eigenvalue method; (a) T_{CR} = 588 °C, a = 10 mm [15] and (b) T_{CR} = 623 °C, a = 35 mm

These figures suggest that effects of thermal loads on buckling and post-buckling behavior are obvious. The post-buckling temperature increases with increasing the hole size, which could be easily recognised by observing figs. 4-7 and also tab. 1. Comparison of the results derived from numerical simulations and available results presented in [14] clearly suggests that good agreements between them is achieved. This agreement is depicted in fig. 5, where thermal buckling and post-buckling responses are presented on diagram on which vertical axis represents temperature and horizontal axis represents relative displacement, w/h. Presented agreements propose that conducted numerical simulations were proven as a credible numeric for thermal buckling and post-buckling prediction and can be use for the purpose of numerical investigation of the geometrical domain between these two observed geometries defined by radius value of the hole

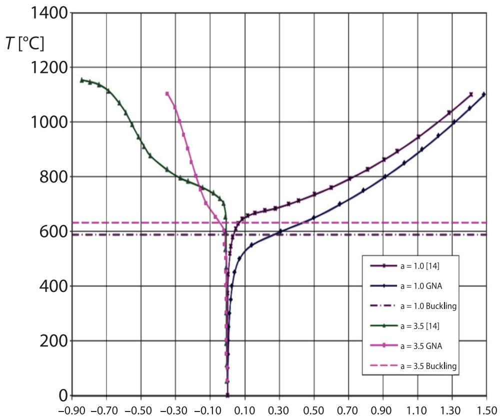


Figure 5. Thermal buckling and post-buckling responses of a curved panel with a hole [15]

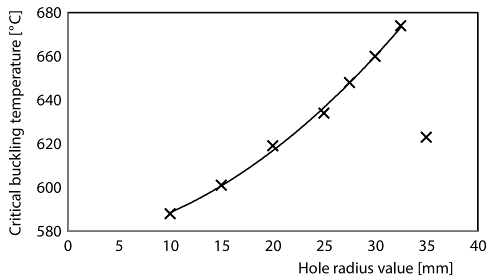


Figure 6. Critical buckling temperature as a function of radius of the hole in a curved panel

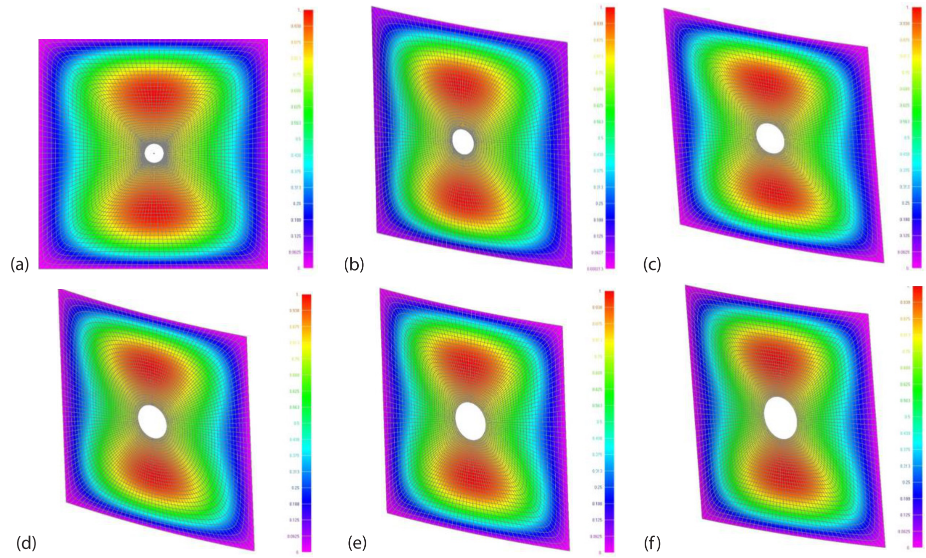


Figure 7. The first buckling mode using linear eigenvalue method; (a) T_{CR} = 601 °C, *a* = 15 mm, (b) T_{CR} = 619 °C, *a* = 20 mm, (c) T_{CR} = 634 °C, *a* = 25 mm, (d) T_{CR} = 648 °C, *a* = 27.5 mm, (e) T_{CR} = 660 °C, *a* = 30 mm, and (f) T_{CR} = 674 °C, *a* = 32.5 mm

of 10 [mm] and 35 [mm]. The main goal of the numerical investigation of the mentioned domain is to study the dependance between radius value of the hole and the critical buckling temperature and to inspect and define nearly the highest critical buckling temperature affected by the maximal radius value after which the loss of stability of composite panel occurs.

The effect of radius value of the hole, *a*, in a curved layered composite panel, as shown in fig. 6 (Material 2), on the critical buckling values of temperature, T_{CR}, are given in tab. 1.

Table 1. The effect of radius value on critical buckling temperatures

T_{CR} [°C]	588	601	619	634	648	660	674	623
a [mm]	10	15	20	25	27.5	30	32.5	35

As results suggest, it is obvious, tab. 1 and fig. 6, that radius value of the hole of about 32.5 mm presents maximal possible value after which the critical behavior of the composite shell structure occurs. It can be concluded from fig. 6 that trend between simulated points could be assumed as polynomial. For the simulated domain, using diagram presented in fig. 6, it is also possible to determine critical buckling temperature for any radius value of the interest which is under the analyzed domain. This means that diagram can be useful as a first iteration in practical thermal buckling analyzes when is necessary to determine critical temperature for exact radius value, but for that purpose, necessary verification procedure has to be conducted for randomly selected hole diameters that differ from those presented in tab. 1, which means that percent discrepancies between values calculated using extracted equation, *i.e.* analytical expression, and simulated (FEM-based) values should be examined.

Also, along with tab. 1, for the purpose of visualization, fig. 7 is presented to depict the effect of radius value on critical buckling temperatures but only for new six numerical points observed within this study.

Additionaly, as a supplementary material *Appendices A* and *B* are implemented to present deformation patterns for curved composite panels with embedded hole for radius value of 10 mm and 35 mm, subjected to thermal loading and bucking and post-bucking behavior of a curved isotropic panel without hole under combined thermal and mechanical loads.

Conclusions

Thermal buckling and post-buckling behaviors of curved laminated composite panels with holes have been examined by employing the finite element technique based on the HOST, which is a tool available in in-house FEM-based software SAMKE, that allows parabolic description of the transverse shear stresses and therefore, the shear correction factors of the usual shear deformation theory are not required. An improved HOST is employed to account for the transverse shear strains by maintaining stress-free top and bottom forces of the panel.

The good agreement between numerical and experimental results was achieved which leads to conclusion that presented finite element results based on HOST can be used as versatile, accurate and trustable numerical method for buckling and post-buckling analyzes of a thin-walled isotropic and composite structural components under thermo mechanical loads. Within this research, after conducted validation, numerical investigation of the geometrical domain between two radius values was carried out to determine the critical temperature at which the loss of stability of a composite panel occurs, depending on the radius of the hole located in it. Furthermore, within this domain, a verified analytical expression for the critical buckling temperature, as a function of radius of the hole in a curved composite panel, can be extracted, which presents common but trustable procedure that can be used as a first iteration in practical thermal buckling analyzes when is necessary to determine critical buckling temperature for exact radius value.

Proposed well-validated numerical approach can be also used for practical instability analysis of the structures made of the fiber reinforced laminates. It should be emphasized that presented results were achieved for particular models, but the whole approach is completely universal and can be applied on any isotropic or composite panel. This approach can be widely used in aircraft and spacecraft industry, and also in design process of the structural components that operates in hostile thermal environments in thermal power plants.

Appendix A. Deformation patterns for curved composite panels with embedded hole subjected to thermal loading

Figures A1 and A2 depict numerically obtained deformations of a curved composite panels with a hole located in it under the thermal load for radius value of 10 mm, fig. A1, and 35 mm, fig. A2, as values (geometries) used for the purpose of validation.

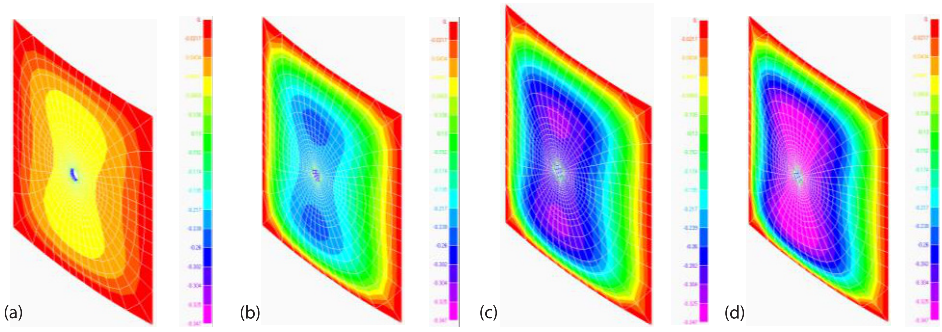


Figure A1. Deformation patterns for curved composite panel with hole value of a = 10 mm [15]; (a) T = 600 °C, (b) T = 850 °C, (c) T = 1000 °C, and (d) T = 1100 °C

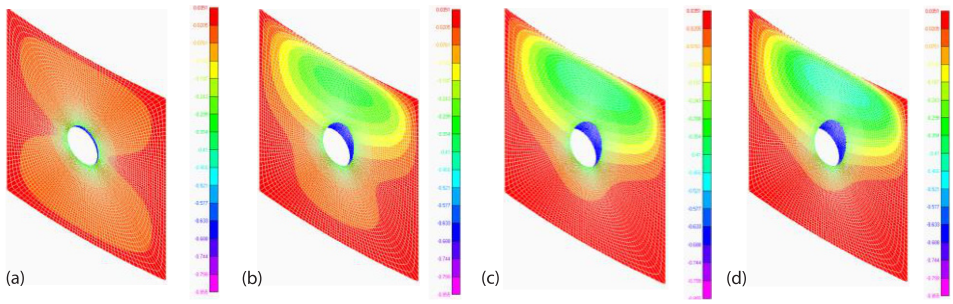


Figure A2. Deformation patterns for curved composite panel with hole value of a = 35 mm [15]; (a) T = 600 °C, (b) T = 850 °C, (c) T = 1000 °C, and (d) T = 1100 °C

Appendix B. Buckling and post-buckling of a curved isotropic panel without hole under combined thermal and mechanical loads

This example considers buckling and post-buckling behavior of a curved isotropic panel without hole subjected to combined thermal load and external mechanical load in form of a pressure (geometry in fig. 2 for Material 1 and same boundary conditions). Computational results derived from FEM simulations and the effects of combined loads to relative displacement of a curved panel are presented in fig. B1.

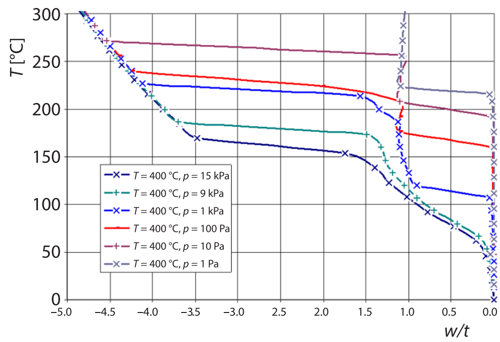


Figure B1. Effect of thermo mechanical load to relative displacement (w/h) of a curved panel
(*for color image see journal web site*)

Figure B1 shows a non-linear analysis in the domain of temperature change up to 400 °C (T = 400 °C). The applied thermal loadings have a significant effect on the thermal buckling and post-buckling responses of considered curved panel. In fact, the structure undergoes buckling at lower temperature when the applied thermal field is uniform through the thickness. Thermal stresses developed due to elevated temperature will lead to buckling failure of these slender structural elements.

Acknowledgment

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Nomenclature

a – nodal displacement vector of the plate, [mm]
a_i – generalized displacement vector in the mid-surface, [mm]
b – non-conforming modes, [–]
d_n – nodal variables, [–]
E – modulus of elasticity, [GPa]
E_L – modulus of elasticity in longitudinal direction, [GPa]
E_T – modulus of elasticity in transversal direction, [GPa]
$[F]$ – global load vektors, [N]
F_1, F_2 – corresponding equivalent load components, [N]
$\{F_M\}$ – mechanical load vector, [–]
$\{F_T\}$ – thermal load vector [–]
G_{LT} – shear module, [GPa]
h – shell/panel thickness, [cm]
$[K]$ – linear stiffness matrix, [–]
$[K_G]$ – geometric stiffness matrices due to initial stress, [–]
$[K_G^*]$ – geometric stiffness matrix due initial state of stress, [–]
$[K_T]$ – geometric stiffness matrices due to thermal stress, [–]
L – length of panel, [cm]
M – is the number of nodes in the element, [–]
$[N_1]$ – non-linear stiffness matrice, [–]
$[N_2]$ – non-linear stiffness matrice, [–]
$N_i\,(r, s)$ – interpolation function associated with the node i and expressed through the normalized co-ordinates (r, s), [–]
R – radius of hole, [cm]
$(1 – r^2)$, $(1 – s^2)$ – incompatible shape functions, [–]
ΔT – temperature rise, [°C]
T_{CR} – critical temperature, [°C]
U_i, V_i – nodal in-plane displacement, [mm]
u, v – translations of the points in the middle plane $(x, y, z = 0)$, [mm]
W – width of panel, [cm]
w – out-of-plane deflection, [mm]
w/h – relative displacement

Greek symbols

α – coefficient of thermal expansion, [1/°C]
α_L – coefficient of thermal expansion in longitudinal direction, [1/°C]
α_T – coefficient of thermal expansion in transversal direction, [1/°C]
δ – vector of degrees of freedom associated to the displacement field in a finite element discretization, [–]
δu – associated mode shape, [–]
λ – least eigenvalue, [–]
ν – Poissons coefficient, [–]
ν_{LT} – Poissons coefficient of orthotropic plate, [–]
Ψ_1, Ψ_2 – rotations of the normals about the y and x-axes, [rad]

Acronym

GNA – general non-linear analysis

References

[1] Jankovic, D., *et al.*, The CFD Calculation of Helicopter Tail Rotor Airloads for Fatigue Strength Experiments, *Journal of Aerospace Engineering*, *30* (2017), 5, 04017032
[2] Maksimovic, S., *et al*., Determination of Load Distributions on Main Helicopter Rotor Blades and Strength Analysis of the Structural Components, *Journal of Aerospace Engineering*, *27* (2014), 6, 04014032
[3] Vasić, Z., *et al.*, Applied Integrated Design in Composite UAV Development, Appl. Compos. Mater., *25* (2018), June, pp. 221-236
[4] Ognjanović, V. O., *et al*., Numerical Aerodynamic-Thermal-Structural Analyses of Missile Fin Configuration During Supersonic Flight Conditions, *Thermal Science*, *21* (2017), 6B, pp. 3037-3049
[5] Vidanovic, N., *et al*., Multidisciplinary Shape Optimization of Missile Fin Configuration Subject to Aerodynamic Heating, *Journal of Spacecraft and Rockets*, *57* (2020) 3, pp. 510-527
[6] Maksimovic, K., *et al*., Determination of Fracture Mechanics Parameters Structural Components with Surface Crack under Thermomechanical Loads, *Scientific Technical Review*, *66* (2016), 3, pp. 27-33

[7] Yeh, F. H., Liu, W. H., Non-Linear Analysis of Rectangular Orthotropic Plates, *International Journal of Mechanical Sciences*, *33* (1991), 7, pp. 563-578
[8] Maksimović, S., *et al*., Numerical and Experimental Stress Analysis of Layered Composite Structures Subject to Mechanical and Hygrothermal Loads, *Structural Integrity and Life*, *19* (2019), 1, pp. 45-49
[9] Mantari, J. I., *et al*., Bending and Free Vibration Analysis of Isotropic and Multilayered Plates and Shells by Using a New Accurate Higher-Order Shear Deformation Theory, Composites – Part: *B Engineering*, *43* (2012), 8, pp. 3348-3360
[10] Maksimović, S., Improved Geometrically Non-Linear Finite Element Analysis and Failure Fiber Reinforced Composite Laminates, *Proceedings*, (Brandt, A. M., Li, V. C., and Marshall, I. H.), 4th Int. Symp. on Brittle Matrix Composites, Warsaw, Poland, 1994, pp. 13-15
[11] Maksimović, S., Instability Finite Element Analysis of Fibre Reinforced Composite Structures Based on the Third Order Theory, *Applied Composite Materials*, *5* (1996), 3, pp. 301-309
[12] Maksimovic, S. *et al*., Experimental and Numerical Stress Analysis of Layered Composite Structures Subject to Hygrothermal Loads, *Proceedings*, 1st Hellenic Conference on Composite Materials and Structures, Xanthi, Greece, 1997
[13] Prabhu, M. R., Dhanaraj, R., Thermal Buckling of Laminated Composite Plates, *Computers and Structures*, *53* (1994), 5, pp. 1193-1204
[14] Madenci, E., Barut, A., Thermal Postbuckling Analysis of Cylindrically Curved Composite Laminates with a Hole, *International Journal for Numerical Methods in Engineering*, *37* (1994), 12, pp. 2073-2091
[15] Bojanić, M., Stability Analysis of Thin-Walled Structures Subjected to Thermal and Mechanical Loads, *Scientific Technical Review*, *LVIII* (2008), 3-4, pp. 21-25
[16] Dawe, D. J., Ge, Y. S., Thermal Buckling of Shear-Deformable Composite Laminated Plates by the Spline Finite Strip Method, *Computer Methods in Applied Mechanics and Engineering*, *185* (2000), 2-4, pp. 347-366
[17] Tauchert, T. R., Temperature and Absorbed Moisture, in: *Buckling and Postbuckling of Composite Plates* (Eds. Turvey, G. J, Marshall, I. H.), Chapter 5, Chapman & Hall, London, UK, 1995. pp. 190-226
[18] Maksimović, S., *et al*., Geometric and Material Non-Linear Analysis of Layered Fiber Reinforced Composite Structures, *Proceedings*, 4th International Conference on Computational Plasticity, Numerical and experimental study, Barcelona, Spain, 1995
[19] Bergan, P. G., Clough, R. W., Convergence Criteria for Iterative Process, *AIAA Journal*, *10* (1972), 8, pp. 1107-1108
[20] Vasović, I, *et al*., Fracture Mechanics Analysis of Damaged Turbine Rotor Discs Using Finite Element Method, *Thermal Science*, *18* (2014), Suppl. 1, pp. S107-S112
[21] Maksimovic, S., Rudic, Z., Analysis of Buckling of Fibrous Composite Shells by Finite Element Method, *Proceedings*, World Conf. on Composite Structures, Nice, France, 1988
[22] Maksimović, S., Failure Analysis of Layered Composite Structures: Computation and Experimental Investigation, *Proceedings*, 16th European Conference of Fracture, Alexandroupolis, Greece, 2006
[23] Norman, F., *et al.*, Postbuckling Behavior of Selected Graphite-Epoxy Cylindrical Panels Loaded in Axial Compression, *Proceedings*, 27th Structures, Structural dynamics and Material conference, San Antonio, Tex., USA, 1986
[24] Reddy, J. N., *An Introduction the Finite Element Method*, Mc. Graw-Hill, New York, USA, 2005
[25] Kim, N. I., *et al*., Coupled Stability Analysis of Thin-Walled Composite Beams With Closed Cross-Section, *Thin-Walled Structures*, *48* (2010), 8, pp. 581-596
[26] Herrmann, J., *et al*., Higher Order Shear Deformation Approach to the Local Buckling Behavior of Moderately Thick Composite Laminates Beams, *International Journal of Structural Stability and Dynamics*, *18* (2018), 11, 1850139
[27] Reddy, B. S., *et al.*, Bending Analysis of Laminated Composite Plates Using Finite Element Method, *International Journal of Engineering, Science and Technology*, *4* (2012), 2, pp.177-190
[28] Maksimovic, S., Buckling and Postbuckling Analysis of Laminated Shell Structures by Finite Elements Based on Third Order Theory, in: *Composite Structures 6* (Ed. I. H. Marshall), Elsevier Applied Science, London, UK, 1991, pp. 89-104
[29] Maksimovic, S., The SAMKE – The Finite Element System, in: *Finite Element Systems* (Ed. A. Niku-Lari), Oxford, Pergamon Press, UK, 1985

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