

**ANALYTICAL SOLUTION OF ELASTIC-PLASTIC STRESSES IN THIN ROTATING DISC MADE UP OF PIEZOELECTRIC MATERIAL**

**ANALITIČKO REŠENJE ELASTOPLASTIČNIH NAPONA U TANKOM ROTIRAJUĆEM DISKU OD PIJEZOELEKTRIČNOG MATERIJALA**

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**Keywords**

- elastic-plastic stress
- mathematical model
- thin rotating disc
- piezoelectric material

*Abstract*

*This paper presents an analytic solution of transitional and plastic stresses in a thin rotating disc made up of piezoelectric material under internal pressure. The stresses in the rotating disc are calculated by applying the transition theory. The transitional and fully plastic stresses are derived with the help of stress strain relations. A nonlinear differential equation is obtained by substituting the resultant relations into the equilibrium equation. The solution of the differential equation with applied boundary conditions gives stresses, pressure and electric displacement. The results obtained are presented graphically and analysed numerically. It has been concluded on the basis of the study that isotropic material is better than piezoelectric material.*

**INTRODUCTION**

Piezoelectric materials are those materials which produce an electric current when some external load is applied to them. This process is also reversible in nature, i.e. if electric current is applied to these materials, their shape is deformed slightly. There are several materials which possess piezoelectric properties such as bones, proteins, quartz and ceramics. Piezoelectric materials have a number of applications such as in sonar, sound detection and generation of high-voltage. These materials are also used in daily life, for example as a cigarette lighter and barbecue-grill igniters. Man-made piezoelectric materials are used in aviation and filters for radios and television. Recently people are more interested in piezoelectric materials due to their electric and magnetic properties. Previously many authors performed studies on piezoelectric materials. Arefi and Rahimi /1/ solved the problem of a thick-walled shell made up of piezoelectric composite material by applying the method of tensor analysis. Dai and Zheng /2/ found an analytic solution for polarized circular cylinder made up of piezoelectric functionally graded material rotating at a constant angular velocity and concluded that functionally graded materials have significant impact. Jafari-Fesharaki

**Ključne reči**

- Lagranžov varijacioni princip
- matematički model
- metoda konačnih razlika
- stanje napon-deformacija

*Izvod*

*U radu je predstavljeno rešenje prelaznih i plastičnih napona u tankom rotirajućem disku od pijezelektričnog materijala, pod dejstvom unutrašnjeg pritiska. Naponi u rotirajućem disku se izračunavaju primenom teorije prelaznih napona. Prelazni naponi i naponi u stanju totalne plastičnosti se određuju iz izraza napona i deformacija. Zame-nom konačnih izraza u jednačinu ravnoteže dobija se nelinearna diferencijalna jednačina. Rešavanjem diferencijalne jednačine primenom graničnih uslova dobijaju se naponi, pritisk i pijezelektrično pomeranje. Dobijeni rezultati su prikazani u vidu dijagrama i analizirani numerički. Na osnovu istraživanja, zaključuje se da je izotropni materijal bolji od pijezelektričnog materijala.*

et. al. /3/ analysed functionally graded piezoelectric hollow cylinder subjected to two-dimensional electromechanical load by applying complex Fourier series and separation of variables. Atrian et. al /4/ discussed the solution of functionally graded piezoelectric thick cylinder under the influence of electric field and mechanical loads by separation of variables. Ghorbanpour et. al. /5/ discussed a thermal rotating disc made up of composite piezoelectric material and demonstrated the effect of non-homogeneity on stresses, displacement and temperature. Rahimi et. al. /6/ discussed the analytical solution of thick circular cylinder made up of piezoelectric functionally graded material subjected to mechanical loads and electric field. Akbarzadeh et al. /7/ calculated dynamic response of functionally graded rectangular plate subjected to thermo-mechanical load by applying Fourier-Laplace method. Renato Caliò et. al. /8/ reviewed the state of art in harvesting of piezoelectric energy. Their work emphasizes on operating modes of material and configurations of devices. Elio /9/ described the phenomenon of forces in piezoelectric materials subjected to electric fields and showed that these materials have capability of producing nonlocal forces of induction. All the above mentioned authors applied classical theory of deformations for solving these problems in the elastic region only.

Borah /10/ has discussed the transition theory by employing the concept of generalized finite strain measures. Many authors /11-26/ have applied transition theory to solve the problems related to different types of solid structures such as cylinders, shells and disc etc., i.e. the stresses in circular cylinder made up of functionally graded material are evaluated by Aggarwal et al. /12/ and found that functionally graded material is better as compared to isotropic material. Sharma et al. /17/ evaluated the thermal shear stresses in torsion of functionally graded cylinder under pressure at inner and outer surface. Sharma and Panchal /18/ and Sharma et al. /19/ used transition theory for finding the stresses in spherical shells and cylinders made up of transversely isotropic material under different loading conditions. Sharma et al. /20/ analysed creep torsion in thick cylinder under the influence of pressure at inner and outer surface and concluded that composite materials are better for design.

In the present paper, elastic-plastic stresses are evaluated in a rotating disc made up of piezoelectric material under internal pressure by applying the concept of transition theory and strain measure theory.

### GOVERNING EQUATIONS

We consider a thin rotating disc having internal and external radii as  $r_1$  and  $r_2$  respectively. The angular velocity of the disc is taken as  $\omega$ . The disc is effectively considered in a state of plane stress, i.e. a thin rotating disk. The displacements co-ordinates in polar form are given by

$$u = r(1-F); \quad v = 0 \quad \text{and} \quad w = dz, \quad (1)$$

where:  $F$  is a function of  $r = \sqrt{(x^2 + y^2)}$ ; and  $d$  is a constant.

With the help of generalized strain measure /10/, we have components of strain as follows

$$\begin{aligned} e_{rr} &= \frac{1}{n} [1 - (rF' + F)^n], \quad e_{\theta\theta} = \frac{1}{n} [1 - F^n], \\ e_{zz} &= \frac{1}{n} [1 - (1-d)^n], \quad e_{r\theta} = e_{\theta z} = e_{zr} = 0. \end{aligned} \quad (2)$$

where:  $n$  is the strain measure; and  $F' = dF/dr$ .

By Hooke's law, the stress-strain relations are given as

$$\begin{aligned} T_{rr} &= (\lambda + 2\mu)[e_{rr} + e_{\theta\theta}] + \lambda(e_{\theta\theta} + e_{zz}) - \epsilon_{11} E_r, \\ T_{\theta\theta} &= (\lambda + 2\mu)e_{\theta\theta} + \lambda(e_{rr} + e_{zz}) - \epsilon_{12} E_r, \\ T_{zz} &= T_{zr} = T_{r\theta} = T_{\theta z} = 0. \end{aligned} \quad (3)$$

The electric displacement equation is

$$D_r = \epsilon_{11} e_{rr} + \epsilon_{12} e_{\theta\theta} + \epsilon_{13} e_{zz} + \eta_{11} E_r, \quad D_\theta = D_z = 0, \quad (4)$$

where:  $\lambda$  and  $\mu$  are Lamé's constants;  $\epsilon_{11}$ ,  $\epsilon_{12}$ ,  $\epsilon_{13}$  are piezoelectric coefficients;  $\eta_{11}$  is dielectric constant; and  $E_r$  is electric field component in radial direction.

$$\frac{d}{dr} \log R = \frac{\left\{ \frac{2\mu}{nr} F^n (1 - (1+P)^n) + \frac{\epsilon_{11} (\epsilon_{11} - \epsilon_{12})}{m\eta_{11} r} [1 - F^n (1+P)^n] - \frac{\epsilon_{12} (\epsilon_{11} - \epsilon_{12}) (1 - F^n)}{m\eta_{11}} + \frac{(\epsilon_{11} - \epsilon_{12}) F^n P}{nr^2 \eta_{11}} - \rho r \omega^2 \right\}}{\frac{2\mu}{nC} (1 - F^n (1+P)^n) + \frac{2\mu(1-C)}{nC} [(1 - F^n) + (1 - (1-d)^n)] + \frac{\epsilon_{11}}{m\eta_{11}} [\epsilon_{11} (1 - F^n (1+P)^n) + \epsilon_{12} (1 - F^n) - \frac{1}{r}]} + B} \quad (11)$$

Applying  $P \rightarrow \pm\infty$  in Eq.(11) and integrating, we get

$$R = Ar^D + B, \quad (12)$$

From the free charge equation, we have

$$D_r = \frac{C_1}{r} = \frac{1}{r}, \quad E_r = \frac{1}{\eta_{11}} \left[ \frac{1}{r} - \epsilon_{11} e_{rr} - \epsilon_{12} e_{\theta\theta} \right]. \quad (5)$$

By using Eqs.(3), (4) and (5), stresses are given as

$$\begin{aligned} T_{rr} &= \frac{2\mu}{Cn} [1 - F^n (1+P)^n] + \frac{2\mu(1-C)}{Cn} [(1 - F^n) + (1 - (1-d)^n)] + \\ &+ \frac{\epsilon_{11}}{m\eta_{11}} \left[ \epsilon_{11} (1 - F^n (1+P)^n) + \epsilon_{12} (1 - F^n) - \frac{1}{r} \right], \\ T_{\theta\theta} &= \frac{2\mu(1-C)}{Cn} [(1 - F^n (1+P)^n) + (1 - (1-d)^n)] + \frac{2\mu}{Cn} (1 - F^n) + \\ &+ \frac{\epsilon_{12}}{m\eta_{11}} \left[ \epsilon_{11} (1 - F^n (1+P)^n) + \epsilon_{12} (1 - F^n) - \frac{1}{r} \right], \end{aligned} \quad (6)$$

where:  $rF' = FP$ ; and  $C = 2\mu/(\lambda + 2\mu)$ .

The equation of equilibrium for rotating disc is given as

$$\frac{d}{dr} (T_{rr}) + \frac{T_{rr} - T_{\theta\theta}}{r} + \rho \omega^2 r = 0. \quad (7)$$

Using Eqs.(6) and (7), the nonlinear differential equation is obtained as

$$\begin{aligned} & - \left\{ \frac{2\mu}{C} F^{n+1} (1+P)^{n-1} P + \frac{\epsilon_{11}^2}{\eta_{11}} F^{n+1} (1+P)^{n-1} P \right\} \frac{dP}{dF} = \\ & = \frac{2\mu}{C} F^{n+1} (1+P)^{n-1} P + \frac{2\mu(1-C)}{C} F^n P + \frac{\epsilon_{11}^2 F^n P}{\eta_{11}} (1+P)^n + \\ & + \frac{\epsilon_{11} \epsilon_{12} F^n P}{\eta_{11}} - \frac{2\mu}{n} [F^n (1 - (1+P)^n)] - \frac{\epsilon_{11} (\epsilon_{11} - \epsilon_{12})}{m\eta_{11}} \times \\ & \times \left[ 1 - F^n (1+P)^n \right] - \frac{\epsilon_{12} (\epsilon_{11} - \epsilon_{12}) (1 - F^n)}{m\eta_{11}} - \frac{\epsilon_{12}}{r m\eta_{11}} - \rho r^2 \omega^2 \end{aligned} \quad (8)$$

The transition points of the above differential equation are  $P \rightarrow \pm\infty$  and  $P \rightarrow -1$ .

The conditions which are to be applied at inner and outer surfaces of the disc are as follows

$$T_{rr} = -p \quad \text{at} \quad r = r_1 \quad \text{and} \quad T_{rr} = 0 \quad \text{at} \quad r = r_2. \quad (9)$$

Transition from elastic to plastic state

According to transition theory /10-26/, a material in the elastic state changes to plastic at critical points  $P \rightarrow \pm\infty$ . For calculating the stresses, transition function is taken as

$$R = T_{rr} + B, \quad (10)$$

where:  $B$  is a constant.

Taking logarithmic differentiation of Eq.(10), we have

$$D = - \frac{2\mu C \eta_{11} + C \epsilon_{11} (\epsilon_{11} - \epsilon_{12})}{2\mu \eta_{11} + C \epsilon_{11}^2}. \quad (13)$$

With the help of Eqs.(12) and (7), the transitional stresses are obtained as follows

$$T_{rr} = Ar^D - B, \quad T_{\theta\theta} = (D+1)Ar^D - B + \rho r^2 \omega^2. \quad (14)$$

By using Eqs.(9) and (13) we have

$$A = \frac{-p}{r_1^D - r_2^D}, \quad B = \frac{-pr_2^D}{r_1^D - r_2^D}. \quad (15)$$

The difference of Eqs.(11) and (12) gives Tresca's yield criteria as

$$|T_{rr} - T_{\theta\theta}| = \left| \frac{-pDr^D}{r_1^D - r_2^D} + \rho r^2 \omega^2 \right|. \quad (16)$$

From Eq.(15), it has been calculated that  $|T_{rr} - T_{\theta\theta}|$  yields maximum value at  $r = r_1$  which results as initial yielding stress.

$$|T_{rr} - T_{\theta\theta}|_{r=r_1} = \left| \frac{-pDr_1^D}{r_1^D - r_2^D} + \rho r_1^2 \omega^2 \right| = Y. \quad (17)$$

From the above equation, the pressure required for initial yielding in piezoelectric material is given as

$$\frac{p}{Y} = \frac{\left( \frac{\rho r_2^2 \omega^2}{Y} - 1 \right) (r_1^D - r_2^D)}{Dr_1^D}. \quad (18)$$

Also, it has been calculated that at  $r = r_2$ , Eq.(15) yields fully plastic yielding stress as given below

$$|T_{rr} - T_{\theta\theta}|_{r=r_2} = \left| \frac{-pDr_2^D}{r_1^D - r_2^D} + \rho r_2^2 \omega^2 \right| = Y_1. \quad (19)$$

Pressure required for fully plastic state in piezoelectric material from the above equation is given as

$$\frac{p}{Y_1} = \left( 1 - \frac{\rho r_2^2 \omega^2}{Y_1} \right) \log \left( \frac{r_1}{r_2} \right). \quad (20)$$

Using the following non-dimensional components in Eqs. (13), (17) and (19), we have

$$R = \frac{r}{r_2}, \quad R_0 = \frac{r_1}{r_2}, \quad \sigma_{rr} = \frac{T_{rr}}{Y}, \quad \sigma_{\theta\theta} = \frac{T_{\theta\theta}}{Y}, \quad \sigma_{rf} = \frac{T_{rr}}{Y_1},$$

$$\sigma_{\theta f} = \frac{T_{\theta\theta}}{Y_1}, \quad P_1 = \frac{p}{Y}, \quad P_2 = \frac{p}{Y_1}, \quad \Omega = \frac{\rho \omega^2 r_1^2}{Y}, \quad \Omega_1 = \frac{\rho \omega^2 r_2^2}{Y_1}.$$

Pressure required for initial yielding in non-dimensional form from Eq.(18) becomes

$$P_1 = \frac{(\Omega - 1)(R_0^D - 1)}{DR_0^D}. \quad (21)$$

Pressure required for fully plastic state in non-dimensional form from Eq.(20) becomes

$$P_2 = (1 - \Omega_1) \log R_0. \quad (22)$$

Transitional stresses from Eq.(14) in non-dimensional form are given as

$$\sigma_{rr} = \frac{P_1(1 - R^D)}{R_0^D - 1}, \quad \sigma_{\theta\theta} = \frac{P_1(1 - (1 + D)R^D)}{R_0^D - 1} + \frac{\Omega R^2}{R_0^2}. \quad (23)$$

When  $D \rightarrow 0$  in Eq.(23), we have fully plastic stresses in non-dimensional form as

$$\sigma_{rf} = \frac{-P_2 \log R}{\log R_0}, \quad \sigma_{\theta f} = \frac{-P_2(1 + \log R)}{\log R_0}. \quad (24)$$

*Particular case*

For comparing the results for piezoelectric and isotropic steel material, the pressure and stresses are calculated for steel (isotropic material) as given below.

From Eq.(21), the pressure required for initial yielding is given by

$$P_1 = \frac{(1 - \Omega)(R_0^{-C} - 1)}{CR_0^{-C}}. \quad (25)$$

From Eq.(22), the pressure required for fully plastic state is given by

$$P_2 = (1 - \Omega_1) \log R_0. \quad (26)$$

Transitional stresses from Eq.(23) are obtained as

$$\sigma_{rr} = \frac{P_1(1 - R^{-C})}{R_0^{-C} - 1}, \quad \sigma_{\theta\theta} = \frac{P_1(1 + (C - 1)R^{-C})}{R_0^{-C} - 1} + \frac{\Omega R^2}{R_0^2}. \quad (27)$$

Fully plastic stresses from Eq.(24) are obtained as follows

$$\sigma_{rf} = \frac{-P_2 \log R}{\log R_0}, \quad \sigma_{\theta f} = \frac{-P_2(1 + \log R)}{\log R_0}. \quad (28)$$

## NUMERICAL DISCUSSION

Figures 1a and 1b have been drawn for thin rotating disc with angular velocity to find the effect of pressure on the piezoelectric material (PZT-4) and isotropic steel material at various radii ratios. It is observed from Fig. 1a that in the case of piezoelectric material, the value of the pressure essential for initial yielding attains its maximum value at the inner surface with angular velocity  $\Omega = 10$ . Also, the pressure for initial yielding is lower for piezoelectric material as compared to the steel (isotropic material). It is also observed that the pressure, essential for full plasticity decreases as radii ratios increase in both types of materials and it has a maximal value at the inner surface. As angular velocity (say,  $\Omega = 20$ ) increases, the necessary pressure for initial yielding and fully plastic state increases for both piezoelectric and steel-isotropic material as observed from Fig. 1b.

Figures 2a and 2b are drawn to observe the variation of angular velocity with pressure at different radii ratios. It is noticed from Fig. 2a that angular velocity required for initial yielding and fully plastic state increases with increasing radii ratios in case of piezoelectric material as well as for the isotropic material. It is also seen that angular velocity is maximum at the outer disc surface. The angular velocity increases with the incremented value of pressure as observed from Fig. 2b.

Figures 3a and 3b are drawn for finding the effect of pressure on stresses in transitional and fully plastic state at different angular velocities. Figure 3a shows that transitional and fully plastic stresses in piezoelectric and isotropic material increase with increasing radii ratio and are maximum at the outer disc surface with angular velocity  $\Omega = 10$ , (say). Also, the transitional stresses are minimum for piezoelectric material as compared to isotropic material.

Transitional and fully plastic stresses increase with increase of angular velocity for both piezoelectric material and steel. Also, these stresses have high values in the case of steel as observed from Fig. 3b.

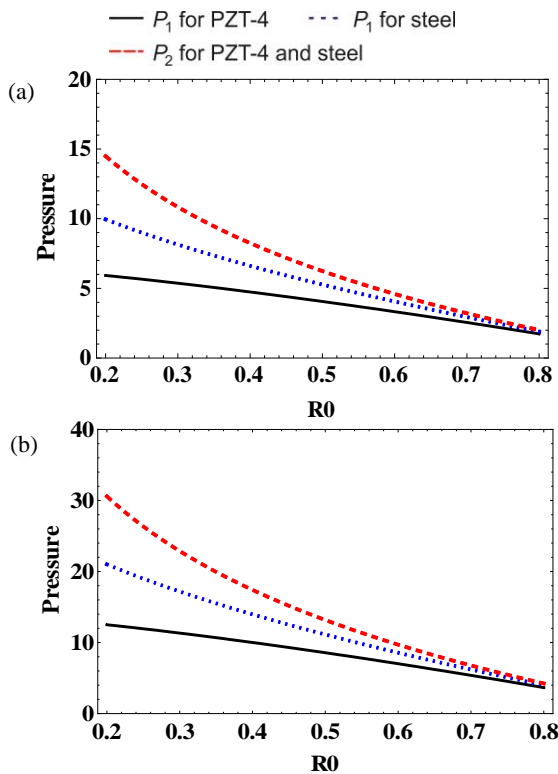


Figure 1. Pressure required for initial yielding and fully plastic state with angular velocities  $\Omega = 10$  and  $20$ , respectively.

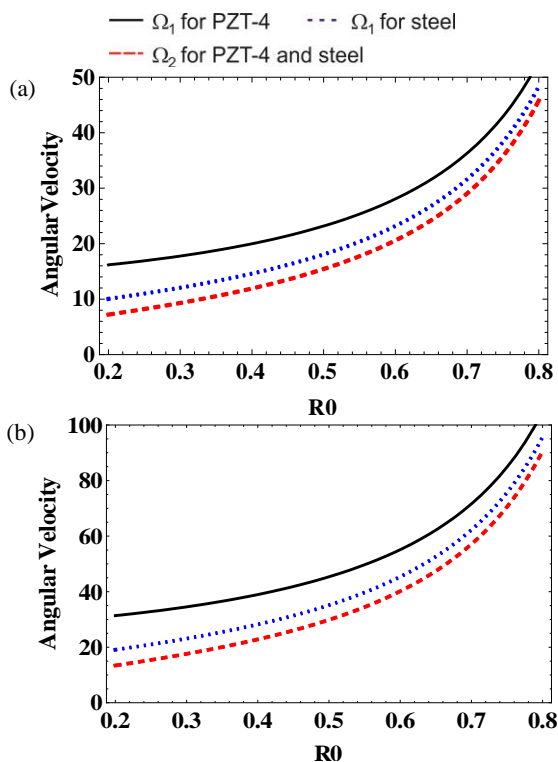


Figure 2. Angular velocity required for initial yielding and fully plastic state with pressures  $P = 10$  and  $20$ , respectively.

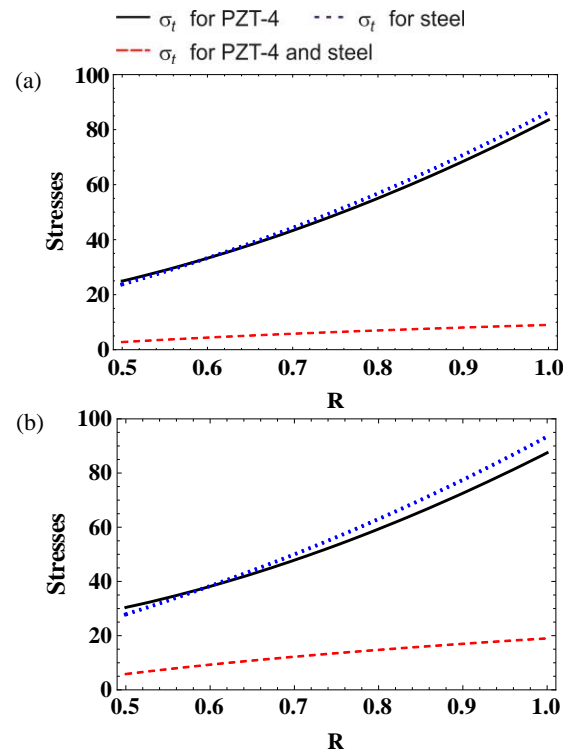


Figure 3. Transitional and fully plastic stresses for piezoelectric and isotropic steel material with angular velocities  $\Omega = 10$  and  $20$ , respectively.

## CONCLUSION

On the basis of all numerical calculations and graphs we come to the conclusion that isotropic material (steel) is a better choice for designing of rotating disc in comparison to the piezoelectric material (PZT-4) as the necessary pressure for initial yielding is lower in case of piezoelectric material.

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