

MATHEMATICAL MODEL OF ENERGY EFFICIENCY IN INTERNAL SPUR GEARS

by

**Predrag S. DOBRATIĆ^{a,*}, Mileta R. RISTIVOJEVIĆ^b, Božidar B. ROSIĆ^b,
Radivoje M. MITROVIĆ^b, and Dragan R. TRIFKOVIĆ^a**

^aMilitary Academy, University of Defence in Belgrade, Belgrade, Serbia

^bFaculty of Mechanical Engineering, University of Belgrade, Belgrade, Serbia

Original scientific paper

<https://doi.org/10.2298/TSCI180118074D>

The impact of geometric parameters of teeth and lubricating oils to the efficiency of involute internal spur gears, when the transverse contact ratio is $2 < \varepsilon_{\alpha} \leq 3$, has been analyzed in this paper. The mathematical model and computer program for determining the current and the effective value of the efficiency have been developed. The influence of the character of load distribution and energy losses due to heating effects during the meshing period is included in the factor of load distribution. The results of computer simulation are given in the form of a diagram of the current values of the efficiency during the meshing period. Also, the values of effective efficiency for the considered cylindrical gear pairs have been calculated.

Key words: *spur gears, mesh teeth, load distribution, energy efficiency, energy losses*

Introduction

Meshing profiles of gears teeth during the contact period is characterized by rolling and sliding, *i. e.* friction that occurs between flanks of teeth. Friction, as a negative occurrence, cannot be completely eliminated, which results in spending a part of power transmitted from the pinion to the gear on overcoming it. The power loss that occurs during the meshing of teeth is shown through efficiency. It is customary to quantify the energy efficiency of gear pairs by analysing this efficiency. Given the increasingly stringent requirements in terms of energy conservation, the efficiency is a very important qualitative and quantitative characteristic in the selection of geometric parameters of gear pairs and appropriate oil for their lubrication.

In order to increase the energy efficiency of the gear pairs in the literature [1-6] adequate theoretical and experimental research have been carried out. In [1] an analytical model of load distribution in simultaneously meshed teeth pairs is developed, when during the contact period the single and double meshed teeth take turns. On the basis of the developed model, the impact of the load distribution on energy efficiency of gear pairs can be analysed. In this literature is shown that load distribution for simultaneously meshed teeth pairs is primarily dependent on the compatibility of accuracy of making of teeth, their rigid-

* Corresponding author, e-mail: dobratipc@yahoo.com

ity and load intensity of gear pairs. For the analysis of energy efficiency of planetary gearboxes the mathematical model for the case of uniform distribution of loads in simultaneously meshed teeth pairs is developed [2], when during the contact period the single and double mesh of teeth take turns. Some researchers focus on the determination of equations used for friction coefficient [7] computation, the rolling friction force and thickness of the oil film [8]. In many studies [9, 10] the influence of oil on the lubrication gear pairs is analysed.

In this paper, a mathematical model for analysing energy efficiency of cylindrical gear pairs with internal teeth is developed, when during the contact period the double and triple mesh of teeth take turns. Based on the developed model for determining efficiency all relevant parameters that have a distinct influence on the efficiency can be identified. The paper analyses the impact of oil and position of the pitch point during the contact period on the energy losses due transformation in thermal energy and on the total efficiency of spur gear pairs with internal teeth. By varying the shape of the teeth profiles [5] the position of the pitch point during the contact period of meshed teeth was being changed.

Load distribution on the simultaneously meshed teeth pairs

For consideration of the load distribution on the simultaneously meshed teeth pairs, load distribution factor [1] is defined, which shows the level of engagement of certain simultaneously meshed teeth pairs in the transmission of the total load of gear pair:

$$K_{ai} = \frac{F_{ni}}{F_n} \quad (1)$$

where F_{ni} [N] is the force transmitted by the considered teeth pair, F_n [N] – the total force of gear pair, and i – the number of simultaneously meshed teeth pairs.

From the aspect of load distribution, two characteristic extreme cases may appear with the simultaneously meshed teeth pairs. The first corresponds to the ideal even distribution of the load and the other a highly uneven distribution of the load, so that the actual distribution of the load is situated between these extreme cases. The actual load distribution depends on the elastic deformation of the simultaneously meshed teeth pairs, shafts and supports, and accuracy of making of teeth. In this load distribution, a set of simultaneously meshed teeth is considered as a statically indeterminate system. From the condition of elastic deformation and variations of the base pitch and the profiles shape of teeth one comes to an analytical expression for the load distribution factor [1].

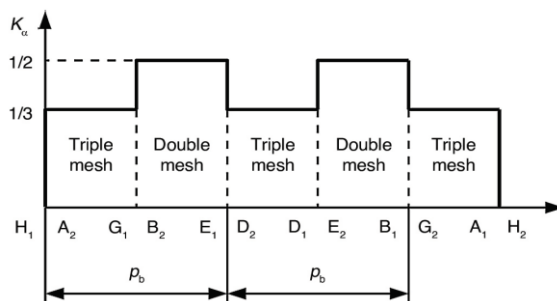


Figure 1. Points of contact at line of action of the profiles meshed teeth pairs in which the load handover is performed

At highly uneven distribution of the load it is characteristic that even when higher number of simultaneously meshed teeth pairs is present, total load is transmitted over a single teeth pair. Load distribution factor then reaches an upper extreme value ($K_{ai} = 1$). In the ideal even load distribution, all simultaneously meshed teeth pairs are equally involved in the transfer of the total load of the gear pair ($K_{ai} = 1/2$ with a double mesh, $K_{ai} = 1/3$ with a triple mesh, and so on), as shown in fig. 1.

Mathematical model for determination of the efficiency

Torque transferring, T_1 , from pinion to gear, is achieved by force by which the flank of the first gear affects the flank of the second gear. Wherein beside the normal force at the point of contact of meshed flanks of teeth, forces of sliding and rolling friction occur. Attacking lines of friction force lie in a common plane of meshed flanks and their directions change when the contact point passes through the pitch point.

Figure 1 shows the points of contact on the action line of the teeth profiles in which handover of loads on the simultaneously meshed teeth pairs are carried out, *i. e.* points of contact in which double and triple mesh of teeth take turns. At the top of the tooth the point of contact A is located. It belongs to the addendum circle gear. Point of contact H is located at the tooth root. It is the last point of the profile tooth which participates in the transfer of the load. Between the points of contact A and H, there are contact points B, D, E, and G, in which the load handover is performed on the simultaneously meshed teeth pairs.

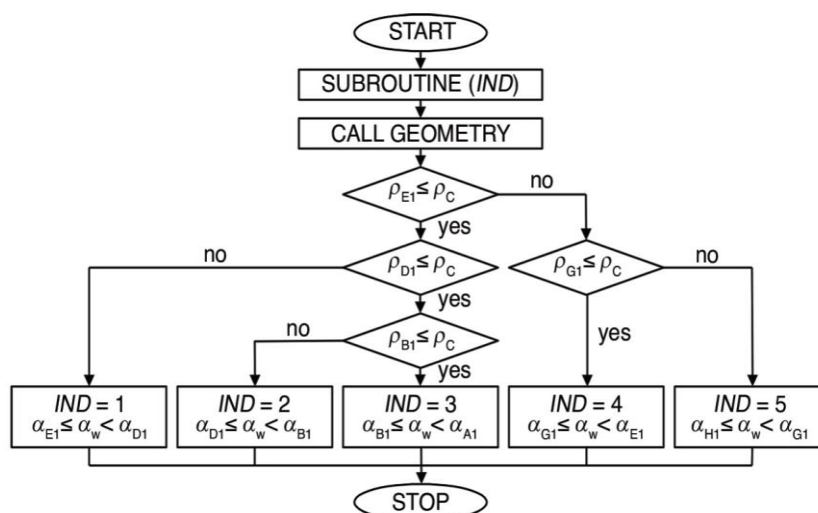


Figure 2. The algorithm for determining the appropriate area of mesh

Depending on the geometric parameters of gear pair, the pitch point may be in the area of double or triple mesh of the considered gear pair. It is therefore necessary to include the variable IND in the mathematical model which defines characteristic points in which the load handover is performed, compared to the pitch point. Figure 2 shows the algorithm for determining the position of the characteristic points of contact in which the load handover is performed. In the presented algorithm, the variable of IND defines appropriate intervals of the load in comparison to the pitch point [2], so:

- $IND = 1$, the pitch point is located in the triple mesh area, wherein the working pressure angle is higher than or equal to the angle in the point E_1 ($\alpha_{E1} \leq \alpha_w$), and the working pressure angle is lower than the angle in the point D_1 ($\alpha_{D1} > \alpha_w$),
- $IND = 2$, the pitch point is located in the double mesh area, wherein the working pressure angle is higher than or equal to the angle in the point D_1 ($\alpha_{D1} \leq \alpha_w$), and the working pressure angle is lower than the angle in the point B_1 ($\alpha_{B1} > \alpha_w$),

- $IND = 3$, the pitch point is located in the triple mesh area, wherein the working pressure angle is higher than or equal to the angle in the point B_1 ($\alpha_{B1} \leq \alpha_w$), and the working pressure angle is lower than the angle in the point A_1 ($\alpha_{A1} > \alpha_w$),
- $IND = 4$, the pitch point is located in the double mesh area, wherein the working pressure angle is higher than or equal to the angle in the point G_1 ($\alpha_{G1} \leq \alpha_w$), and the working pressure angle is lower than the angle in the point E_1 ($\alpha_{E1} > \alpha_w$), and
- $IND = 5$, the pitch point is located in the triple mesh area, wherein the working pressure angle is higher than or equal to the angle in the point H_1 ($\alpha_{H1} \leq \alpha_w$), and the working pressure angle is lower than the angle in the point G_1 ($\alpha_{G1} > \alpha_w$).

To determine the efficiency of gear pairs, the following assumptions have been adopted:

- friction losses in the bearings are small in comparison to the losses due to sliding and rolling friction on the flanks of teeth meshed, which is, given the exclusive application of the rolling bearings with these transmitters, very close to reality,
- rotational speed, *i. e.* angular frequency of meshed gears is constant, and
- torque acting on the working machine during operation of the gear unit power is constant.

The intensity of the sliding friction force, acting on the flanks of teeth meshed during the contact period depends on the normal force and the friction coefficient [6, 7] at the considered point of contact:

$$F_\mu(\xi) = \mu(\xi)F_n \quad (2)$$

$$\mu(\xi) = 0.0127 \log \left(\frac{29.66}{b} \frac{F_n}{\eta v_s(\xi) v_r^2(\xi)} \right) \quad (3)$$

where $\mu(\xi)$ is the friction coefficient, b [m] – the face width of gear pair, η [Pa·s] – the dynamic viscosity of lubricating oil, $v_s(\xi)$ [ms⁻²] – the sliding velocity of tooth profile of the pinion compared to the gear, and $v_r(\xi)$ [ms⁻²] – the rolling velocity of tooth profile of the pinion compared to the gear.

The intensity of the rolling friction force, acting on the teeth flanks of meshed gears during the contact period, at a current point of contact can be displayed depending on the thickness of the oil film [8] separating the flanks of the meshed teeth of the gear pair:

$$F_r(\xi) = Ch(\xi)b \quad (4)$$

$$h(\xi) = 1.6\alpha^{0.6}[\eta v_r(\xi)]^{0.7} E^{0.003} \frac{R^{0.43}}{F_n^{0.13}} \quad (5)$$

where $C = 9 \cdot 10^7$ is the constant of proportionality, $h(\xi)$ [m] – the thickness of the oil film, α [Pa⁻¹] – the pressure viscosity coefficient, R [m] – the equivalent radius of curvature of meshed teeth profiles, and E [Nm⁻²] – the equivalent module of elasticity.

Presuming that parameters of the gear pair are selected so that the pitch point (point C in fig. 3) is in the area of the double mesh, *i. e.* when $IND = 1$. With that in mind, fig. 3 shows the forces acting on the simultaneously meshed teeth profiles in comparison to a fixed co-ordinate system attached to the axis of rotation of gear pair.

At the point of contact the normal force has a constant direction in comparison to the co-ordinate system attached to the axis of rotation of meshed gears, and friction forces are in the tangent plane of meshed flanks of teeth.

The intensity of the normal force, which acts on simultaneously meshed profiles of teeth, is determined from the condition that the sum of torque to the pinion axis O_1 is equal to zero:

$$T_1 - K_{\alpha 1} F_n \frac{3d_{b1}}{2} + F'_{i1} \xi - F'_{r2} p_1 - F'_{r3} p_2 = 0 \quad (6)$$

$$F_n = \frac{T_1 - F_{r1} \xi - F_{r2} p_1 - F_{r3} p_2}{K_{\alpha 1} \frac{3d_{b1}}{2} - \mu_1 K_{\alpha 1} \xi + \mu_2 K_{\alpha 2} p_1 + \mu_3 K_{\alpha 3} p_2} \quad (7)$$

where $\rho_{H1} \leq \xi < \rho_{G1}$ is the area of the triple mesh and p_b [m] – the base pitch:

$$p_1 = \xi + p_b, \quad p_2 = \xi + 2p_b$$

$$F'_{i1} = \mu_1 F_{n1} - F_{r1} = \mu_1 K_{\alpha 1} F_n - F_{r1}, \quad F'_{i1} = \mu_1 F_{n1} + F_{r1} = \mu_1 K_{\alpha 1} F_n + F_{r1}$$

$$F'_{r2} = \mu_2 F_{n2} + F_{r2} = \mu_2 K_{\alpha 2} F_n + F_{r2}, \quad F'_{r2} = \mu_2 F_{n2} - F_{r2} = \mu_2 K_{\alpha 2} F_n - F_{r2}$$

$$F'_{i3} = \mu_3 F_{n3} + F_{r3} = \mu_3 K_{\alpha 3} F_n + F_{r3}, \quad F'_{i3} = \mu_3 F_{n3} - F_{r3} = \mu_3 K_{\alpha 3} F_n - F_{r3}$$

and d_{b1} [m] – the base circle diameter of the pinion.

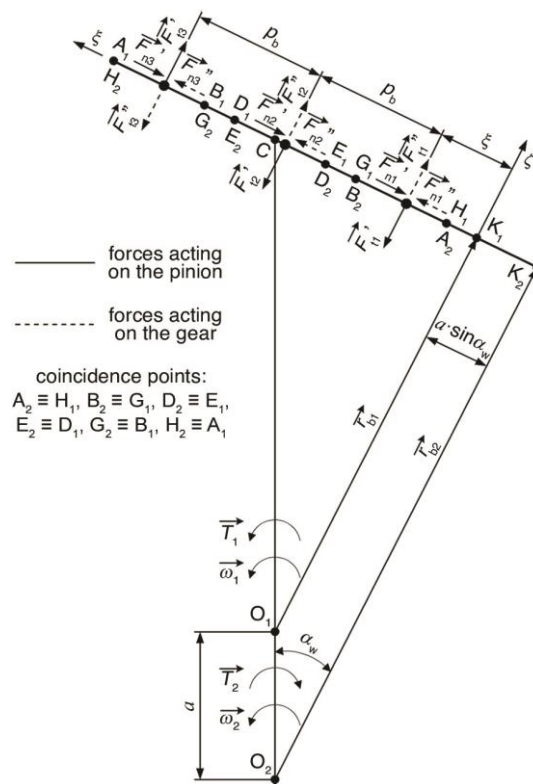


Figure 3. The load acting on the simultaneously meshed teeth profiles for $IND = 1$

The intensity of the normal force cannot be explicitly determined based on the eq. (7), since the coefficient of friction and rolling friction force are the functions of the normal force. Therefore, it is necessary to assume an initial value for the intensity of the normal force which is in the first iteration:

$$F_{np1} = \frac{2T_1}{d_{b1}} \quad (8)$$

Interactive method for determining the intensity of the normal force acting on the flanks of teeth of meshed gears, lasts until the requirement in terms of desired accuracy is met:

$$\left| \frac{F_{n(y+1)} - F_{n(y)}}{F_{n(y)}} \right| \leq \varepsilon \quad (9)$$

where $\varepsilon = 0.01$ is the present accuracy and y is the order number of iteration.

To determine the efficiency for a given gear pair, it is necessary to know the torque, T_2 , which acts on the gear. Starting from the condition that the sum of torque to the axis O_2 of the gear is zero, this torque is:

$$T_2 = K_{\alpha 2} F_n \frac{3d_{b2}}{2} - F_{i1}^* p_3 + F_{i2}^* p_4 + F_{i3}^* p_5 \quad (10)$$

where d_{b2} [m] is the base circle diameter of the gear, a [m] – the centre distance of the gear pair, α_w [°] – the working pressure angle, $p_3 = a \sin \alpha_w + \xi$, $p_4 = a \sin \alpha_w + \xi + p_b$, and $p_5 = a \sin \alpha_w + \xi + 2p_b$.

In the observed point of contact the torques acting on the gear pair is determined. Accordingly, the current values of the efficiency given gear pair in the observed point of contact can be determined:

$$\eta_\xi = \frac{T_2}{T_1} \frac{1}{u} \quad (11)$$

where T_1 [Nm] is the torque acting on the pinion, T_2 [Nm] – the torque acting on the gear, and u – the gear ratio.

In a completely analogous way one determines the normal force and torque acting on the gear in the area of the double mesh $\rho_{G1} \leq \xi < \rho_{E1}$. In doing so, the intensities of the normal force and torque acting on the gear are given by the following expressions:

$$F_n = \frac{T_1 - F_{r1} \xi - F_{r2} p_1}{K_{\alpha 1} d_{b1} - \mu_1 K_{\alpha 1} \xi + \mu_2 K_{\alpha 2} p_1} \quad (12)$$

$$T_2 = K_{\alpha 2} F_n d_{b2} - F_{i1}^* p_3 + F_{i2}^* p_4 \quad (13)$$

In a completely analogous way one determines the normal force and torque acting on the gear in the area of the triple mesh $\rho_{E1} \leq \xi < \rho_C$. It should be noted here that the directions of sliding speed and sliding friction force change when the contact point goes through the pitch point. Therefore, the meshing teeth profiles are first considered in the area before the point of contact passes through the pitch point. In doing so, the intensities of the normal force and torque acting on the gear are given by the following expressions:

$$F_n = \frac{T_1 - F_{r1}p_6 + F_{r2}\xi - F_{r3}p_1}{K_{\alpha 1} \frac{3d_{b1}}{2} - \mu_1 K_{\alpha 1} p_6 - \mu_2 K_{\alpha 2} \xi + \mu_3 K_{\alpha 3} p_1} \quad (14)$$

$$T_2 = K_{\alpha 2} F_n \frac{3d_{b2}}{2} - F_{i1}^* p_7 - F_{i2}^* p_3 + F_{i3}^* p_4 \quad (15)$$

where $p_6 = \xi - p_b$, and $p_7 = a \sin \alpha_w + \xi - p_b$.

If the meshing teeth profiles are examined in the area after the point of contact passes through the pitch point $\rho_C \leq \xi < \rho_{D1}$, the intensities of the normal force and torque acting on the gear are:

$$F_n = \frac{T_1 - F_{r1}p_6 - F_{r2}\xi - F_{r3}p_1}{K_{\alpha 1} \frac{3d_{b1}}{2} - \mu_1 K_{\alpha 1} p_6 + \mu_2 K_{\alpha 2} \xi + \mu_3 K_{\alpha 3} p_1} \quad (16)$$

$$T_2 = K_{\alpha 2} F_n \frac{3d_{b2}}{2} - F_{i1}^* p_7 + F_{i2}^* p_3 + F_{i3}^* p_4 \quad (17)$$

If the meshing teeth profiles is considered after the point of contact is out of the area of the triple mesh and re-entering into the area of the double mesh $\rho_{D1} \leq \xi < \rho_{B1}$, the intensities of the normal force and torque acting on the gear are:

$$F_n = \frac{T_1 - F_{r1}p_6 - F_{r2}\xi}{K_{\alpha 1} d_{b1} - \mu_1 K_{\alpha 1} p_6 + \mu_2 K_{\alpha 2} \xi} \quad (18)$$

$$T_2 = K_{\alpha 2} F_n d_{b2} - F_{i1}^* p_7 + F_{i2}^* p_3 \quad (19)$$

On the other side, when the contact of meshed teeth profiles is out of the area of the double mesh and re-entering into the area of the triple mesh $\rho_{B1} \leq \xi < \rho_{A1}$, these variables are:

$$F_n = \frac{T_1 - F_{r1}p_8 - F_{r2}p_6 - F_{r3}\xi}{K_{\alpha 1} \frac{3d_{b1}}{2} - \mu_1 K_{\alpha 1} p_8 + \mu_2 K_{\alpha 2} p_6 + \mu_3 K_{\alpha 3} \xi} \quad (20)$$

$$T_2 = K_{\alpha 2} F_n \frac{3d_{b2}}{2} - F_{i1}^* p_9 + F_{i2}^* p_7 + F_{i3}^* p_3 \quad (21)$$

where $p_8 = \xi - 2p_b$, and $p_9 = a \sin \alpha_w + \xi - 2p_b$.

Therefore, an effective efficiency for a given gear pair may be determined based on the following relation:

$$\eta_{ef} = \frac{1}{l} \int_{H_1}^{A_1} \eta_{\xi} d\xi = \frac{1}{l} \int_{H_1}^{A_1} \frac{T_2}{T_1} \frac{1}{u} d\xi = \frac{1}{l T_1 u} \int_{H_1}^{A_1} T_2 d\xi \quad (22)$$

where l [m] is the length of action line.

Table 1. Normal force F_n and torque T_2 for $IND = 2$

		$IND = 2$	F_n [N]
			T_2 [Nm]
Mesh area	Triple	$\rho_{H1} \leq \xi < \rho_{G1}$	$\frac{T_1 - F_{r1} \xi - F_{r2} p_1 - F_{r3} p_2}{K_{a1} \frac{3d_{b1}}{2} - \mu_1 K_{a1} \xi + \mu_2 K_{a2} p_1 + \mu_3 K_{a3} p_2}$
			$K_{a2} F_n \frac{3d_{b2}}{2} - F_{t1}'' p_3 + F_{t2}'' p_4 + F_{t3}'' p_5$
	Double	$\rho_{G1} \leq \xi < \rho_{E1}$	$\frac{T_1 - F_{r1} \xi - F_{r2} p_1}{K_{a1} d_{b1} - \mu_1 K_{a1} \xi + \mu_2 K_{a2} p_1}$
			$K_{a2} F_n d_{b2} - F_{t1}'' p_3 + F_{t2}'' p_4$
	Triple	$\rho_{E1} \leq \xi < \rho_{D1}$	$\frac{T_1 - F_{r1} p_6 - F_{r2} \xi - F_{r3} p_1}{K_{a1} \frac{3d_{b1}}{2} - \mu_1 K_{a1} p_6 + \mu_2 K_{a2} \xi + \mu_3 K_{a3} p_1}$
			$K_{a2} F_n \frac{3d_{b2}}{2} - F_{t1}'' p_7 + F_{t2}'' p_3 + F_{t3}'' p_4$
	Double	$\rho_{D1} \leq \xi < \rho_C$	$\frac{T_1 - F_{r1} p_6 + F_{r2} \xi}{K_{a1} d_{b1} - \mu_1 K_{a1} p_6 - \mu_2 K_{a2} \xi}$
			$K_{a2} F_n d_{b2} - F_{t1}'' p_7 - F_{t2}'' p_3$
		$\rho_C \leq \xi < \rho_{B1}$	$\frac{T_1 - F_{r1} p_6 - F_{r2} \xi}{K_{a1} d_{b1} - \mu_1 K_{a1} p_6 + \mu_2 K_{a2} \xi}$
			$K_{a2} F_n d_{b2} - F_{t1}'' p_7 + F_{t2}'' p_3$
	Triple	$\rho_{B1} \leq \xi < \rho_{A1}$	$\frac{T_1 - F_{r1} p_8 - F_{r2} p_6 - F_{r3} \xi}{K_{a1} \frac{3d_{b1}}{2} - \mu_1 K_{a1} p_8 + \mu_2 K_{a2} p_6 + \mu_3 K_{a3} \xi}$
			$K_{a2} F_n \frac{3d_{b2}}{2} - F_{t1}'' p_9 + F_{t2}'' p_7 + F_{t3}'' p_3$

It is necessary to consider the scenario of meshing when the gear pair parameters are chosen so that the pitch point is in the area of double mesh that is between points D_1 and B_1 . In a completely analogous manner, as in the previous case, one can determine the normal force F_n and torque T_2 for $IND = 2$. In tab. 1 are given the final expressions for the normal force F_n and torque T_2 for $IND = 2$.

The next scenario is for case when the gear pair parameters are chosen so that the pitch point is in the area of double mesh that is between points G_1 and E_1 . In a completely analogous manner, as for the two previous cases, one can determine the normal force F_n and torque T_2 for $IND = 4$. In tab. 2 are given the final expressions for the normal force F_n and torque T_2 for $IND = 4$.

Cases $IND = 3$ and $IND = 5$ are theoretically possible, but they are almost unachievable in practice due to the geometry of meshed teeth pairs. Therefore, they are not taken into consideration in this presentation.

Table 2. Normal force F_n and torque T_2 for $IND = 4$

$IND = 4$			F_n [N]	
			T_2 [Nm]	
Mesh area	Triple	$\rho_{H1} \leq \zeta < \rho_{G1}$	$\frac{T_1 - F_{r1} \zeta - F_{r2} p_1 - F_{r3} p_2}{K_{a1} \frac{3d_{b1}}{2} - \mu_1 K_{a1} \zeta + \mu_2 K_{a2} p_1 + \mu_3 K_{a3} p_2}$	
			$K_{a2} F_n \frac{3d_{b2}}{2} - F_{t1}'' p_3 + F_{t2}'' p_4 + F_{t3}'' p_5$	
	Double	$\rho_{G1} \leq \zeta < \rho_C$	$\frac{T_1 - F_{r1} \zeta - F_{r2} p_1}{K_{a1} d_{b1} - \mu_1 K_{a1} \zeta + \mu_2 K_{a2} p_1}$	
			$K_{a2} F_n d_{b2} - F_{t1}'' p_3 + F_{t2}'' p_4$	
		$\rho_C \leq \zeta < \rho_{E1}$	$\frac{T_1 + F_{r1} \zeta - F_{r2} p_1}{K_{a1} d_{b1} + \mu_1 K_{a1} \zeta + \mu_2 K_{a2} p_1}$	
			$K_{a2} F_n d_{b2} + F_{t1}'' p_3 + F_{t2}'' p_4$	
	Triple	$\rho_{E1} \leq \zeta < \rho_{D1}$	$\frac{T_1 - F_{r1} p_6 - F_{r2} \zeta - F_{r3} p_1}{K_{a1} \frac{3d_{b1}}{2} - \mu_1 K_{a1} p_6 + \mu_2 K_{a2} \zeta + \mu_3 K_{a3} p_1}$	
			$K_{a2} F_n \frac{3d_{b2}}{2} - F_{t1}'' p_7 + F_{t2}'' p_3 + F_{t3}'' p_4$	
		Double	$\rho_{D1} \leq \zeta < \rho_{B1}$	$\frac{T_1 - F_{r1} p_6 - F_{r2} \zeta}{K_{a1} d_{b1} - \mu_1 K_{a1} p_6 + \mu_2 K_{a2} \zeta}$
				$K_{a2} F_n d_{b2} - F_{t1}'' p_7 + F_{t2}'' p_3$
	Triple	$\rho_{B1} \leq \zeta < \rho_{A1}$	$\frac{T_1 - F_{r1} p_8 - F_{r2} p_6 - F_{r3} \zeta}{K_{a1} \frac{3d_{b1}}{2} - \mu_1 K_{a1} p_8 + \mu_2 K_{a2} p_6 + \mu_3 K_{a3} \zeta}$	
			$K_{a2} F_n \frac{3d_{b2}}{2} - F_{t1}'' p_9 + F_{t2}'' p_7 + F_{t3}'' p_3$	

Results

Results of research conducted in this paper are shown in diagram form in figs. 4-9. The flow of changes of the current value of efficiency of spur gear pairs during the contact period is shown in these diagrams for two types of oil, tab. 3. All diagrams are developed for internal gear pairs with the number of revolutions of the pinion $n_1 = 1000$ rpm, torque on the pinion $T_1 = 150$ Nm, standard module $m_n = 4$ mm, face width gears $b = 50$ mm, pressure angle $\alpha_0 = 20^\circ$, number of teeth of the pinion $z_1 = 50$, and gear $z_2 = 200$. The shape of the profiles of simultaneously meshed teeth pairs was varied by selecting the appropriate shifting coefficient of teeth profiles [5], tab. 4.

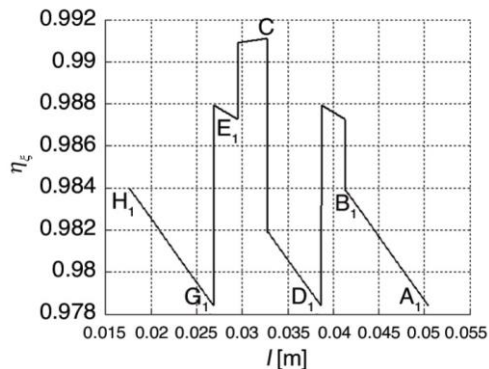
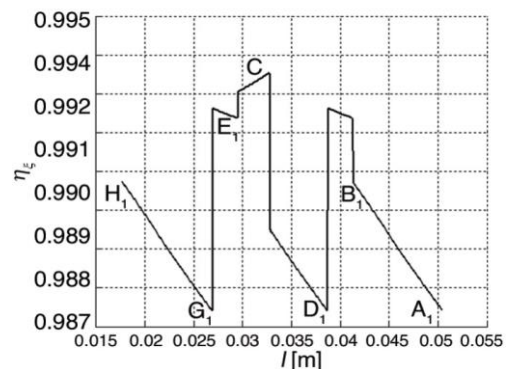
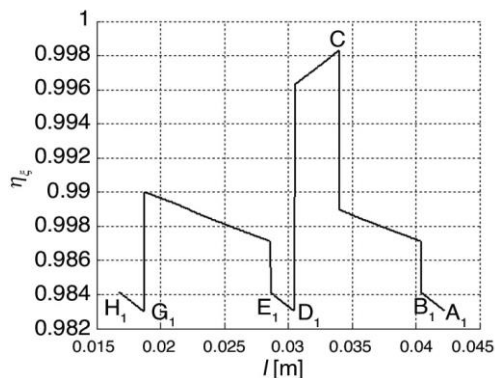
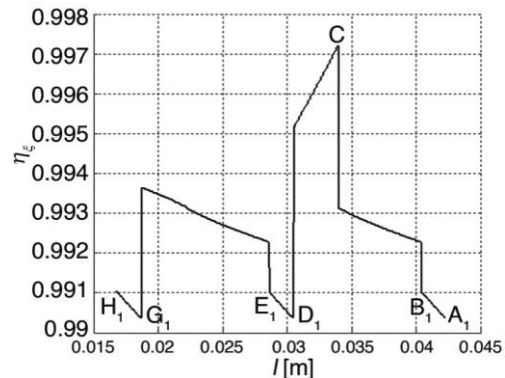
The values of effective efficiency are in the range of 0.9820 to 0.9875 for the first oil [9] (figs. 4, 6, and 8) and in the range of 0.9889 to 0.9918 for the second oil [10] (figs. 5, 7, and 9), tab. 3. The energy efficiency of all shown gear pairs are in the range of 98.20% to 99.18%.

Table 3. Dynamic viscosity, η , pressure viscosity coefficient, α , and ambient temperature, T_0

Oil	η [Pas]	α [Pa ⁻¹]	T_0 [°C]
First	0.075	$2.190 \cdot 10^{-8}$	40
Second	0.036	$1.344 \cdot 10^{-8}$	60

Table 4. Transverse contact ratio, ϵ_α , shifting coefficient of teeth profiles of the pinion, x_1 , and gear, x_2

IND	ϵ_α [-]	x_1 [-]	x_2 [-]
1	2.78	-0.3	0.3
2	2.16	-0.4	-0.3
4	2.53	0	0.5

**Figure 4. Diagram of change of the current value of the efficiency for $IND = 1$ and first oil ($\eta_{ef}^f = 0.9820$)****Figure 5. Diagram of change of the current value of the efficiency for $IND = 1$ and second oil ($\eta_{ef}^s = 0.9889$)****Figure 6. Diagram of change of the current value of the efficiency for $IND = 2$ and first oil ($\eta_{ef}^f = 0.9875$)****Figure 7. Diagram of change of the current value of the efficiency for $IND = 2$ and second oil ($\eta_{ef}^s = 0.9918$)**

By comparing the diagrams and results of these six combinations, figs. 4-9, it follows that the highest values of the current and effective efficiency are achieved in gear pair in the case when is $IND = 2$, tab. 4. In this gear pair, lubricated with second oil, tab. 3, the pitch point is located in the area of the double mesh between points D_1 and B_1 . The energy efficiency of this combination of gear pair and oil is 99.18% due to the low transverse contact ratio this of gear pair, as it shown also in literature [10], as well as characteristics of the considered oil. Bearing in mind the fact that the shear stress between the oil layers is proportional to the oil viscosity, higher efficiency were obtained using the second oil in all three variants of gear pairs.

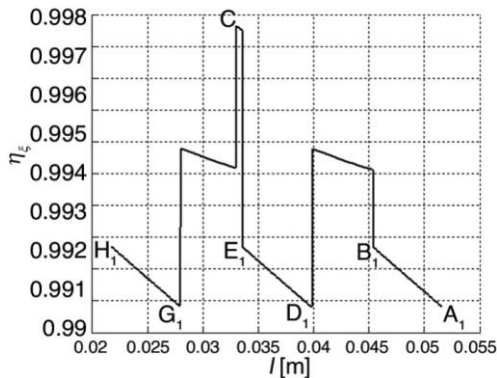


Figure 8. Diagram of change of the current value of the efficiency for $IND = 4$ and first oil ($\eta_{ef}^f = 0.9821$)

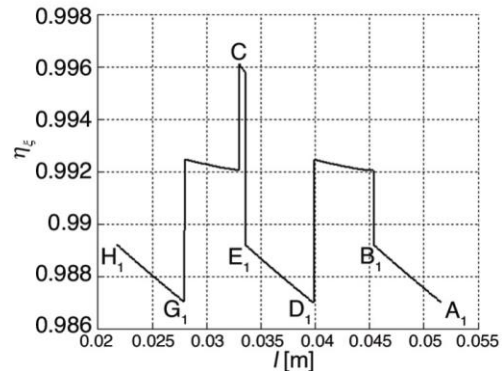


Figure 9. Diagram of change of the current value of the efficiency for $IND = 4$ and second oil ($\eta_{ef}^s = 0.9893$)

The research of influence of geometric and kinematic values of the gear pairs and the oil viscosity on energy losses ($\Delta\eta_\xi = 1 - \eta_\xi$) are shown in the diagrams in fig. 10. The diagrams show the ratio between the average values of energy losses due transformation in thermal energy in the area of double and triple mesh for considered oils (first-f and second-s) and gear pairs, tabs. 3 and 4. Energy losses appear as result of overcoming the resistance that occurs in the oil as well as due to friction on the flanks of the simultaneously meshed teeth pairs.

Based on obtained results, fig. 10, it is shown that the lowest difference in energy losses is generated in the area of meshing of the teeth flanks where the rolling friction is dominant (e. g. 33% for $IND = 2$). In this area is located the pitch point, i. e. the point in which the sliding velocity of teeth flanks is zero. The largest difference in energy losses is found in the areas of the triple mesh (e. g. 78% for $IND = 2$), when pitch point is not located in these areas. Contrary to the areas of the triple mesh, in the areas of the double mesh lower difference in energy losses are obtained (e. g. 63% for $IND = 2$) because of smaller number of contact surfaces.

Simultaneous influence of teeth geometry, load, oil viscosity, sliding and rolling velocity on the energy efficiency of cylindrical gear pairs with straight teeth is shown in the diagrams in figs. 11-13. The two previously indicated oils were considered [9, 10], tab. 3. The teeth geometry is changed by selecting appropriate shifting coefficient of teeth profiles [5], tab. 4. Torque on the pinion, T_1 , varied in the interval 50-250 Nm. The impact of sliding and rolling velocity was considered through the change of the number of revolutions of the pinion, n_1 , in the interval 500-1500 rpm.

Based on the obtained results shown in diagrams in figs. 11-13 it can be concluded that when oil has lower dynamic viscosity and pressure viscosity coefficient the considered gear pairs have higher energy efficiency (from minimum of 0.14% for $IND = 2$, $T_1 = 250$ Nm,

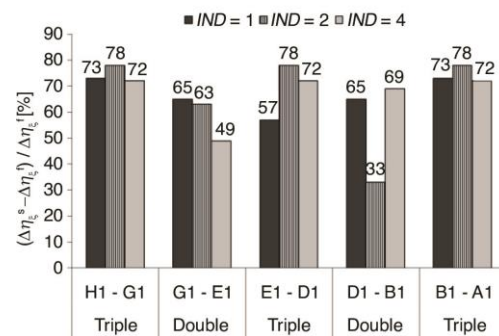


Figure 10. The ratio of average values of energy losses due transformation in thermal energy per mesh area for considered oils and gear pairs

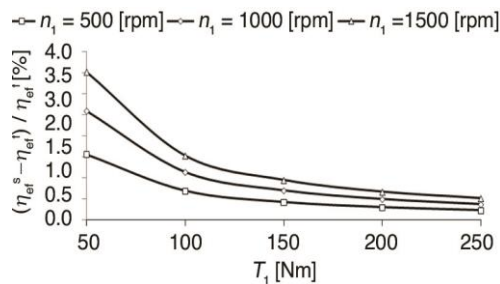


Figure 11. The ratio of effective efficiency for considered oils and $IND = 1$

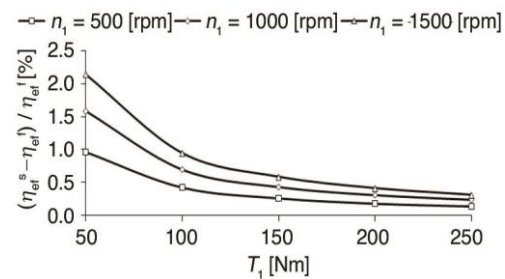


Figure 12. The ratio of effective efficiency for considered oils and $IND = 2$

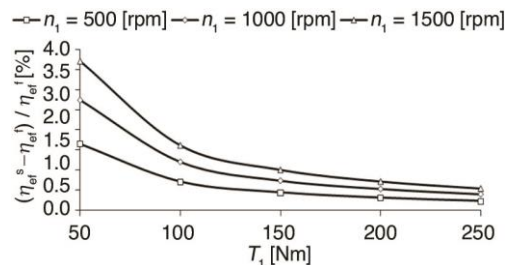


Figure 13. The ratio of effective efficiency for considered oils and $IND = 4$

and $n_1 = 500$ rpm to maximum of 3.71% for $IND = 4$, $T_1 = 50$ Nm, and $n_1 = 1500$ rpm, figs. 12 and 13) compared to the oil which has higher dynamic viscosity and pressure viscosity coefficient in the considered range of load and number of revolutions. Thereby, with the increase of torque this difference in the energy efficiency for the considered oils decreases (e. g. from 3.51% to 0.52% for $IND = 1$ and $n_1 = 1500$ rpm, fig. 11) and it decreases much more in the area of the high intensity of the load.

However, with higher number of revolutions this difference in the energy efficiency for the considered oils increases (e. g. from 1.65% to 3.71% for $IND = 4$ and $T_1 = 50$ Nm, fig. 13). From the aspect of teeth geometry the best energy efficiency is achieved in gear pair for the case when $IND = 2$, because it has the lowest difference in the energy efficiency for the considered oils (from minimum of 0.14% for $T_1 = 250$ Nm and $n_1 = 500$ rpm to maximum of 2.15% for $T_1 = 50$ Nm and $n_1 = 1500$ rpm, fig. 12).

Conclusions

The graphic presentation of the results obtained shows that changes of the efficiency follow the character of changes of load distribution on the simultaneously meshed teeth pairs. Based on the received results it follows that the current value of the efficiency is maximal in the pitch point. Higher values of the efficiency can be found in the area of the double mesh, and lower values in the area of the triple mesh. Energy losses caused by the rolling friction are much lower than the energy losses caused by sliding friction.

The paper shows that when oil has lower dynamic viscosity and pressure viscosity coefficient the considered gear pairs have higher energy efficiency compared to the oil which has higher dynamic viscosity and pressure viscosity coefficient in the considered range of load and number of revolutions. With the increase of torque this difference in the energy efficiency of the considered oils decreases, and with higher number of revolutions this difference increases.

While energy losses due its transformation in thermal energy of single gear pair are quite small, energy efficiency of gearbox that consists of few gear pairs may be significant. The total energy efficiency of gearbox formed of few gear pairs is product of energy efficiency of every single gear pair. Accordingly, any improvement in energy efficiency that can be achieved in each gear pair significantly contribute to energy efficiency of entire gearbox.

The developed mathematical model in this paper and the results of research obtained can be used in designing energy efficient of gearboxes. Also, it can be used for further research in order to form more complex models to analyse the energy efficiency of gearboxes.

References

- [1] Ristivojević, M. R., Mitrović, R. M., Load Distribution - Gear Pairs and Roller Bearings (in Serbian), Faculty of Mechanical Engineering, University of Belgrade, Belgrade, Serbia, 2002
- [2] Rosić, B. B., Planetary Gears - Cylindrical Internal Pairs (in Serbian), Faculty of Mechanical Engineering, University of Belgrade, Belgrade, Serbia, 2003
- [3] Blagojević, I. A., *et al.*, Energy Efficiency Improvement by Gear Shifting Optimization, *Thermal Science*, 17 (2013), 1, pp. 91-105
- [4] Blagojević, I. A., *et al.*, The Effects of Gear Shift Indicator Usage on Fuel Efficiency of a Motor Vehicle, *Thermal Science*, 21 (2017), 1B, pp. 707-713
- [5] Baglioni S., *et al.*, Influence of the Addendum Modification on Spur Gear Efficiency, *Mechanism and Machine Theory*, 49 (2012), Mar., pp. 216-233
- [6] Anderson, N. E., Loewenthal, S. H., Efficiency of Nonstandard and High Contact Ratio Involute Spur Gears, *Journal of Mechanisms, Transmissions, and Automation in Design*, 108 (1986), 1, pp. 119-126
- [7] Benedict, G. H., Kelly, B. W., Instantaneous Coefficients of Gear Tooth Friction, *ASLE Transactions*, 4 (1961), 1, pp. 59-70
- [8] Dowson, D., Higginson, G. R., *Elastohydrodynamic Lubrication*, SI ed., Pergamon Press Ltd., Oxford, UK, 1977
- [9] Wang, Y., *et al.*, Transient Thermoelastohydrodynamic Lubrication Analysis of an Involute Spur Gear, *Tribology International*, 37 (2004), 10, pp. 773-782
- [10] Kahraman, A., *et al.*, Development of a Generalized Mechanical Efficiency Prediction Methodology for Gear Pairs, Department of Mechanical Engineering, The Ohio State University, Columbus, O., USA, 2005