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NUMERICAL ASSESSMENT OF STRESS INTENSITY FACTORS AT TIPS OF MULTI-SITE CRACKS IN UNSTIFFENED PANEL

NUMERIČKO ODREĐIVANJE FAKTORA INTENZITETA NAPONA NA VRHU VIŠESTRUKIH PRSLINA U NEOJAČANIM PANELIMA

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- unstiffened panel
- fatigue crack growth

Ključne reči

- višestruke prsline
- neojačani panel
- zamorni rast prsline

Abstract

The aim of this paper is to establish and demonstrate diverse capacity and performances of different numerical methods for calculating stress intensity factors' (SIFs) histories versus crack length for problems involving multiple, interacting cracks resulting from multiple site damage (MSD). A typical aero structural configuration is analysed: unstiffened flat panel made of aluminium Al 2024-T3, containing 11 holes, each of which is a site for crack growth. Analysed model makes a complex configuration with 22 cracks, propagating at the same time. The computations are carried out in FRANC2D software which is FEM based and with superposition based approximate method. The comparison of the results has shown that obtained solutions can be used for SIF predictions with acceptable accuracy.

Izvod

Cilj ovog rada je da se ustanove i prikažu raznovrsni kapaciteti i performanse različitih numeričkih metoda za proračun istorije faktora intenziteta napona (FIN) u zavisnosti od dužine prsline u problemima vezanim za višestruke, povezane prsline koje su nastale kao posledica oštećenja na više mesta (MSD). Tipična konfiguracija konstrukcije letelica je analizirana: neukrućeni ravni panel napravljen od legure Al 2024-T3, sa 11 rupa, od kojih je svaka predstavljala mesto rasta prsline. Analizirani model se sastoji od složene konfiguracije sa 22 prsline, koje se istovremeno šire. Proračun je urađen u FRANC2D softveru, koji se zasniva na MKE i uz pomoć aproksimativne metode superpozicije. Poređenje rezultata ovih metoda je pokazalo da se dobijena rešenja mogu koristiti za dovoljno precizno predviđanje FIN.

INTRODUCTION

Multiple site damage (MSD) is a typical problem for ageing aircraft, which is characterized by the interaction of a major crack with several short cracks located at various sites of the same structural element. It often occurs in longitudinal and circumferential riveted lap joints, starting when the fuselage pressure cycling fatigue loads lead to crack initiation and propagation at multiple rivet locations. MSD cracks may interact and sudden crack link-up may occur, reducing the overall structural integrity of the structure, /1/. Prediction of the crack growth rate and residual strength of the structure with MSD require an accurate calculation of stress intensity factors (SIFs).

Many studies and analyses of this phenomenon have been carried out in recent decades. Overviews on analytical methods for MSD are provided in many scientific papers, such as /2/.

Still, the mutual influence of the adjacent cracks additionally increases the complexity of stress intensity factor determination. This is the main reason for introducing approximation methods and procedures that enable faster and simpler determination of SIFs in supporting aircraft structures with multiple cracks.

Kastratović et al. proposed an approximation method for the determination of stress intensity factors in case of multiple site damage based on existing solution for SIF in case of two unequal cracks in infinite plate subjected to remote uniform stress, /3/. Kastratović et al. had published a research on a finite element calculation of stress intensity factors in structures with MSD using approximate procedure. The results are compared against values obtained by FRANC2D software with acceptable accuracy, /4/.

So, depending on the analysed geometry, as well as the available time and resources, FEM along with approximate methods must be employed in order to provide fairly accurate solutions for SIFs, thus eliminating the need for expensive tests (which is why the experimental results reported in literature are very limited). The aim of this paper is to show how SIF calculations for structures with multiple cracks can be performed with the use of FEM and approximate numerical method based on the principle of superposition.

In this study, the stress intensity factor versus crack length histories are calculated for a typical aero structural configuration: unstiffened flat panel made of aluminium Al 2024-T3, containing 11 holes, each of which is a site for crack growth. The analysed model makes a complex configuration with 22 cracks, propagating at the same time, whereas stress intensity factors are computed along the crack front for each of the 22 cracks. The computations are carried out with FRANC2D software which is FEM based and with an approximate method, /3/.

In this research a FE model of a flat, unstiffened panel (dimensions $L_1 \times L_2 = 609.6 \times 863.6$ mm, thickness = 1.6 mm, Fig. 1) with 11 fastener holes (radii $r = 3.23$ mm at distances $b = 25.4$ mm) subjected to uniform uni-axial tensile stress of 200 MPa is created to examine effects of MSD on fatigue crack growth rates.

Typical material properties for 2024-T3 aluminium – Young's modulus of 73000 MPa, Poisson's ratio of 0.33, are used in the analyses. Uniform tensile stresses are applied at the top and bottom edges of the model. Each hole in the panel had two radial cracks, numbered from 1 to 22 in Fig.1.

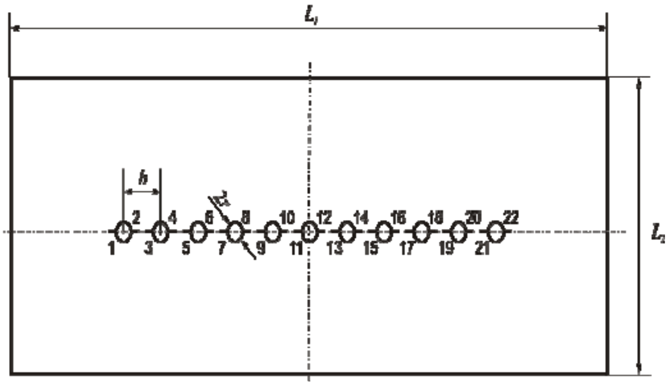

Figure 1. Analysed configuration with multiple cracks.

SIFs calculation for multiple cracks using 2D FEM

The SIFs for analysed configuration are also calculated in FRANC2D/L software, /5/, which has the crack growth simulation capability. FRANC2D/L calculates SIF values at the tip of 2D cracks using the J-integral method, determines the appropriate crack growth increment, extends the crack, remeshes around new crack tip, and then performs a next solution step with the new FE mesh. This procedure is performed 15 times in order to simulate incremental crack growth. The elements used in this template are quarter-point singular elements.

Approximate method for SIFs calculation in case of multiple cracks

The SIFs for the analysed configuration are calculated as well by approximate method presented in /3/. This approximate procedure for calculating stress intensity factors is based on the principle of superposition. According to this procedure, the SIF for opening mode of analysed crack in any given configuration with n cracks (Fig. 2) can be estimated as:

$$K_{IjA,B} = c_{1b,d} \cdot K_{I1} + \cdots + c_{jb,d} \cdot K_{Ij} + \cdots + c_{nb,d} \cdot K_{In} = \sum_{i=1}^{n} c_{ib,d} \cdot K_{Ii} \tag{1}$$

where: $K_{IjA,B}$ - represents stress intensity factor for tip A, or B of the analysed crack in presence of all other cracks in the configuration; K_{It} - individual stress intensity factor of all cracks in the configuration, i.e., stress intensity factors of auxiliary configurations; $c_{ib,d}$ - the coefficient that takes into consideration influence of i-th crack on stress intensity factor of analysed crack, (the influential coefficient of the analysed crack on itself is $c_{jb,d} = 1$).

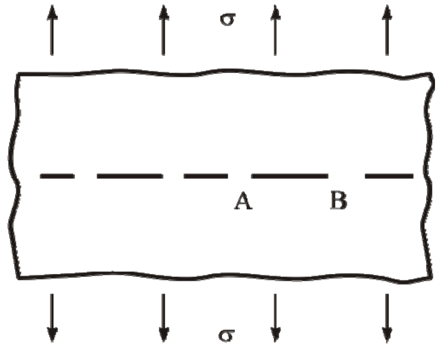

Figure 2. Configuration with multiple cracks.

In this manner the analysed complex 2D or 3D configuration can be represented as a combination of several simpler (auxiliary) configurations. The number of those configurations is equal to the number of cracks, such that every auxiliary configuration usually contains only one crack. Hence, the determination of SIF at analysed crack tip is reduced to determination of influence that every crack in the initial configuration has on the analysed one. This influence is here, as it is shown, represented with corresponding coefficients. These coefficients are estimated based on the configuration with two unequal cracks in an infinite plate subjected to remote uniform stress, with detailed calculation presented in /3/, which gives:

$$c_{ib} = (\beta_{jB} - 1) \cdot \sqrt{\frac{a_j}{a_i}} \tag{2}$$

$$c_{id} = (\beta_{jA} - 1) \cdot \sqrt{\frac{a_j}{a_i}} \tag{3}$$

where: β_{jB} - geometry factor of analysed crack for tip B, in the case when only the analysed and influencing cracks are present in the configuration (this geometry factor is known and can be calculated with equations in /6/); β_{jA} - geometry factor of analysed crack for tip A, in the case when only analysed and influencing cracks are present in the configuration (this geometry factor is known and can be calculated with equations presented in /6/); a_j - half length of the analysed crack; a_i - half length of the influential crack.

The influential coefficients are estimated for a vast number of crack lengths and distances between them, i.e. for their combinations, in /7/. Here, eleven auxiliary configurations are used, which are all the same: thin plate with central circular hole with two radial cracks subjected to uniform uniaxial tensile stress, with well-known Bowie's solution for stress intensity factor /8/.

It should be noted that the SIFs are calculated for models with different crack sizes for all the cracks in the configuration, but with same crack increment for all cracks, because the service data showed that in MSD all cracks are roughly the same length, /9/, (so called 'catch-up' phenomenon).

Analysis of results

The results obtained using classical FEM (FRANC2D/L software) are compared to the previously described approximate method. Results are presented through normalised stress intensity factors (geometry factor $\beta = \dfrac{K_I}{\sigma\sqrt{\pi a}}$) for 6 selected cracks, and are shown in Fig. 3. Factor β is a function of the stress intensity factor for mode I (K_I), stress σ and crack length a.

These six cracks are selected because of their unique positions. That is, at these positions the influences of adjacent cracks' interaction are either minimal (at the first and the eleventh hole; cracks 1, 2, 21 and 22 respectively) or maximal (the fifth hole; cracks 11 and 12).

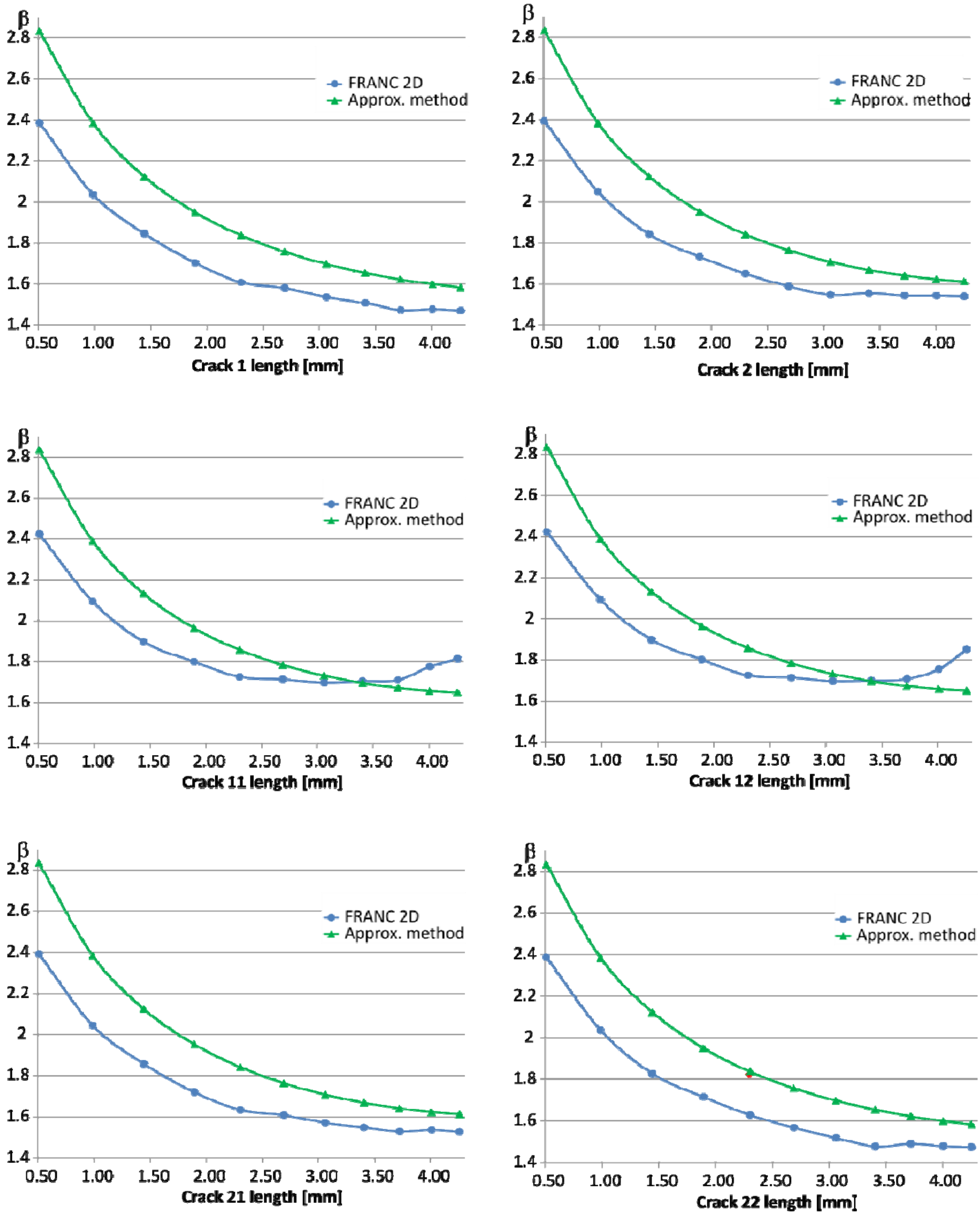

Figure 3. Comparison of SIF values obtained by three different methods (σ = 200 MPa).

Comparing the results obtained by approximate procedure and FEM (FRANC2D/L) it can be seen that SIFs histories for selected cracks differ up to 16.7% for all cracks. The maximal differences occur for initial crack size, due to the proximity of the hole boundary. For the cracks on the first two holes, the difference between approximate procedure and FEM results decreases with crack growth to 5.9% (crack 1), i.e. 4.0% (crack 2). The cracks 21 and 22 at the eleventh hole exhibit a similar trend. For cracks at the fifth hole, this difference starts to decrease with crack growth to 0.6% (crack 11), i.e. 1.1% (crack 12), but with further crack growth, it increases up to 6.3% (crack 11), i.e. 8.2% (crack 12). In all of these cases, the approximate method gives slightly higher results then FEM, except for larger crack sizes (greater than 3 mm) for central cracks (crack 11 and 12) This can be explained by the fact that the approximation method defines the influential coefficients that take into consideration cracks' interaction effect for every crack in the analysed structure, which generally increases the stress intensity factor.

CONCLUSION

As technology and computer sciences develop and become more available, the researchers are trying to introduce and apply new computational methods and techniques in order to find SIFs solutions for complex geometries with multiple cracks. This study represents an effort in that direction. Here, the SIFs 2D computations are carried out in FRANC2D software which is a FEM based, and with the approximate method which is based on superposition. These calculations are conducted for a typical aero structural configuration with MSD. The analysed model is a complex configuration with 22 cracks that propagate at the same time, whereas the stress intensity factors are computed along the crack fronts for all 22 cracks.

Analysis of the results has shown that all the presented methods can provide stress intensity factors of the analysed configuration with an acceptable accuracy.

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