SERBIAN JOURNAL OF ELECTRICAL ENGINEERING
Vol. 14, No. 1, February 2017, 161-176

UDC004.78:621.38]:681.586:621.317 **DOI: 10.2298/SJEE1701161R**

Optimal Source Localization Problem Based on TOA Measurements

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Abstract: Determining an optimal emitting source location based on the time of arrival (TOA) measurements is one of the important problems in Wireless Sensor Networks (WSNs). The nonlinear least-squares (NLS) estimation technique is employed to obtain the location of an emitting source. This optimization problem has been formulated by the minimization of the sum of squared residuals between estimated and measured data as the objective function. This paper presents a hybridization of Genetic Algorithm (GA) for the determination of the global optimum solution with the local search Newton-Raphson (NR) method. The corresponding Cramer-Rao lower bound (CRLB) on the localization errors is derived, which gives a lower bound on the variance of any unbiased estimator. Simulation results under different signal-to-noise-ratio (SNR) conditions show that the proposed hybrid Genetic Algorithm–Newton-Raphson (GA–NR) improves the accuracy and efficiency of the optimal solution compared to the regular GA.

Keywords: Genetic Algorithm, Localization, Signal-to-noise ratio, Time of Arrival, Wireless Sensor Networks.

1 Introduction

Localization has attracted a significant amount of attention in many applications such as telecommunications, navigation, target tracking, environmental monitoring and biomedical health monitoring [1 – 2].

The localization algorithms use various techniques such as the time of arrival (TOA), the time difference of arrival (TDOA), the received signal strength (RSS), or the angle of arrival (AOA). Between them, the time-based techniques are preferred because of their high ranging accuracy and relatively simple hardware structure. The focus of the present paper is an emitting source localization problem, where the distance between the emitting source and the set of sensors is generally estimated by the TOA measurements. This method requires synchronization between the emitting source and all sensors in the measurements and high precision timing. The Gaussian noise is widely used in

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the most localization algorithms and it is a good assumption for line-of-sight (LOS) scenarios.

The least squares (LS) and the maximum likelihood (ML) are powerful methods which can be employed successfully to estimate the location of the emitting source. In practical implementation, due to the TOA measurement errors, the localization problem is formulated as an optimization problem known as the least squares that minimizes the sum of squared residuals between the estimated and the measured distances. There are two categories of the LS problems, closed-form linear least squares and nonlinear least squares. The problem of finding the nonlinear least squares estimates in the literature is known as the unconstrained nonlinear optimization problem [3]. The gradient-based unconstrained optimization methods such as, the Newton-Raphson, the Gauss-Newton, and the Levenberg-Marquardt methods are very effective in the process of finding the optimal solution to a given NLS optimization problem. Between them, the Newton-Raphson iterative method is widely used to estimate parameters with high localization accuracy [4]. However, the results from nonlinear optimization methods are largely dependent on the initial values.

The LLS technique is widely used in TOA localization approach because of its computational efficiency, therefore it can be used to obtain an initial position for estimation algorithms, such as the NLS approach. Thus, the NLS localization problem is transformed into the LLS problem [5 – 6].

The GA is one of the most powerful evolutionary methods in the applications to many optimization problems. However, numerical results show that the GA has a slow convergence speed and the premature convergence of the solution. Therefore, in order to improve the performance of the regular GA, various hybridizations of GA with conventional local search methods have been proposed for this problem [7].

The procedure of the hybrid GA is composed of two phases: the exploration of the search space, and the solution improvement (local search) phase. The first phase of the search process starts with the GA by randomly generating initial population by exploring the search space and finding the global or near global optimum solution of a given objective function. In this case, the GA is characterized by a slow convergence rate at the later stage of the search process. In the second phase, a local search algorithm is used to find the absolute optimum by exploiting the neighborhood of the initial point determined by the GA in order to accelerate the convergence and improve the accuracy of the solution obtained by the GA algorithm. The gradient based Newton-Raphson method has been incorporated into the hybrid algorithm, which results in improvement of the global optimal solution obtained from the GA algorithm. Therefore, the hybrid GA–NR method has been proposed in this sense.

The Cramer–Rao lower bound provides a lower bound on the variance of any unbiased estimator [8] and it is a very useful tool for evaluating the localization accuracy. The root mean square error (RMSE) performance of the proposed TOA based estimators is compared to the CRLB and presented in this paper.

The rest of the paper is organized as follows. Section 2 considers the emitting source localization problem in WSNs based on the TOA measurements. Section 3 describes emitting source localization problem which is modeled as a least squares estimation problem with NLS method. Section 4 introduces the proposed hybrid GA–NR algorithm, along with the GA as the evolutionary computation global search optimization technique and the NR local search technique. Section 5 provides the CRLB for analysis of TOA measurement data. Section 6 gives the simulation results and the performance analysis of the proposed optimization methods for different signal-to-noise ratio levels. Finally, the conclusion and the future work are drawn in Section 7.

2 Problem Formulation

Determining the location of emitting source is a challenging problem for the most WSN applications. In this section, the two-dimensional (2-D) emitting source localization problem is considered using the TOA measurements.

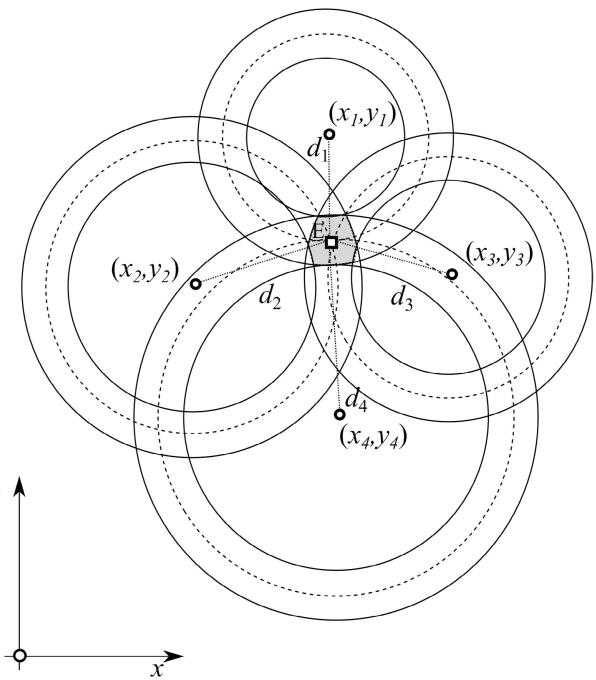


Fig. 1 – *Geometrical model based on TOA measurements.*

The TOA localization method finds the location of an emitting source by using the TOA measurements from each sensor. This localization method requires at least three sensors at known positions $\boldsymbol{x}_l = [x_l, y_l]^{\mathrm{T}}$, $l \in \{1,2,...,N\}$, where $[\cdot]^{\mathrm{T}}$ denotes matrix transpose operation, as shown in Fig. 1.

This paper assumes that the distance measurement errors $\{n_l\}$ are independent Gaussian random variables with zero mean and known variance σ_l^2, i.e., $\mathcal{N}(0,\sigma_l^2)$ and it is a good assumption for LOS conditions. In practical situations where noise exist, three or more than three circles do not intersect at the same point. These circles form a small region, which can be regarded as the size of the time measurement errors. Then, the distances, $\{r_l\}$ between the emitting source at $\boldsymbol{x} = [x, y]^{\mathrm{T}}$ and the each sensor at $\boldsymbol{x}_l = [x_l, y_l]^{\mathrm{T}}$, $l \in \{1,2,...,N\}$, based on the TOA measurements, in the presence of noise, are modeled as follows

$$r_l = c \cdot t_l = d_l + n_l, l \in \{1,2,...,N\}, \tag{1}$$

where c is the speed of light, t_l is the TOA of the signal to the l-th sensor, and d_l is the distance between the true emitting source location and l-th sensor, given by

$$d_l = \|\boldsymbol{x} - \boldsymbol{x}_l\|_2 = \sqrt{(x - x_l)^2 + (y - y_l)^2}, \quad l \in \{1,2,...,N\}, \tag{2}$$

where $\|\cdot\|_2$ stands for the 2-norm.

Equation (1) can be written in a vector form as

$$\boldsymbol{r} = \boldsymbol{d}(\boldsymbol{x}) + \boldsymbol{n}, \tag{3}$$

where $\boldsymbol{d}(\boldsymbol{x}) = [d_1,...,d_N]^{\mathrm{T}}$ is the vector of distances between the true emitting source location and each of the sensor. The multilateration localization techniques, in the absence of measurement errors, determine the location of an emitter source E from the intersection of a set of circles, with radii $\{d_l\}$ and centers, $[x_l, y_l]^T$, $l \in \{1,2,...,N\}$, as shown in Fig. 1.

3 Least Squares Methods

In this section, we use the NLS method to estimate the unknown location of the emitting source by minimizing the sum of the squares residuals, $R_{es,l}$, between the estimated and the measured distances, in the least square sense. The main goal of the localization problem based on the TOA measurements is to estimate $\boldsymbol{x} = [x, y]^T$. Therefore, the estimation of the emitting source location can be formulated as a problem of minimizing the objective function $J_{NLS}(\boldsymbol{x})$ which is defined as the sum of the squared residuals, given as

$$\min_{\mathbf{x}\in R^2} J_{NLS}(\boldsymbol{x}) = \min_{\mathbf{x}\in R^2} \sum_{l=1}^{N} R_{es,l}^2(\boldsymbol{x}). \tag{4}$$

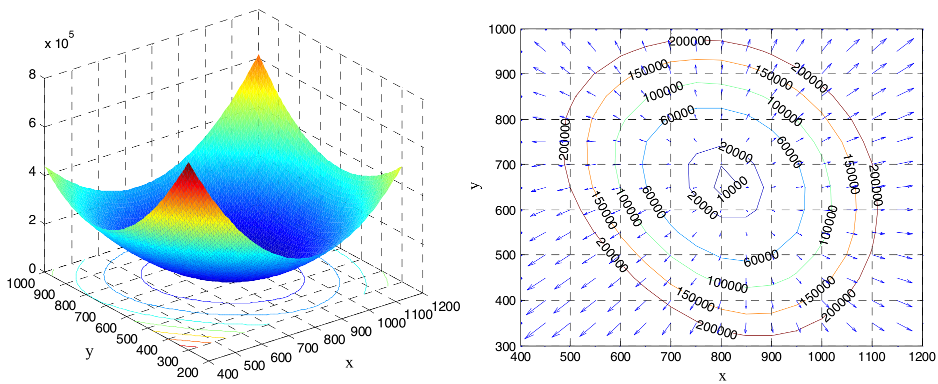
The residual $R_{es,l}(\boldsymbol{x})$ is defined as the difference between the measured distance r_l at l-th sensor, and the estimated distance obtained from the Euclidean distance between the position of l-th sensor and estimated location of an emitting source, $\|\boldsymbol{x}_l - \boldsymbol{x}\|$. Thus, the residual is expressed as

$$R_{es,l}(\boldsymbol{x}) = r_l - \sqrt{(x - x_l)^2 + (y - y_l)^2}. \tag{5}$$

The optimal emitting source location for the proposed nonlinear optimization problem can be formulated as

$$\boldsymbol{x}^* = \underset{\mathbf{x}\in R^2}{\operatorname{argmin}} J_{NLS}(\boldsymbol{x}). \tag{6}$$

Figs. 2a and 2b show the objective function $J_{NLS}(\boldsymbol{x})$ of the NLS optimization problem and corresponding 2D contour plot, with the gradient vectors of the objective function, respectively.



(a) (b)

Fig. 2 – (a) *Graph of objective function* $J_{NLS}(\boldsymbol{x})$;
(b) *Contour with gradient vectors of the objective function.*

Notice that $J_{NLS}(\boldsymbol{x})$ is a convex function with a single unique minimum, as shown in Fig. 2. This problem can be successfully solved by iterative numerical gradient search algorithms such as the Newton–Raphson method [4].

4 Hybrid GA-NR Algorithm

In this section, the proposed hybrid GA–NR is presented in the context of application to the emitting source localization problem. The main goal behind the hybridizations of different optimization algorithms is one of the most efficient strategies to combine the advantages of each algorithm in a way to avoid their disadvantages [7]. The GA as a powerful stochastic global optimization algorithm starts to explore the search space by randomly generating initial solutions under boundary limits, and locates the global or near global optimal solution. A local search algorithm is employed to find the optimal solution by exploiting the neighborhood of the initial solution obtained by the GA.

In this regard, the GA algorithm and the efficient NR direct local search method are combined in order to form the hybrid GA–NR algorithm to improve the effectiveness and accuracy of the solution obtained from the GA. Therefore, in this section, the GA and the NR method are introduced, respectively.

4.1 Genetic Algorithm (GA)

Genetic algorithm is a highly popular metaheuristic optimization method that is based upon mechanics of genetics and natural selection [9] which can be successfully applied to solve the NLS minimization problems. The minimization problem (3) can be modified by introduction of bound-constraints, which can be written as

$$\min_{\boldsymbol{x}^l \leq \mathbf{x} \leq \boldsymbol{x}^h} J_{NLS}(\boldsymbol{x}), \tag{18}$$

where $\boldsymbol{x}$ is a vector of decision variables, $\boldsymbol{x}^l$ and $\boldsymbol{x}^h$ are the lower and upper bounds of $\boldsymbol{x}$ respectively. The bounds are introduced in order to prevent the evaluation of the objective function for unfeasible solutions during the search process.

The algorithm starts with the randomly selected population of N_p individuals on the feasible solution space. Unlike gradient based optimization procedures, the GAs operate on coded representation of the optimization parameters. Each individual is represented by a chromosome and encoded as a fixed length binary vector, whose length is determined by the number and the encoding accuracy of the optimization parameters. The proposed GA procedure can be divided, according to the evolutionary operators, into selection, crossover and mutation.

The process of selection uses the fitness values of chromosomes to determine which individuals are chosen for the mating, and eliminates the chromosomes which don't have necessary attributes for the efficient solving of the optimization problem. Chromosomes are selected according to the roulette wheel selection, where an individual with smaller objective function value is more likely selected in a minimization problem. The selection probability P_i of the *i*-th individual is proportional to the quality of its original fitness, which can be formulated as

$$P_i = \frac{F_i}{\sum_{j=1}^{N_p} F_j}, \quad i \in \{1, \ldots, N_p\}, \tag{19}$$

where F_i is the corresponding fitness value. The cumulative probability C_i of the *i*-th individual is determined according to the following expression

$$C_i = \sum_{j=1}^{N_p} F_j. \tag{20}$$

Once all the individuals in the population are assigned a corresponding cumulative probability, they undergo the selection process where appropriate individual is chosen according to a random number r which has to satisfy the following relation

$$C_{i-1} < r \le C_i, i \in \{1, 2, \ldots, N_p\}, \tag{21}$$

where r is the generated random number between 0 and 1.

One of the main components of GA, the crossover operator combines and exploits the available information to influence the search direction. The Crossover operator is performed by combining the information of the two previously selected individuals to form an offspring that shares positive traits of both individuals. Two point crossover is performed by selecting two random crossover points c_1 and c_2 along the chromosome length, as shown in Fig 3. Therefore, the encoded binary values enclosed by these points are exchanged between selected chromosomes interchangeably.

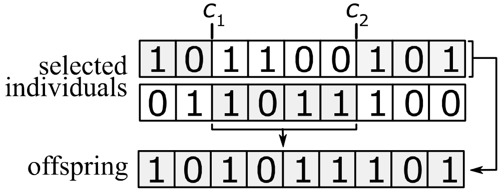


Fig. 3 – *Two point crossover.*

Mutation is a GA operator that introduces new unexplored solutions in the GA population, and keeps algorithm from being trapped in the local optima, which makes the algorithm faster in achieving better solutions. To avoid destruction of the valuable information for the optimization process, only the small percentage of the population is mutated. The mutation rate is defined as the percentage of the total number of genes from the population whose values are mutated. The process of mutation is performed by random binary changes in a chromosome.

The process of selection, crossover and mutation is repeated and population is evolved over successive iterations towards the global optimal solution of the given optimization problem until the termination criterion is satisfied. Although the maximum number of iterations is generally used as a termination criterion in evolutionary algorithms, additionally the relative error between the two consecutive iterations of the average population fitness is applied. The GA procedure continues until the average population fitness converges to a stable value which can be defined as

$$\left| \frac{f_{avg}^{(k)} - f_{avg}^{(k-1)}}{f_{avg}^{(k)}} \right| \le \varepsilon , \tag{22}$$

where ε is a small positive real number, in which

$$f_{avg}^{(k)} = \frac{1}{N_p} \sum_{i=1}^{N_p} f_i , \tag{23}$$

represents the average fitness value of the entire population in kth iteration. The elitism concept is applied in GA to ensure that the individuals with the best objective function value within a population remain unaltered from one generation to the next.

4.2 Newton-Raphson optimization method

The Newton–Raphson method is one of the most well-known and powerful numerical methods [10] for the solution of NLS problems. At each iteration, this method requires the computation of the gradient of the objective function $\nabla J_{NLS}(\boldsymbol{x})$ and Hessian matrix $\nabla^2 J_{NLS}(\boldsymbol{x})$, which can be expressed as follows

$$\nabla J_{NLS}(\boldsymbol{x}) = \begin{bmatrix} \dfrac{\partial J_{NLS}(\boldsymbol{x})}{\partial x} \\ \dfrac{\partial J_{NLS}(\boldsymbol{x})}{\partial y} \end{bmatrix} \in R^2 , \tag{24}$$

$$\nabla^2 J_{NLS}(\boldsymbol{x}) = \begin{bmatrix} \dfrac{\partial^2 J_{NLS}(\boldsymbol{x})}{\partial x^2} & \dfrac{\partial^2 J_{NLS}(\boldsymbol{x})}{\partial x \partial y} \\ \dfrac{\partial^2 J_{NLS}(\boldsymbol{x})}{\partial y \partial x} & \dfrac{\partial^2 J_{NLS}(\boldsymbol{x})}{\partial y^2} \end{bmatrix} \in R^{2\times 2} . \tag{25}$$

The objective function $J_{NLS}(\boldsymbol{x}+\Delta\boldsymbol{x})$ is approximated by a quadratic form of the Taylor series expansion at the current iteration as follows:

$$J_{NLS}(\boldsymbol{x}+\Delta\boldsymbol{x}) \approx \varphi(\boldsymbol{x}) = J_{NLS}(\boldsymbol{x}) + \Delta\boldsymbol{x}^T \left[\nabla J_{NLS}(\boldsymbol{x})\right] + \frac{1}{2}\Delta\boldsymbol{x}^T \nabla^2 J_{NLS}(\boldsymbol{x})\Delta\boldsymbol{x}, \tag{26}$$

where $\Delta\boldsymbol{x}$ is a small change at the given point $\boldsymbol{x} \in \mathrm{R}^2$.

Writing optimality conditions for the function $\varphi(\boldsymbol{x})$ of (26), that is

$$\nabla\varphi(\boldsymbol{x}) = \nabla J_{NLS}(\boldsymbol{x}) + \nabla^2 J_{NLS}(\boldsymbol{x})\Delta\boldsymbol{x} = \boldsymbol{0}. \tag{27}$$

Then, the step size is obtained as follows

$$\Delta\boldsymbol{x} = -\left[\nabla^2 J_{NLS}(\boldsymbol{x})\right]^{-1} \nabla J_{NLS}(\boldsymbol{x}). \tag{28}$$

As a result, the Newton-Raphson method can be applied iteratively as follows

$$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} + \Delta\boldsymbol{x}^{(\mathrm{k})}. \tag{29}$$

where $(\cdot)^{(k)}$ is $(\cdot)$ in the *k*th iteration. The iterative process of the NR method is repeated until the norm of the gradient of the objective function $J_{NLS}(\boldsymbol{x})$ satisfies the following relation

$$\left\|\nabla J_{NLS}\left(\boldsymbol{x}^{(k+1)}\right)\right\| \le \varepsilon, \tag{30}$$

where ε is a sufficiently small positive constant.

4.3 Hybrid GA-NR algorithm

The proposed hybrid GA–NR method is based on the global search GA and local search NR method in order to improve the performance of GA. The flowchart of hybrid GA–NR method is shown in Fig. 4.

The basic idea of the hybridization GA algorithm with the NR local search method is to integrate their advantages and avoid their disadvantages in order to improve the efficiency and accuracy simultaneously. The NR method is a very efficient local search procedure, but its convergence is extremely sensitive to the choice of the initial point. Hence, the GA, a global search algorithm, is applied in order to deal with the selecting initial point in the search space of the localization problem for the NR local search algorithm without trapping in local optimum, and to achieve a faster convergence rate.

The proposed hybrid GA–NR method for the NLS localization problem is then applied to estimate the position of the emitting source in the LOS environments.

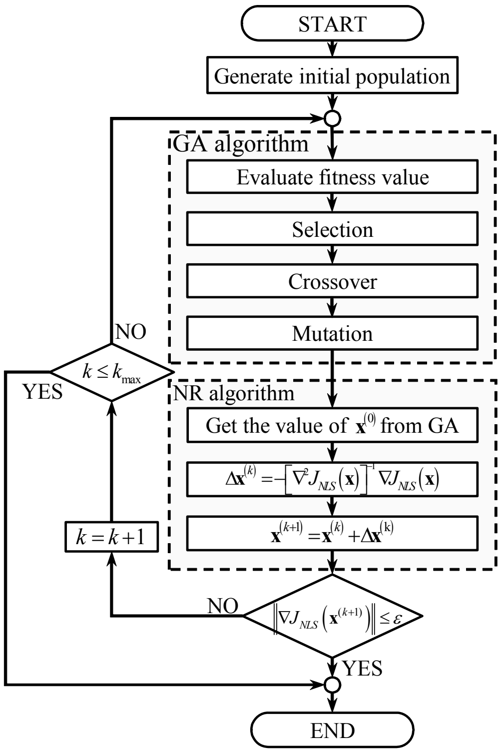


Fig. 4 – *Flowchart of hybrid GA-NR algorithm.*

5 Cramer-Rao Lower Bound

In general, the Cramer-Rao lower bound gives the theoretical lower bound on the variance (or covariance matrix), and provides a benchmark to compare the localization performance of any unbiased estimator [8].

The CRLB can be obtained using the inverse of the Fisher information matrix (FIM) $\boldsymbol{I}(\boldsymbol{x})$ which can be defined as

$$\boldsymbol{I}(\boldsymbol{x}) = E\left[\left(\frac{\partial \ln\left(f\left(\boldsymbol{r}|\boldsymbol{x}\right)\right)}{\partial \mathbf{x}}\right)\left(\frac{\partial \ln\left(f\left(\boldsymbol{r}|\boldsymbol{x}\right)\right)}{\partial \boldsymbol{x}}\right)^{\mathrm{T}}\right] = -E\left[\frac{\partial^2 \ln\left(f\left(\boldsymbol{r}|\boldsymbol{x}\right)\right)}{\partial \boldsymbol{x} \partial \boldsymbol{x}^{\mathrm{T}}}\right], \quad (31)$$

where $E(\cdot)$ is the expectation operator and $f(\boldsymbol{r}|\boldsymbol{x})$ is a joint density function that has the following form

$$f(\boldsymbol{r}|\boldsymbol{x})=\frac{1}{(2\pi)^{N/2}|\boldsymbol{C}|^{1/2}}\exp\left(-\frac{1}{2}(\boldsymbol{r}-\boldsymbol{d}(\boldsymbol{x}))\right)^{T}\boldsymbol{C}^{-1}\left(-\frac{1}{2}(\boldsymbol{r}-\boldsymbol{d}(\boldsymbol{x}))\right),\quad(32)$$

where $\boldsymbol{C}=\mathrm{diag}\{\sigma_1^2\cdots\sigma_N^2\}$ is N dimensional covariance matrix. Then, the partial derivative of the natural logarithm of (32) with respect to

$$\frac{\partial\ln(f(\boldsymbol{r}|\boldsymbol{x}))}{\partial\boldsymbol{x}}=-\frac{1}{2}\frac{\partial}{\partial\boldsymbol{x}}\left((\boldsymbol{r}-\boldsymbol{d}(\boldsymbol{x}))^{\mathrm{T}}\boldsymbol{C}^{-1}(\boldsymbol{r}-\boldsymbol{d}(\boldsymbol{x}))\right).\quad(33)$$

Thus, the first element of the FIM can be derived as follows

$$I_{xx}=E\left[\left(\frac{\partial\ln f(\boldsymbol{r}|\boldsymbol{x})}{\partial x}\right)\left(\frac{\partial\ln f(\boldsymbol{r}|\boldsymbol{x})}{\partial x}\right)^{\mathrm{T}}\right]=\left(\frac{\partial\boldsymbol{d}(\boldsymbol{x})}{\partial x}\right)^{\mathrm{T}}(\boldsymbol{C}^{-1})^{T}\left(\frac{\partial\boldsymbol{d}(\boldsymbol{x})}{\partial x}\right).\quad(34)$$

Therefore, the elements of the Fisher information matrix

$$\mathrm{CRLB}=\boldsymbol{I}^{-1}=\frac{1}{|\boldsymbol{I}|}\begin{bmatrix}I_{yy} & -I_{xy}\\ -I_{yx} & I_{xx}\end{bmatrix},\quad(35)$$

can be obtained as follows

$$I_{xx}=\sum_{i=1}^{N}\frac{(x-x_l)^2}{d_l^2\sigma_i^2},\quad(36)$$

$$I_{xy}=I_{yx}=\sum_{i=1}^{N}\frac{(x-x_l)(y-y_l)}{d_l^2\sigma_i^2},\quad(37)$$

$$I_{yy}=\sum_{i=1}^{N}\frac{(y-y_l)^2}{d_l^2\sigma_i^2}.\quad(38)$$

The covariance matrix of any unbiased estimator $\hat{\boldsymbol{x}}$ of $\boldsymbol{x}$ is limited by inverse of the FIM as

$$\mathbf{cov}(\hat{x},\hat{y})\geq(\boldsymbol{I}(x,y))^{-1}.\quad(39)$$

Then, the relation between the CRLB based on the TOA measurements with independent zero-mean Gaussian noise and variance can be expressed as

$$E\left[(\hat{x}-x)^2+(\hat{y}-y)^2\right]\geq\frac{I_{xx}+I_{yy}}{I_{xx}I_{yy}-I^2{}_{xy}},\quad(40)$$

where $\hat{x}, \hat{y}$ are the estimated values of $x, y,$ respectively and I_{ij} represents the element of the FIM $\boldsymbol{I}(x, y)$ in the i-th row and j-th column.

6 Simulation Results

Numerical simulations have been conducted to evaluate the localization performance of the proposed hybrid GA–NR method by comparing it with the derived CRLB for given TOA localization problem. The simulation results of the proposed algorithm have been compared to the regular GA.

The geometry of the considered localization system is composed of four sensors positioned at $\boldsymbol{x}_1 = [0,0]^T$ m, $\boldsymbol{x}_2 = [3000,0]^T$ m, $\boldsymbol{x}_3 = [0,3000]^T$ m and $\boldsymbol{x}_4 = [3000,3000]^T$ m, while the emitting source is located at $\boldsymbol{x} = [700,800]^T$ m, as shown in Fig.5.

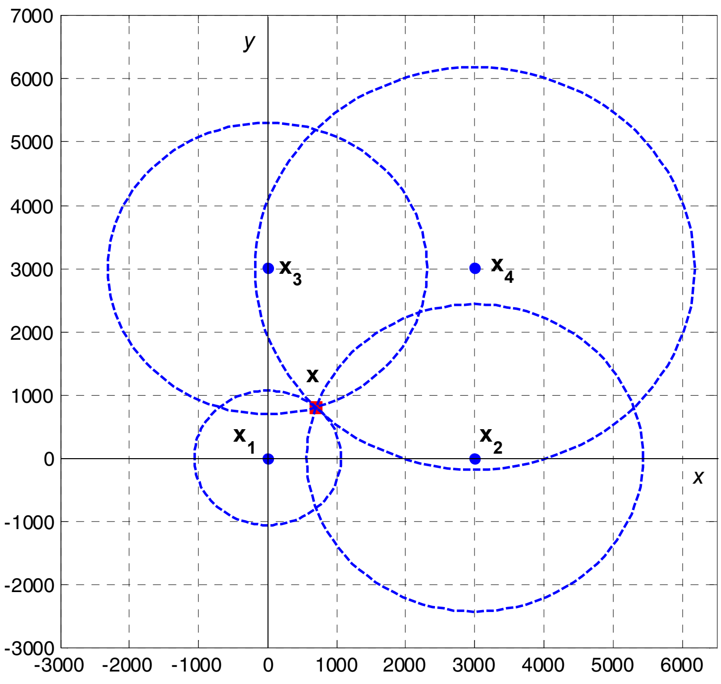


Fig. 5 – *Simulated emitting source localization geometry.*

The TOA measurements are generated by adding to the true values zero-mean Gaussian variable. Monte Carlo simulations, based on $N = 200$ independent simulation runs, have been carried out to compare the localization performance of considered approaches in terms of the root mean square error (RMSE), which is defined as

$$RMSE = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left\|\hat{\boldsymbol{x}}(n) - \boldsymbol{x}\right\|_2^2}, \tag{41}$$

where $\hat{\boldsymbol{x}}(n)$ is an estimated emitting source position for the n-th simulation run and $\mathbf{x}$ is a true emitting source position. In this work, MATLAB software is used to simulate the corresponding optimization methods, as well as the effects of measurement errors under different SNR conditions on the accuracy of estimation.

The cumulative distribution functions (CDFs) of the error between the estimated and the true emitting source position, for the considered hybrid GA–NR and regular GA methods are illustrated in Fig.6 and compared with the CRLB to evaluate the localization performance.

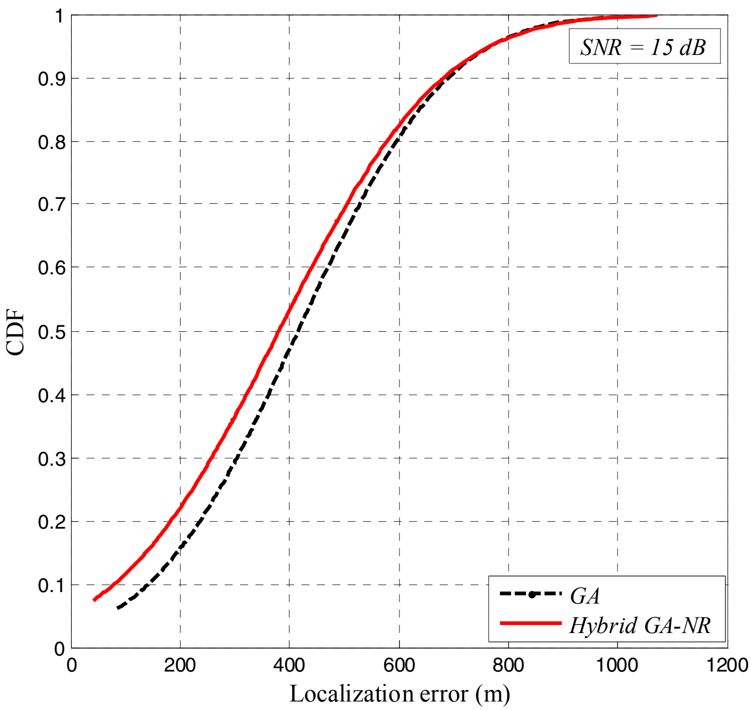


Fig. 6 – *CDFs of the localization error for SNR* = 15 dB.

The results presented in Fig.6 for the case when SNR is 15 dB, show that the hybrid GA–NR estimator has a superior performance in comparison to the regular GA, mainly because the local search NR estimator is initialized with potentially good solution provided by the GA.

In Fig. 7 the CDFs of the hybrid GA–NR and the regular GA estimators are compared to the CRLB, when SNR is set to 50 dB.

From Fig. 7 it is observed that when the SNR is larger, the CDFs of the both considered estimators have similar performance, although the performance of the regular GA is slightly worse.

Comparing the results presented in Fig. 6 and Fig. 7 it should be noted that the accuracy of the localization degrades in the case of low SNR. Moreover, it is observed that the proposed hybrid GA–NR method is quite accurate, even in large noise conditions.

Finally, the effect of the measurement noise variance value on the localization performance of the regular GA and hybrid GA–NR methods is investigated and compared to the derived CRLB, where the RMSE of the estimated emitting source position is plotted on Fig. 8, as a function of SNR.

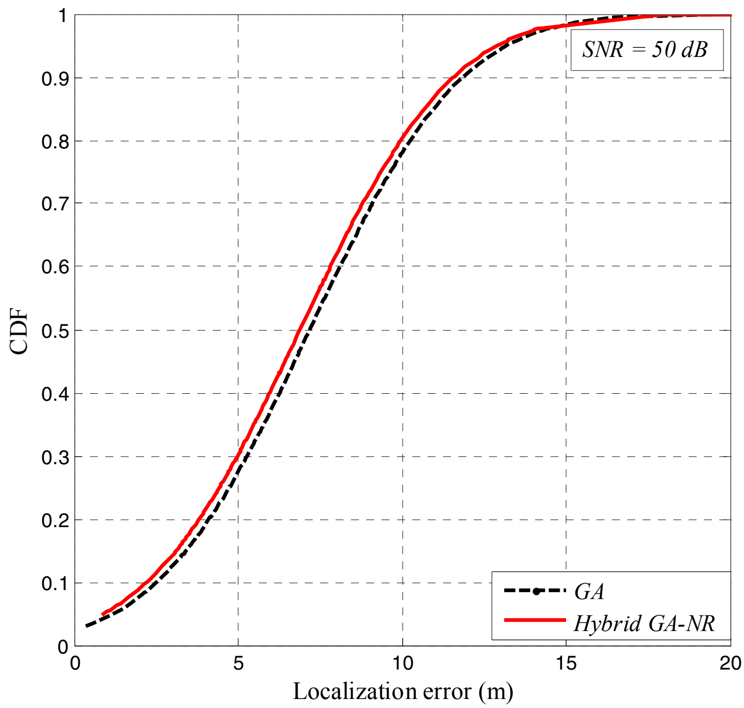


Fig. 7 – *CDFs of the localization error for SNR* = 50 dB.

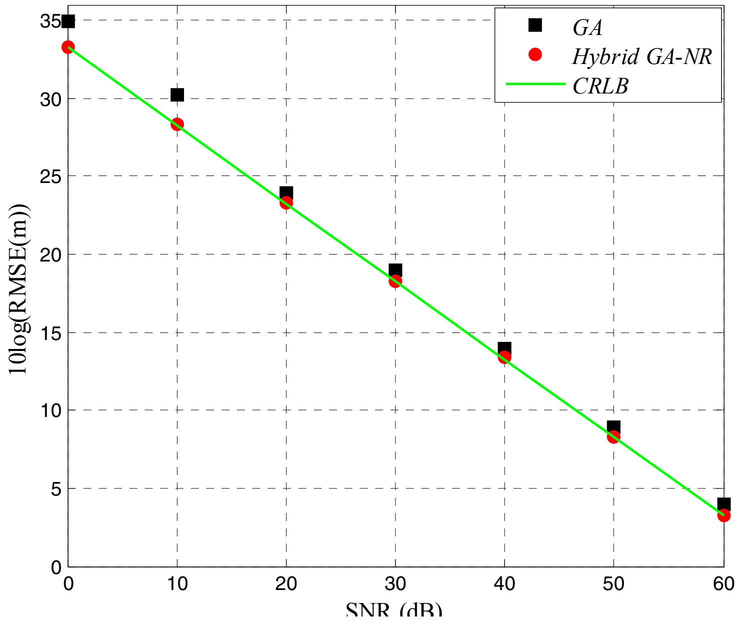


Fig. 8 – *Comparison of RMSE versus SNR levels.*

From the simulation results depicted in Fig. 8 it is observed that the proposed hybrid GA–NR method outperforms regular GA and provides performance very close to the CRLB. Moreover, it can be seen that the GA exhibits significant deviation of the localization accuracy from the CRLB as the SNR decreases. Compared to the regular GA, the improved localization performance of the GA–NR method is a consequence of the hybridization with a local search NR method, which is able to refine solutions obtained by GA, leading to a higher accuracy.

7 Conclusion

This paper presents the source localization problem that is implemented as the unconstrained optimization problem where the hybrid GA–NR method has been proposed and successfully applied and the dispatch results are compared directly with those of the GA algorithm. In this work, the localization problem is formulated as the NLS estimation problem based on the TOA measurements. Simulation results indicate that hybrid GA-NR algorithm successfully localizes the emitting source and has a superior localization performance to GA in terms of its convergence characteristics with different levels of SNR. The main advantages of the proposed hybrid GA–NR method are the faster convergence speed and the higher accuracy of the obtained solution.

Further investigation may use the CRLB to evaluate the optimal sensors placement necessary for the high-performance localization. The proposed localization problem can be formulated as the optimization model of ML estimator where the objective function is nonconvex and determination of global optima is a difficult task.

8 Acknowledgement

The work of M. Bjelica and M. Simić was supported in part by Serbian Ministry of Education and Science under Grant TR32028.

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