# LASER DOPPLER VELOCIMETRY AND CONFINED FLOWS

by

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*Finding the mode, in which two component laser Doppler velocimetry can be applied to flows confined in cylindrical tubes or vessels, was the aim of this study. We have identified principle issues that influence the propagation of laser beams in laser Doppler velocimetry system, applied to flow confined in cylindrical tube. Among them, the most important are influences of fluid and wall refractive indices, wall thickness and internal radius ratio and beam intersection angle. In analysis of the degrees of these influences, we have applied mathematical model, based on geometrical optics. The separation of measurement volumes, that measure different velocity components, has been recognized as the main drawback. To overcome this, we propose a lens with dual focal length – primary focal length for the measurement of one velocity component and secondary focal length for the measurement of the other velocity component. We present here the procedure for calculating the optimal value of secondary focal length, depending on experimental set-up parameters. The mathematical simulation of the application of the dual focal length lens, for chosen cases presented here, confirmed the accuracy of the proposed procedure.*

Key words: *laser Doppler velocimetry, confined flow, cylindrical tube, fluctuation, geometrical optics*

## Introduction

Laser Doppler velocimetry (LDV) was, for a long time [1], one of a few standard testing techniques in fluid dynamics. Being an absolute measurement technique (no calibration required) and non-intrusive tool, it has been readily used in laboratory conditions for measurements in flows. Its main advantage, as compared to classical techniques, is that it makes almost no interference with turbulence structures of a flow – it does not harm even small vortices. Furthermore, by its nature, along with the detailed description of velocity values, LDV provides root mean square (RMS) of them, which represents the velocity fluctuation in the measurement point. The LDV measurement promptly provides the turbulence level in that point, thus simplifying the analysis of turbulence and is especially suitable in an open flow research, where nothing impacts a calibration constant. Each

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measurement volume position is easily defined (there is no dislocation of measurement volume, certain displacement of traversing system produces equal displacement of measurement volume) and 2-D and 3-D LDV systems easily provide reliable results. However, their application in confined flow measurements has some restrictions, and requires certain corrections. Some of them are linked to the geometrical optics *i. e.* the refraction and reflection of laser beams on wall surfaces in a defined geometry. When flow parameters are measured through a flat wall, minimum corrections of measurement results should be made (such as the corrections of the position of measurement volume). In many technical problems, flows in cylindrical geometry (tubes, vessels, pumps, …) have to be investigated [2, 3]. In that case, if an axial velocity component is measured in points of a radial plane (a plane that contains tube axis) laser beams of LDV system act as if they pass through a flat wall, and stay in the same plane. The measurement of radial velocity component involves more complex analysis and correction calculations [4, 5]. Modifications of cylindrical geometry with the flat surface of an external wall has been made as the attempt to diminish undesired effects of laser beam refractions [6-8], but these corrections cannot be generalized and each geometry requires particular analysis. The detailed analysis of LDV laser beam propagation in the case of water flow in cylindrical tube with flat external wall is reported by Zhang [9]. Many water flows are also investigated by 2-D LDV measurements with cylindrical tube placed within a square cross-section duct filled with water [10]. That way, problems with laser beam refraction are reduced, but they are not avoided. The LDV measurements through a window as an optical access [11] are very common. If the window is flat, the tube cylindrical geometry is to a various degrees "damaged". If the window continues the cylindrical curve of the wall, the measurement is equivalent to the one in fully transparent cylindrical tube [12]. In ref. [13], it was demonstrated that at 2-D LDV measurement in simple cylindrical tubes, measurement volumes for different velocity components are separated less than in the case of cylindrical tube with flat external wall, but it is still present.

The aim of this paper is to present the analysis of some influences on the measurement volumes separation and to propose a solution that overcomes the separation of different velocity components measurement volumes. In next chapter the influences of refractive indices of flowing fluid and wall material, wall thickness, light wavelength, and angle of beam intersection on measurement volumes separation and tilt of the measurement direction are analyzed. The proposed method of reduction of the problem of measurement volume separation by using slightly different focal lengths for axial and radial components is also shown in next chapter. The simulation of application of the proposed solution is presented and analyzed in the chapter *Discussion and results*. The conditions required for reliable application of 2-D LDV systems are defined.

## Set-up analyses and method

The experimental set-up, considered in this paper, consists of a horizontal tube with a circular cross-section, flowing fluid within it, and a 2-D LDV system positioned to measure the axial $\vec{u}$ and radial $\vec{v}$ velocity component along the vertical diameter of the cross-section (fig. 1). Angles $\theta$ are half angles of a beam intersection in air. They are determined by the focal length of a transmitting lens. It is expected that the transmitting lens is traversing the measurement volume along the vertical diameter (dash-dot vertical line in fig. 1). However, the refraction of laser beams produces the dislocation of measurement volumes.

A mathematical model of LDV laser beam propagation is explained in detail in ref. [13]. According to [13], it is impossible to derive one mathematical expression describing

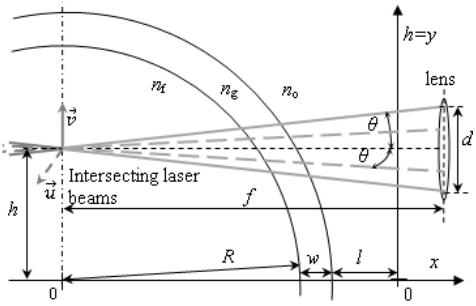


**Figure 1. Positions of lens and LDV laser beams measuring axial $\vec{u}$ and radial $\vec{v}$ velocity component**

certain beams propagation properties (such as intersection angle, co-ordinates of measurement volume, tilt angle of measurement volume). Complete procedure requires more than seventy expressions to be calculated in order to find those parameters. Therefore, the influence of components of the experimental set-up to LDV laser beams propagation cannot be studied using only analytical methods, but numerical one. Due to the complexity of this phenomenon, there are many possible ways of its numerical analysis. Here, one set of experimental set-up parameters is chosen as initial, presented in tab. 1.

**Table 1. Initial set of set-up parameter values**

| Parameter value | Description |
|---|---|
| $R$ = 1 m | Internal radius of a tube |
| $w$ = 0.05 m | The thickness of a tube wall |
| $f$ = 1.5 m | The focal length of a lens |
| $d$ = 0.3 m | Beam separation at lens |
| $n_0$ = 1 | Refractive index of surrounding fluid |
| $n_g$ = 1.4878 | Refractive index of wall glass |
| $n_f$ = 1 | Refractive index of an internal fluid |

Then, each of these parameters has been individually varied and beam propagation properties have been calculated. The only fixed parameter value, in the analysis presented in this paper, is the refractive index of air as surrounding stationary fluid $n_0$ = 1 (though the cases with some other surrounding fluid could also be of interest). For the refractive index of wall material, the one of plexiglas at the light wavelength of $\lambda$ = 660 nm was chosen [14]. Though, in real 2-D LDV systems, the wavelengths of a laser beam pairs, measuring different velocity components, are not equal, the changes of refractive indices with the change of the wavelength of the beam is small enough, that its influence to the propagation of the beam is considered negligible.

The initial set of parameters represents air flow in a tube with a wall thickness that equals 5% of tube radius. Charts in fig. 2 present some of beam propagation properties in that case. The propagation of laser beams follows geometric similarity laws, and depends only on the ratio of the wall thickness and the tube radius. This means that the relative positions (of a center) of measurement volumes in this case (fig. 2(a)) and measurement volumes tilt angles (fig. 2(b)) are the same for all tubes with the same ratio of tube wall thickness and tube internal radius.

The right curve in fig. 2(a) presents the measurement volumes positions for a radial velocity component measurement (normalized positions *x/R* and *y/R*, *i. e.* relative distances from the center of a tube are presented). It can be seen that the measurement volume in the center of a tube is not dislocated. Up to 0.6*R*, the measurement volume dislocation is less than 1% of a tube radius *R*. On the other hand, the measurement volume that measures the axial velocity component (left curve) in the center of a tube is dislocated by 1.6% of tube internal radius *R* (for the initial set of parameters). This dislocation is approximately constant up to 0.8*R*. Figure 2(b) shows that measurement volumes measuring radial (squares) and axial (diamonds) velocity components are tilted with respect to the LDV system symmetry axis. In fig. 2(b) tilt angles *vs.* relative *y* positions are presented. They are tilted at almost the same angles. That angles are less than 1° up to 0.65*R* and rise up to the 4.8° close to the tube wall. For air, as an internal moving fluid, the tilt angles are positive, while for liquids, as it will be shown further in the text, they are negative. That inclination affects only the direction of measured radial velocity component, and measured values should be corrected according to it. The measurement volume for the axial velocity component, though tilted, still measures the velocity values of the same direction.

The greatest impact on a laser beam propagation has the change of the refractive index of internal (moving) fluid $n_f$. If $n_f = 1.002$, the shapes of charts, like these in fig. 2, have the same shape, with slightly different numerical values – the separation of measurement volumes in the center of a tube is 1.85% of *R*, and tilt angles are less than 1° up to 0.7*R* and rise up to the 4.5° close to the tube wall. Since most of the gases have a refractive index less than 1.002 [15], it can be considered that charts in fig. 2 are valid for gases.

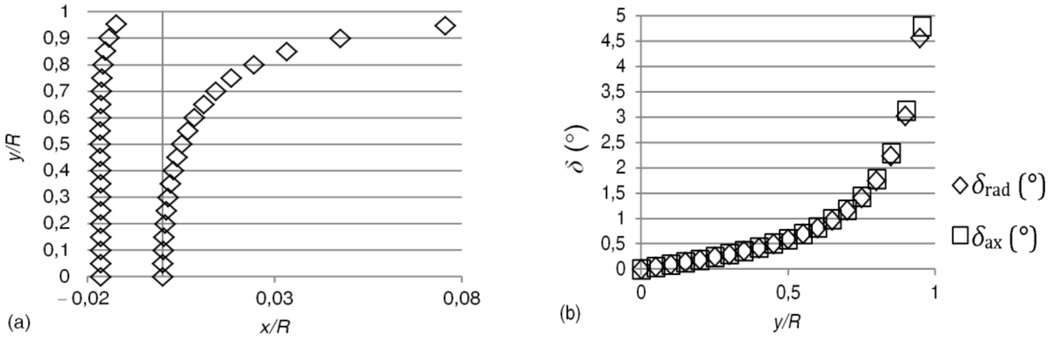


**Figure 2. The dislocated positions (a) and tilt angles (b) of measurement volumes for the initial set of parameters**

The further increase of refractive index causes considerable change of the shape of the curves in charts presenting the measurement volume positions and their tilt angles, (figs. 3 and 4). Like in fig. 2 in figs. 3-5, and figs. 7-9, normalized positions of the measurement volumes *x/R* and *y/R* are presented, *i. e.* their relative distances from the center of a tube (left curve for axial and right curve for radial velocity component), and the tilt angles *vs.* relative *y* positions are presented. The measurement volume positions and their tilt angles are changed even for internal fluid refractive index of $n_f = 1.025$ (figs. 3(a) and 3(b)). Up to that fluid refraction index value, separation of measurement volume rises (4.2% of *R* at $n_f = 1.025$) and their tilt angles rapidly fall, becoming negative in the central region of a tube. These trends continue as the internal fluid refractive index value rises (figs. 3(c)-3(f), fig. 4). In addition to that, a few more phenomena can be noticed. First, the dislocation of a radial velocity component measurement volume, in the region close to the tube wall, gets smaller as the internal fluid refractive index rises, and *x* - component of its position become negative, making it closer to the measurement volume of the axial velocity component. They even overlap in one point for each value of internal fluid refractive index greater than 1.05. Second, the rise of internal fluid refractive index causes the decrease in *y* component in the position of both measurement volumes (for radial and axial velocity component) and for all positions of transmitting lens. This way, the region near the tube wall remains unapproached *i. e.* measurement volume can be positioned only up to a certain value of *y* – up to 0.9*R*, for $n_f = 1.05$ (fig. 3(e)), up to 0.8*R*, for $n_f = 1.33$ (fig. 4(a)), up to 0.7*R*, for $n_f = 1.5$ (fig. 4(c)). Third, negative angles of the measurement volume inclination increase with the rise of fluid refractive index (up to 2.4°, for $n_f = 1.05$ – fig. 3(f), up to 22°, for $n_f = 1.33$ – fig. 4(b), up to 29°, for $n_f = 1.5$ – fig. 4(d)). Furthermore, while the tilt angles of measurement volumes for small values of fluid refractive index are almost the same for each position of transmitting lens, for larger refractive indices the negative tilt of measurement volume measuring axial velocity component is greater than that of a radial velocity component, and that difference increases with the increase of the fluid refractive index value.

To represent the cases with liquid fluid flows, the refractive index of water $n_f = 1.33$, and approximately of benzene $n_f = 1.5$ [16] at $\lambda = 540$ nm are chosen. In the case of liquid flowing fluid, too large separation of measurement volumes, that measure different velocity components, and great tilt of measurement volume for radial velocity component, makes the application of 2-D LDV systems inappropriate, without adequate modification.

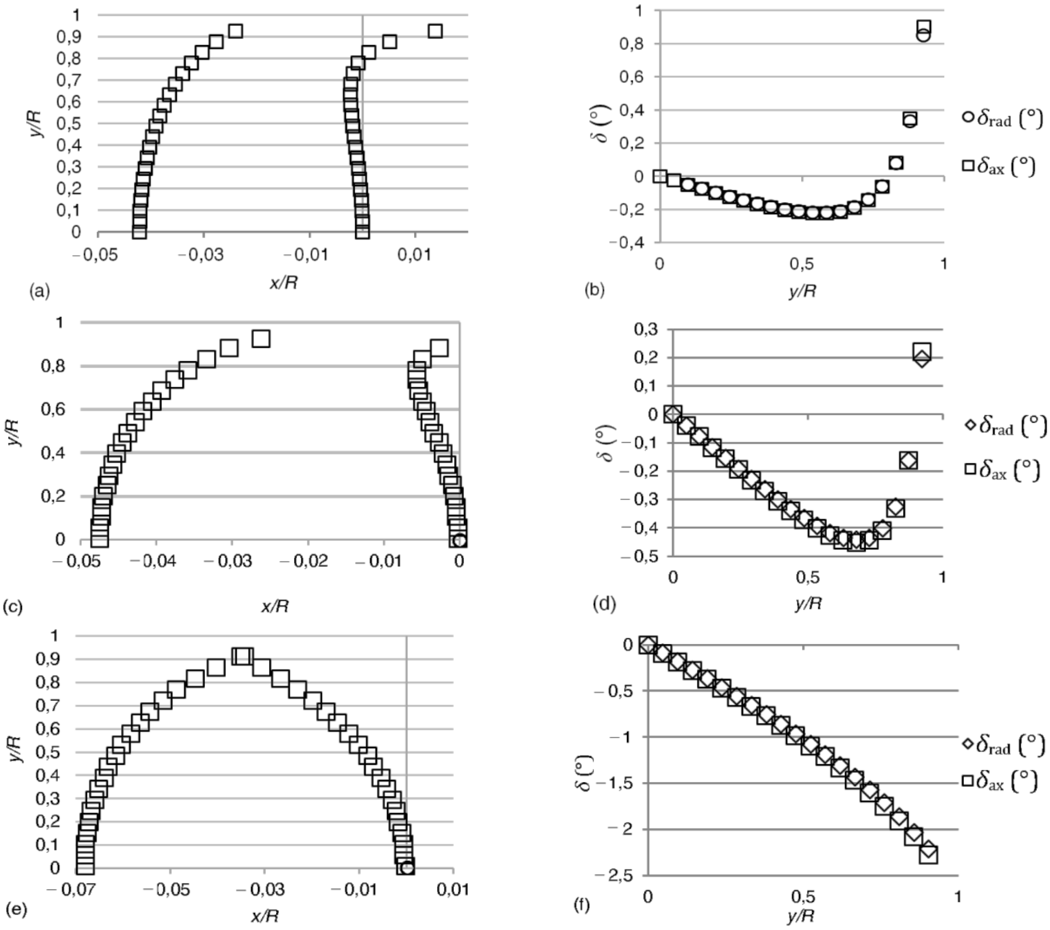


**Figure 3. The dislocated positions and tilt angles of measurement volumes for $n_f$ = 1.025 (a) and (b), for $n_f$ = 1.03 (c) and (d) and for $n_f$ = 1.05 (e) and (f)**

The impact of tube wall refractive index $n_g$ on the propagation of LDV laser beams is less pronounced than that of fluid flow refractive index. This is illustrated by the results of simulation of propagation of LDV laser beams in the case where all parameters are like in the initial set of parameters (tab. 1) except the $n_g$. The chosen values for refractive index of glasses, low ($n_g$ = 1.4) and high ($n_f$ = 1.8) are implemented in the calculation (fig. 4). Maximum difference between the positions of measurement volumes that measure a radial velocity component for the same position of emitting lens is 1.3% of $R$; and for axial velocity component is 0.8% of $R$. The difference between the inclination angles of measurement volumes for the different glasses and for the same positions of emitting lens is less than 0.8°, for both velocity components.

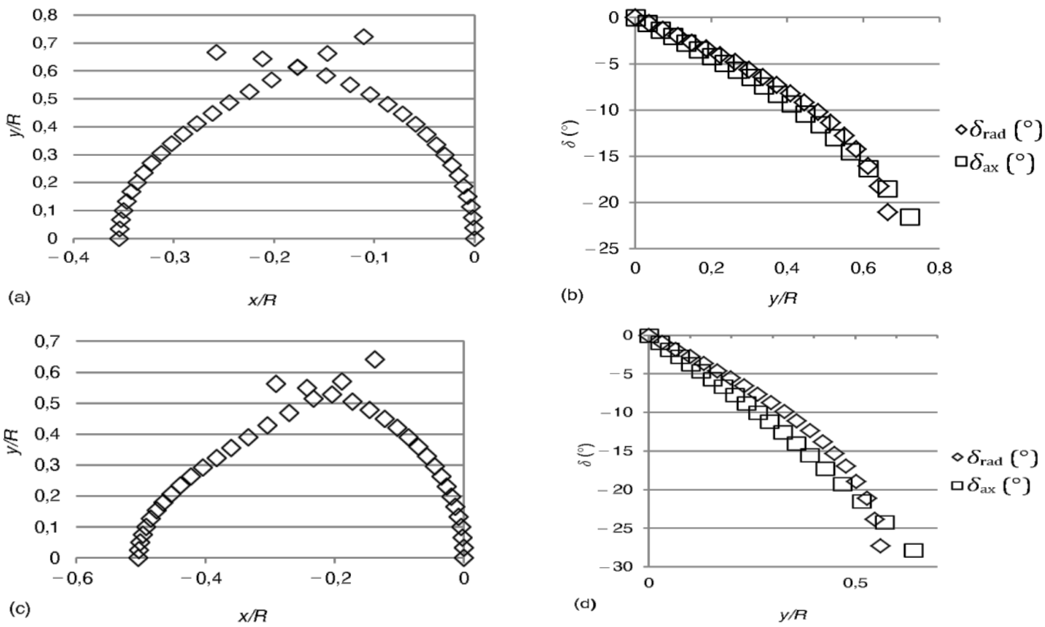


**Figure 4. The dislocated positions and tilt angles of measurement volumes for $n_f$ = 1.33 (a) and (b), for $n_f$ = 1.5 (c) and (d)**

Even less pronounced are the influences of laser beam wavelength $\lambda$, and half angle of the beam intersection $\theta$. The charts that represent these influences are very similar to those in fig. 5 except for more evident overlapping of the markers. For present analysis, one component of $Ar^+$ ion laser ($\lambda$ = 454.6 nm) and one component of $Nd^{3+}$:YAG laser ($\lambda$ = 785 nm) are chosen. The differences between the positions of the measurement volumes of the same kind for the two wavelengths, and for the same position of transmitting lens, are less than 0.06% of $R$ for both velocity components. Chosen half angles of beam intersection, in this analysis, are $\theta$ = 1° and $\theta$ = 18.4°. The differences between the positions of the measurement volumes that measure radial velocity component, for the two half angles of beam intersection, and for the same position of transmitting lens, are almost zero in the central region of a tube, and rise up to the 2.7% of $R$ in the vicinity of a tube wall. For axial velocity component, maximum difference between the positions of the measurement volumes is in the center of a tube and it is less than 1% of $R$.

It is foreseeable that the thicker the tube wall, the greater the measurement volume dislocation is. This is also approved by the study presented here. It is found that, for the initial set of parameters (tab. 1), by varying only the thickness of the tube wall $w$ (or $w/R$), the separation of measurement volumes $\Delta l$, due to beam refraction, is equal $\Delta l$ = 0.3297$w$. The uncertainty of linear regression coefficient is $5.8 \cdot 10^{-14}$, indicating the strong linearity of the relationship. At the distance of 0.5$R$ from the center of a tube cross-section, the separation of measurement volumes $\Delta l$ is equal $\Delta l$ = 0.417$w$. The uncertainty of linear regression coefficient 0.0026, shows weaker linearity of this relationship. Having in mind a small influence of other parameters except $n_f$ and $w$, previous consideration gives the rule of thumb that: if the working fluid in the cylindrical tube is air, or some other gas, the separation between the two measurement volumes of 2-D LDV system is slightly less than a third of a tube wall thickness in the center of the cross-section, and slightly more than 2/5 of a tube wall thickness at half radius distance from the center of the cross-section.

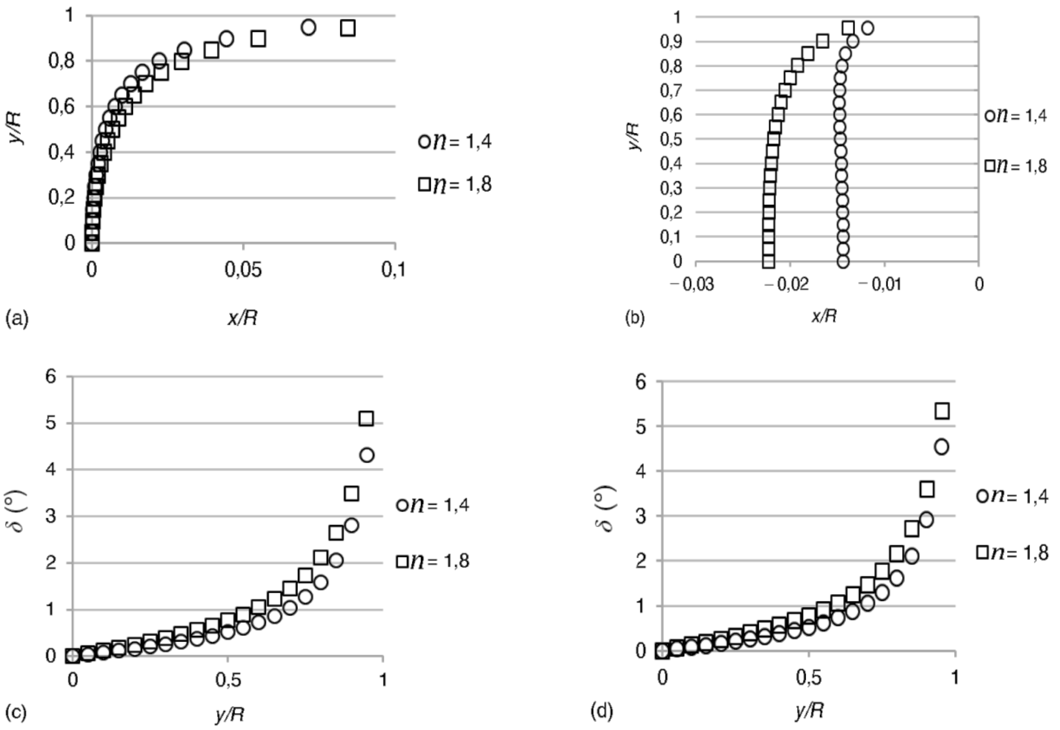


**Figure 5. The case of tube with wall glass refractive index of $n_g$ = 1.4 (circles) and $n_g$ = 1.8 (squares); the dislocated positions of measurement volumes for radial velocity component (a), for axial velocity component (b); the tilt angles of measurement volumes for radial velocity component (c), for axial velocity component (d)**

One of the main drawbacks of application of 2D - LDV system, in the case of cylindrical tube, is the separation of measurement volumes. Measurement volume, that measures radial velocity component, is close to the lens focus and the vertical diameter of a tube cross-section up to certain point, depending mainly on a working fluid refractive index and the wall thickness of the tube. This means that the dislocation, in the central region of a tube, is small. On the other hand, the maximum dislocation of a measurement volume that measures axial velocity component is in the center of a tube cross-section. One of possible solutions is to shift the axial velocity component measurement volume towards the vertical diameter of a tube cross-section by using a lens with a dual focal length. The idea is to change the primary focal length *f* of the lens in the region where laser beams for axial velocity component (here the horizontal pair of laser beams) pass through the lens. In fig. 6(a), grey segments represent the parts of the lens that should have different radius of lens surface than that of the rest of the lens. Local changes in focal length can be obtained by adding a small change in the diameter lens in places where the outputs of horizontal laser beams are. Thus, if the primary focal length of a lens is $f$, the secondary focal length of grey segments should be $f_1$.

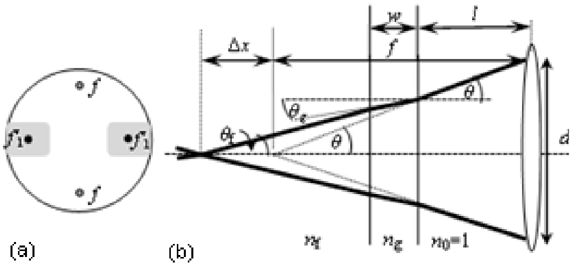


**Figure 6. (a) Lens with dual focal length: primary focal length *f* for vertical pair of laser beams, and secondary focal length $f_1$ for horizontal pair of laser beams; (b) Propagation of horizontal pair of LDV laser beams in the plain with tube axis**

In order to find the way to calculate the value of secondary focal length $f_1$, such that minimizes measurement volume separation, fig. 6(b) was used. In the plane containing the horizontal pair of laser beams and tube axis, laser beams pass through a tube wall as if it was a flat wall, fig. 6(b). Thick black lines in fig. 6(b), present propagation of LDV laser beams in that case. Let us assume that $f$ is primary focal length of a simple lens. Then, measurement volume is dislocated by $\Delta x$.

Using notations presented in fig. 6(b), it follows that:

$$[\Delta x + (f - l - w)]\tan\theta_f + w\tan\theta_g + l\tan\theta = f\tan\theta \tag{1}$$

where $l$ is the shortest distance between the lens and the tube, $\theta_g$ is refraction angle in glass, $\theta_f$ is the half angle of beam intersection in flowing fluid. If the internal flowing fluid is air, then $\theta_f = \theta$, and eq. (1) is reduced to:

$$\Delta x = w(1 - \tan\theta_g / \tan\theta) \tag{2}$$

Taking into account Snell's law ( $n_0 \sin\theta = n_g \sin\theta_g$ ), and the fact that $\tan\theta = d/(2f)$, eq. (2) becomes:

$$\Delta x = w\{1 - 1/[n_g^2 - d^2(n_g^2 - 1)/(4f^2)]^{1/2}\} \tag{3}$$

To compensate measurement volume dislocation $\Delta x$, the sum of new – coinciding secondary – focal length $f_{1c}$ (it makes the two measurement volumes coincide) and new measurement volume dislocation $\Delta x_1 = \Delta x(f_{1c})$ must be equal to the initial – primary – focal length $f$: $f = f_{1c} + \Delta x_1$. Applying to this eq. (3), the following equation can be obtained:

$$1/[n_g^2 - d^2(n_g^2 - 1)/(4f_{1c}^2)]^{1/2} = 1 - (f - f_{1c})/w \tag{4}$$

Numerical solution of eq. (4) gives focal length $f_{1c}$ for the measurement of axial velocity components which ensures that measurement volumes for both velocity components are coinciding in the center of a tube cross-section. If the internal fluid is not air, but arbitrary fluid, taking into account Snell's law ( $n_g \sin\theta_g = n_f \sin\theta_f$ ), eq. (1) can be written as:

$$f - l = (f_{1c} - l)t(n_f, f_{1c}) + w[1 - t(n_f, f_{1c})/t(n_g, f_{1c})] \tag{5}$$

where $t(n_i, f_{1c}) = [n_i^2 - d^2(n_i^2 - 1)/(4f_{1c}^2)]^{1/2}$ .

By solving the eq. (5), $f_{1c}$, a coinciding secondary focal length, can be obtained. That way, overlapping of the two 2-D LDV measurement volumes, for any fluid, can be accomplished.

## Discussion and results

The simultaneous measurement of two velocity components requires the overlapping of the two measurement volumes. A perfect case is when the centers of measurement volumes are at the same point. Yet, 2-D measurement is feasible as long as the overlapping parts of measurement volumes are large enough to create a sufficient data rate. Defining a sufficient overlapping volume is very intricate because it depends on a hardly predictable and usually non-uniform seedings concentration, then on a laser beam intensity and its distribution within a beam, and on the signal processing settings. Nevertheless, an ultimate restriction can be found having in mind the separation distances of centers of the two measurement volumes with respect to the measurement volume dimensions (particularly the length) and the inclination angle. In simplified analysis, it can be assumed that measurement volumes are symmetrical (possible shape distortion of measurement volume is out of the scope of this paper) and that dimensions of the two measurement volumes are approximately equal. If the inclination angles are zero, like they are in the centers of a tube cross-section (presented in previous section), the separation of measurement volume centers cannot be greater than the measurement volume length, in order to overlap. Therefore, if length of measurement volume is greater than the separation distance of measurement volume centers, which can be seen in charts like fig. 2(a), 2-D LDV measurement is feasible. In practice, this could happen if the internal fluid is gas. In liquid flowing fluids, huge separation of measurement volumes discards 2-D LDV application.

The application of transmitting lens with a dual focal length, in the case of the initial set of parameters (coinciding secondary focal length is $f_{1c}$ = 1.4837 m), positions of measurement volumes in fig. 7(a) are obtained. This confirms the accuracy of eq. (4). With the modification of a lens, the overlapping of measurement volumes in the center of a tube cross-section is accomplished. The distance between measurement volume centers is less than 1% of $R$ up to 0.6$R$, for the initial case (flowing fluid is air, and $w = 0.05R$). A similar case but with different wall thicknesses produce measurement volume distances presented in fig. 8. If $w = 0.01R$, the distance between measurement volume centers is less than 1% of $R$ up to 0.9$R$ (fig. 8(a)), and if $w = 0.1R$, the distance between measurement volume centers is less than 1% of $R$ up to 0.6$R$ (fig. 8(b)). Up to those heights, inclination angles are less than 1°, for both cases, not affecting the reasoning explained in previous paragraph.

In a case presented in fig. 7, secondary focal length $f_{1c}$ is shifted with respect to primary focal length $f$ for shift $\Delta f = f - f_{1c} = \Delta f_c$. This focal length shift caused the movement of a left "axial" curve in fig. 2(a) towards $y$ axis. In the case of fig. 2(a), minimum separation of measurement volumes is 1.6% of $R$, and in the case of fig. 7(a) it is less than 1% of $R$, up to 0.62$R$. If this "axial" curve is moved farther, the application of larger focal length shift $\Delta f = f - f_1 = 1.4 \cdot \Delta f_c$, positions of measurement volumes are like in fig. 7(b), and measurement volume separation is less than 1% of $R$ up to 0.63$R$. It means that the proper increase of a frequency shift can extend the region of measurement volumes overlapping, and thus extending the region of application of 2-D LDV. The decision on how much to increase the focal length shift, depends on the ratio of measurement volume length and internal radius $R$, and desired minimum overlapping of measurement volumes.

In order to make the two measurement volumes overlap, in the center of a tube cross-section, in the case of liquids, a left "axial" curve in fig. 4(a) should be moved toward y-axis using a transmitting lens with a dual focal length ($f_{1c}$ = 1.234 m). The result is in fig. 9(a), where measurement volume separation is less than 1% of $R$ up to 0.21$R$. If this "axial" curve is moved nearer, the application of smaller focal length shift $\Delta f = f - f_1 = 0.98\Delta f_c$, positions of measurement volumes are like in fig. 9(b), and a measurement volume separation is less than 1% of $R$ up to 0.25$R$. Therefore, the application of transmitting lens with dual focal length, enables the use of 2D LDV in liquid fluid flows, as concerns the positions of measurement volumes.

The choice of the focal length shift $\Delta f$ depends on measurement requirement. If the priority is high quality measurement in the center of a tube cross-section, focal length shift should be equal the coinciding one $\Delta f = \Delta f_c$. Whereas, if the priority is to extend the measurement region around the center of the tube cross-section, focal length shift should be greater than the coinciding one $\Delta f > \Delta f_c$ for gaseous fluids, and smaller than the coinciding one $\Delta f < \Delta f_c$ for liquids.

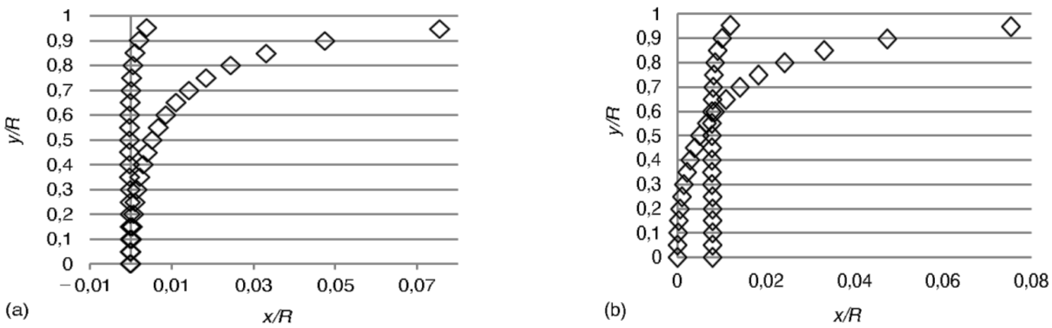


**Figure 7. Positions of measurement volumes for initial case and transmitting lens with dual focal length applied; secondary focal length shift is: (a) $\Delta f = \Delta f_c$, (b) $\Delta f = 1.4\,\Delta f_c$**

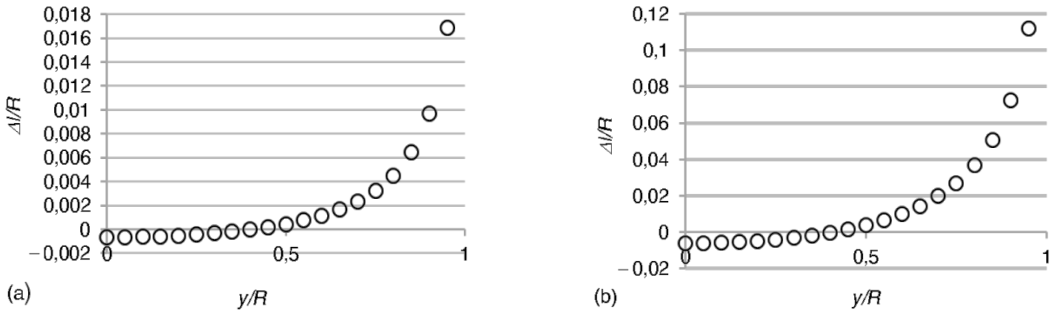


**Figure 8. Relative distances between measurement volumes for axial and radial velocity components versus the height of the center of the transmitting lens for ratios of wall thickness and internal radius of (a) 1%, and (b) 10%**

For any set of LDV system parameters, secondary focal length can be calculated using the method explained here. Results for two cases are presented in tab. 2 (other parameters are like in tab. 1).

**Table 2. Coinciding secondary focal lengths $f_c$**

| Case | $R$ (m) | $w$ (mm) | $f$ (m) | $d$ (m) | $f_c$ (m) |
|---|---|---|---|---|---|
| 1 | 0.125 | 4 | 0.3 | 0.06 | 0.2987 |
| 2 | 0.2 | 5.5 | 0.3 | 0.06 | 0.2982 |

In the application of lens with dual focal length in 2D LDV, special attention has to be paid to signal processing settings because the slight change of focal length for "axial" pair of beams cause the change of their intersection angle and likewise the change of calibration constant for axial velocity component.

Presented calculations are valid if the imperfections of a tube wall, such as uneven surface curvature radius and wall glass impurities, are negligible. Interference effects, laser stability effects, stochastic beam intensity fluctuations due to flow particles passing through the beams have also significant impact on measurement volume and therefore on LDV measurement quality, but, apart from fringe distortion and oscillation of measurement volume position, they do not change average positions of measurement volumes. Extension to 3-D LDV measurements, by the using of fifth laser beam through the same lens, requires further geometrical analysis. On the other hand, realization of 3-D velocity measurements with additional separate probe (additional lens with separate pair of beams) could have benefit of mathematical model used here and in [13].

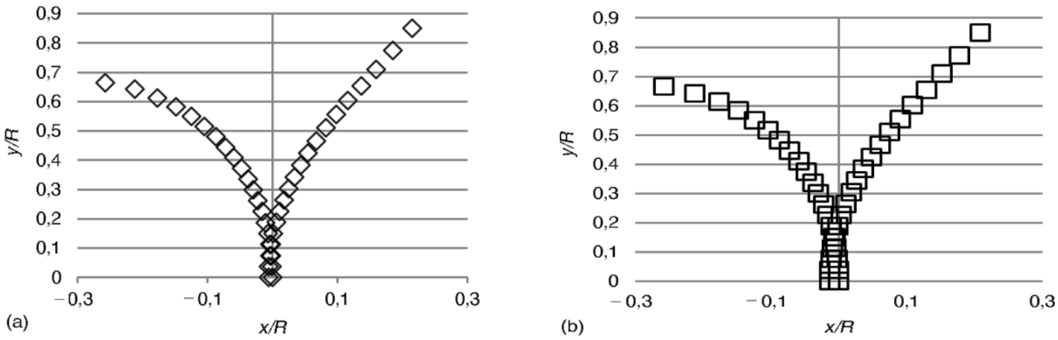


**Figure 9. Positions of measurement volumes when the flowing fluid is water and transmitting lens with dual focal length is applied; secondary focal length shift is (a) $\Delta f = \Delta f_c$, (b) $\Delta f = 0.98\Delta f_c$**

## Conclusions

The study, presented here, demonstrated that, in flows within cylindrical geometry, flowing fluid refractive index and the ratio of wall thickness and internal radius have strong impact on variations of 2-D LDV laser beam propagation, and measurement volume dislocation and separation. The changes of refractive indices of wall materials, laser wavelength, and angles of intersection have considerably weaker impact.

The analysis of the results of the mathematical simulation of application of the lens with a dual focal length in 2-D LDV system, in the case of a flow confined within a cylindrical geometry, shows that it can accomplish overlapping of the two measurement volumes, and that the method for the calculation of secondary focal length, presented here, is accurate enough. The lens surface with modified focal length is small related to surface of transmitting/receiving lens. Because of that, the modifications didn't influence on the receiving and focusing scattering light on photo-detector.

We have also shown here that the small increase or decrease of the shift from a primary to a secondary focal length, with respect to coinciding - one that is necessary for overlapping of the measurement volume in the center, enables extension of the region of a reliable 2-D LDV measurement (the increase is for gaseous fluids, and the decrease is for liquids).

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## Nomenclature

$d$ – beam separation at lens, [m]
$f$ – focal length of a lens, [m]
$h$ – the height of the center of transmitting lens, [m]
$l$ – the shortest distance between transmitting lens and a tube, [m]
$\Delta l$ – separation of measurement volumes, [m]
$n$ – refractive index, [–]
$R$ – internal tube radius, [m]
$w$ – the thickness of the wall, [m]
$\Delta x$ – the dislocation of measurement volume for axial velocity component, [m]

*Greek symbols*

$\delta$ – inclination angle of measurement volume, [rad]
$\theta$ – half angle of beam intersection, [rad] or [°]
$\lambda$ – laser beam wavelength, [m]

*Subscripts*

ax – axial
c – coinciding
g – wall glass
f – internal moving fluid
rad – radial
0 – surrounding fluid
1 – secondary (focal length)

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