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Determination of Geometrical Parameters for Semi-Rolling Bevel Precessional Gears With Straight Teeth

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Abstract. The aim of this paper is to analyse the semi-rolling bevel precessional gear with straight teeth. The given method of cutting pinion and gear teeth significantly simplifies its manufacture technology. Authors have given a system of equations to study the analysis of generating pinion tooth surfaces and geometry of meshing for semi-rolling bevel precessional gear with straight teeth. A computer program was developed to enable implement formulas provided in the paper. By using this program authors obtained contact lines on the pinion tooth surface and number of tooth pairs in gears mesh.

1. Introduction

The process of generation of a bevel gear [1, 2, 3] in comparison with cylindrical and worm (spiroid [4]) gears is more difficult. During the last decades, many papers [5, 6, 7] have been directed to the determination the optimal geometry of bevel gears which provide the required loading capacity for operating conditions. So we propose to use gearboxes based on bevel precessional gear (figure 1). Bevel precessional gears are applied in compact gearboxes for oil-and-gas equipment for transmitting rotation and torque [8, 9]. The most important characteristics for the quality of bevel precessional gearboxes are efficiency (about 0.9), wide range of gear ratios, low starting torque which is crucial for wide range of operating temperatures [8, 9]. Currently, the most fully researched bevel precessional gears are those with double-concave and double-convex teeth [8, 9]. The process of gear cutting is not only labor-intensive technology, but also is incredibly complex. It requires four re-settings of the cutting machine, as each tooth surface is cut separately. In this article we study semi-rolling bevel precessional gears with straight teeth. Despite the absence of localized contact in the studied gear, as a result of multiple teeth contact, it is rival product while manufacturing high-loaded gearboxes which operating under low angular velocity and short term duty.

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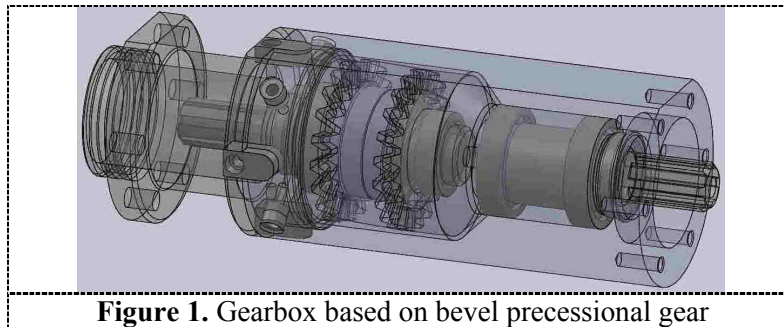


Figure 1. Gearbox based on bevel precessional gear

2. Analysis of gears mesh in the semi-rolling bevel precessional gears with straight teeth

Figure 2 describes design scheme of scheme of the semi-rolling bevel precessional gear with straight teeth. Cutting of the gear tooth space is done by a cutting tool with a straight-line cutting edge. Gear remains fixed during gear cutting.

Prior to gear cutting each of the following the gear tooth space, the gear rotates at an angle equal to tooth pitch angle, i.e. a method of «unit division» is realized. As cutting tools we can use cutters (cutting without generating on a gear-shaping machine), end-mill type cutters or side milling cutters (cutting on a milling machine with a turntable).

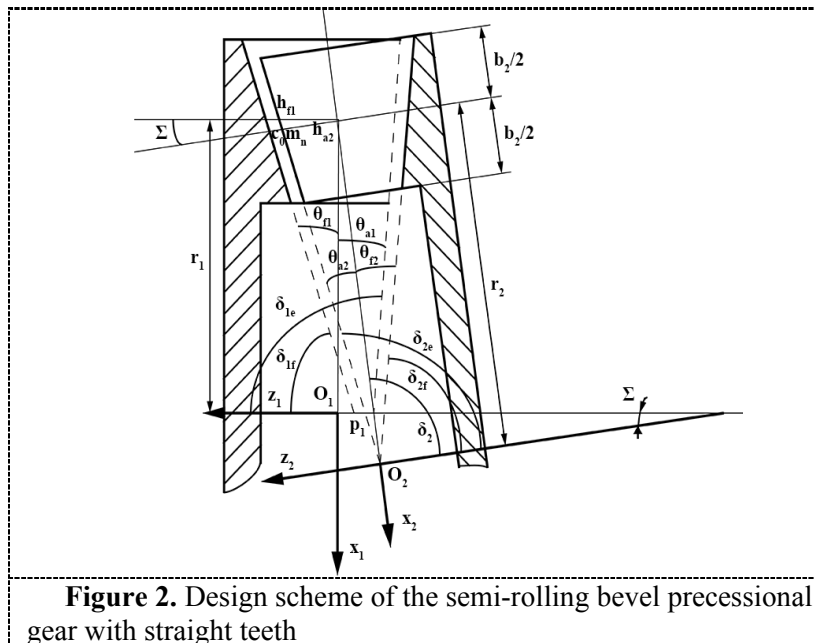


Figure 2. Design scheme of the semi-rolling bevel precessional gear with straight teeth

From the described gear cutting method it follows that the gear tooth surface is a plane. The reference surface of tooth is a plane that goes through point M (center of action) parallel to the plane $x_2o_2y_2$ (perpendicular to the axis z_2). The coordinate system $S_2(x_2, y_2, z_2)$ is rigidly connected to the gear. In the coordinate system $S_p(x_p, y_p, z_p)$ tooth surface, which axis y_p is directed along the normal towards the gear tooth surface, is a plane and calculated by equation (1):

$$x_p = u \quad y_p = 0 \quad z_p = h \quad (1)$$

where u is the line coordinate along the tooth length; h is the line coordinate along the tooth profile.

Let us express the position vector $\overline{r_p}$ of the tooth surface («plane») in the coordinate system $S_p(x_p, y_p, z_p)$ as a row matrix $\tilde{r}_p = \|\|x_p; y_p; z_p; 1\|\|$, elements of which are coordinates x_p, y_p, z_p , and the position vector of the tooth surface $\overline{r_2}$ in the system $S_2(x_2, y_2, z_2)$, - as a row matrix $\tilde{r}_2 = \|\|x_2; y_2; z_2; 1\|\|$, elements of which are defined by coordinates (x_2, y_2, z_2) . Then tooth surface in the system $S_2(x_2, y_2, z_2)$ is described as the formula:

$$\tilde{r}_2 = \tilde{A}_{2p} \cdot \tilde{r}_p \quad (2)$$

where \tilde{A}_{2p} is the square matrix (4 x 4) [7, 8, 9], which is define coordinate transformation from coordinate system S_p to coordinate system S_2 .

Solving equation (2), we obtain:

$$\left. \begin{aligned} x_2 &= u \cdot \cos \theta_{f2} - h \cdot \sin \theta_{f2} \cdot \cos \alpha_n - r_2 \\ y_2 &= -h \cdot \sin \alpha_n + t \\ z_2 &= u \cdot \sin \theta_{f2} + h \cdot \cos \theta_{f2} \cdot \cos \alpha_n \end{aligned} \right\} \quad (3)$$

where θ_{f2} is the gear dedendum angle; α_n is the pressure angle; r_2 is the mean cone distance of gear; t is a half of the tooth thickness.

In the studied semi-rolling bevel precessional gear with straight teeth pinion tooth surface is an envelope in a one-parameter motion. The imaginary generating surface is a gear tooth surface. Figure 3 describes the coordinate systems between the imaginary gear and work gears. The coordinate system $S_2(x_2, y_2, z_2)$ is rigidly connected to the gear and the coordinate system $S_1(x_1, y_1, z_1)$ is rigidly connected to the pinion during generation pinion surface.

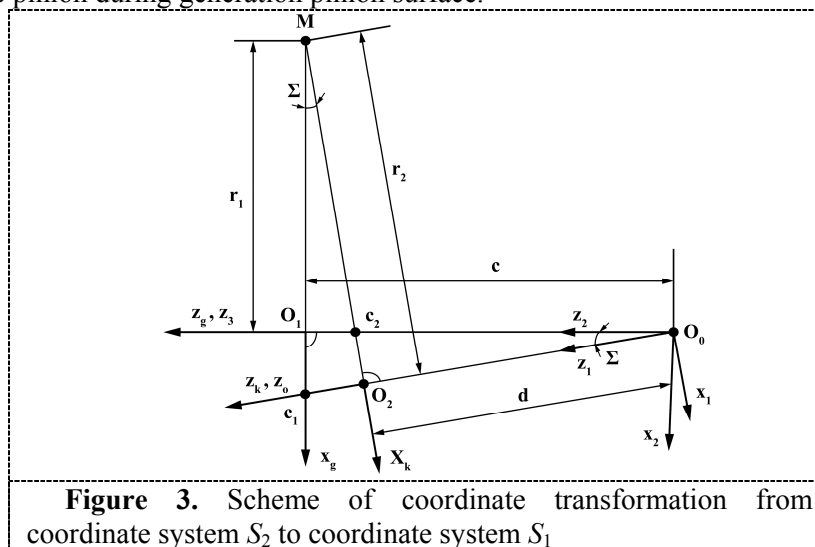


Figure 3. Scheme of coordinate transformation from coordinate system S_2 to coordinate system S_1

Due to the machine kinematics of gear cutting, when the gear tooth rotates around its axis at an angle φ_2 pinion rotates around its axis at an angle φ_1 , related to the angle φ_2 by formulas:

$$\begin{aligned} \varphi_1 &= \varphi_2 \cdot i \\ i &= Z_2^* / Z_1^* \end{aligned} \quad (4)$$

where i is the gear ratio of the machine generating train, z_2^* is a number of gear teeth, z_1^* is a number of pinion teeth.

The coordinates of current point of the gear tooth surface (1) can be determined by two independent parameters: u and h , that is $\tilde{r}_2 = \tilde{r}_2(u, h)$.

Due to the rolling motion when generating the pinion tooth surface, the matrix of relative motion \tilde{A}_{12} is a function of parameter φ_1 : $\tilde{A}_{12} = \tilde{A}_{12}(\varphi_1)$. A position vector \tilde{r}_1 of the pinion tooth surface in the coordinate system $S_1(x_1, y_1, z_1)$ (figure 3), in matrix form is determined as:

$$\tilde{r}_1 = \tilde{A}_{12} \cdot \tilde{r}_2 \quad (5)$$

where $\tilde{r}_1 = \|\|x_1, y_1, z_1, 1\|\|$ is a row matrix, composed of coordinate projections of the position vector \overline{r}_1 of the pinion tooth surface.

According to figure 2 elements a_{ij} , $i = \overline{1,4}$, $j = \overline{1,4}$ of matrix \tilde{A}_{12} have the form:

$$\begin{aligned} a_{11} &= \cos \varphi_1 \cdot \cos \Sigma \cdot \cos \varphi_2 + \sin \varphi_1 \cdot \sin \varphi_2 \\ a_{12} &= -\cos \varphi_1 \cdot \cos \Sigma \cdot \sin \varphi_2 + \sin \varphi_1 \cdot \cos \varphi_2 \\ a_{13} &= \cos \varphi_1 \cdot \sin \Sigma \\ a_{14} &= d \cdot \cos \varphi_1 \cdot \sin \Sigma \\ a_{21} &= -\sin \varphi_1 \cdot \cos \Sigma \cdot \cos \varphi_2 + \cos \varphi_1 \cdot \sin \varphi_2 \\ a_{22} &= \sin \varphi_1 \cdot \cos \Sigma \cdot \sin \varphi_2 + \cos \varphi_1 \cdot \cos \varphi_2 \\ a_{23} &= -\sin \varphi_1 \cdot \sin \Sigma \\ a_{24} &= -d \cdot \sin \varphi_1 \cdot \sin \Sigma \\ a_{31} &= -\sin \Sigma \cdot \cos \varphi_2 \\ a_{33} &= \cos \Sigma \\ a_{34} &= d \cdot \cos \Sigma - c \\ a_{41} &= a_{42} = a_{43} = 0 \\ a_{44} &= 1 \end{aligned} \quad (6)$$

The values c and d included in the elements a_{ij} are calculated according to the formulas (1):

$$\begin{aligned} c &= r_1 \cdot (i - \cos \Sigma) \cdot (\sin \Sigma)^{-1}; \\ d &= r_1 \cdot (i \cdot \cos \Sigma - 1) \cdot (\sin \Sigma)^{-1} \end{aligned} \quad (7)$$

where r_1 is the mean cone distance of pinion, Σ is a shaft angle.

Solving formula (5), we obtain:

$$\tilde{r}_1(u, h, \varphi_1) = \tilde{A}_{12}(\varphi_1) \tilde{r}_2(u, h) \quad (8)$$

As the tooth surface can have only two independent parameters, for mathematical description of the pinion tooth surface it is necessary to relate an additional relation among parameters φ_1 , u and h . In the theory of gearing [10] such relation is referred to equation of meshing:

$$f(u, h, \varphi_1) = 0 \quad (9)$$

If the equation of meshing is known, the pinion tooth surface, as an envelope to the family of gear tooth surface, is described as follows [10, 11]:

$$\begin{aligned} \tilde{r}_1(u, h, \varphi_1) &= \tilde{A}_{12}(\varphi_1) \tilde{r}_2(u, h) \\ f(u, h, \varphi_1) &= 0 \end{aligned} \quad (10)$$

In this study, to determine the equation of meshing, we used the method suggested in the book [9]. Pinion tooth surface will be an envelope to the family of gear tooth surfaces in relative motion with parameter φ_1 only when relative velocity \overline{V}_φ is perpendicular to the unit vector \overline{m}_2 of generating surface (gear tooth surface). This condition corresponds to the fact that the scalar product \overline{V}_φ and \overline{m}_2 is equal to zero [10, 11]. Then, the equation of meshing may be derived as follows:

$$\begin{aligned} \tilde{r}_1(u, h, \varphi_1) &= \tilde{A}_{12}(\varphi_1) \tilde{r}_2(u, h) \\ f(u, h, \varphi_1) &= 0 \end{aligned} \quad (11)$$

Projections of m_{2x} , m_{2y} , m_{2z} on the unit vector \overline{m}_2 of gear tooth surface (3), determined as:

$$\begin{aligned} m_{2x} &= -\sin \theta_{f2} \cdot \sin \alpha_n; \\ m_{2y} &= \cos \alpha_n; \\ m_{2z} &= \cos \theta_{f2} \cdot \sin \alpha_n. \end{aligned} \quad (12)$$

Where \tilde{A}_{21} is a square matrix (4 x 4), which is inverse to matrix \tilde{A}_{12} (6); $\frac{d\tilde{A}_{12}}{d\varphi_1}$ is a square matrix (4 x 4), determined by differentiation of matrix \tilde{A}_{12} (6) with respect to φ_1 , \tilde{r}_2 is a column matrix, composed by coordinates of position vector gear tooth surface (3).

Relative velocity \tilde{V}_φ with parameter φ_1 is determined by equation, which based on the work [9]:

$$\tilde{V}_\varphi = \tilde{A}_{21} \cdot \frac{d\tilde{A}_{12}}{d\varphi_1} \cdot \tilde{r}_2 \quad (13)$$

Equation (13) yield:

$$\begin{aligned} V_{\varphi x} &= -f_3 \cdot (i^{-1} - \cos \Sigma) - \sin \Sigma \cdot \sin \varphi_2 \cdot (f_2 + d) \\ V_{\varphi y} &= f_1 \cdot (i^{-1} - \cos \Sigma) - \sin \Sigma \cdot \cos \varphi_2 \cdot (f_2 + d) \\ V_{\varphi z} &= \sin \Sigma \cdot (f_1 \cdot \sin \varphi_2 + f_3 \cdot \cos \varphi_2) \end{aligned}$$

Here

$$\begin{aligned} f_1 &= u \cdot \cos \theta_{f2} - h \cdot \sin \theta_{f2} \cdot \cos \alpha - r_2 \\ f_2 &= u \cdot \sin \theta_{f2} + h \cdot \cos \theta_{f2} \cdot \cos \alpha_n \\ f_3 &= t - h \cdot \sin \alpha_n \end{aligned}$$

As a result, formulas determining the pinion tooth surface described as:

$$\begin{aligned} x_1 &= A_1 \cdot \cos \varphi_1 + B_1 \cdot \sin \varphi_1 \\ y_1 &= -A_1 \cdot \sin \varphi_1 + B_1 \cdot \cos \varphi_1 \\ z_1 &= \sin \Sigma \cdot (f_3 \cdot \sin \varphi_2 - f_1 \cdot \cos \varphi_2) + \cos \Sigma \cdot (f_2 + d) - c \\ \varphi_2 &= \arcsin \left[-C_\varphi \cdot (\sqrt{A_\varphi^2 + B_\varphi^2})^{-1} \right] - \xi \end{aligned} \quad (14)$$

Here,

$$\begin{aligned} A_1 &= \cos \Sigma \cdot (f_1 \cdot \cos \varphi_2 - f_3 \cdot \sin \varphi_2) + \sin \Sigma \cdot (f_2 + d) \\ B_1 &= f_3 \cdot \sin \varphi_2 - f_1 \cdot \cos \varphi_2 \\ A_\varphi &= \sin \alpha_n \cdot \sin \Sigma \cdot (u + d \cdot \sin \theta_{f2} - r_2 \cdot \cos \theta_{f2}) \\ B_\varphi &= -\sin \Sigma \cdot (f_2 \cdot \cos \alpha_n - f_3 \cdot \sin \alpha_n \cdot \cos \theta_{f2} + d \cdot \cos \alpha_n) \\ C_\varphi &= (i^{-1} - \cos \Sigma) \cdot (f_1 \cdot \cos \alpha + f_3 \cdot \sin \theta_{f2} \cdot \sin \alpha) \end{aligned}$$

where angle ξ is determined based on its trigonometric functions:

$$\begin{aligned} \sin \xi &= B_\varphi \cdot (\sqrt{A_\varphi^2 + B_\varphi^2})^{-1} \\ \cos \xi &= A_\varphi \cdot (\sqrt{A_\varphi^2 + B_\varphi^2})^{-1} \end{aligned}$$

3. Conclusion

These formulas enable us, on the one hand, to perform the analysis of generating pinion tooth surfaces, and, on the other hand, to study geometry of meshing in the semi-rolling bevel precessional gearing with straight teeth, which is «matched» according to the way it generation of gear tooth surfaces.

By using formulas (14) and MathCAD software a computer program was developed to study the position and contact lines in gears mesh of the semi-rolling bevel precessional gear with straight teeth. The program showed its contact lines with the pinion tooth surface and the gear tooth surface of the

semi-rolling bevel precessional gearing with straight teeth for a number of fixed values of the angle of action ($\varphi_1 = -0,172; -0,115; -0,057; 0,0; 0,088; 0,177; 0,265$) (figure 4). Pinion and gear design parameters are given in table 1.

Table 1. Pinion and gear design parameters

| | Number of teeth | Normal module (mm) | Pressure angle (degree, °) | Face width (mm) | Pitch angle (degree, °) | Addendum factor | Clearance coefficient |
|---------------|-----------------|--------------------|----------------------------|-----------------|-------------------------|-----------------|-----------------------|
| Pinion | 64 | 5.0 | 20 | 0.593 | 88 | 1 | 0.5 |
| Gear | 65 | 5.0 | 20 | 0.796 | 90 | 1 | 0.5 |

In the studied semi-rolling bevel precessional gear with straight teeth a change in angle φ_1 from $\varphi_{1\min} = -0,172$ to $\varphi_{1\max} = 0,265$ is according to a maximum angle of action. Taking into account the value of the tooth pitch angle on the pinion ($t_1 = 2 \cdot \pi / z_1^* = 0,09817$), we determined that $(-\varphi_{1\min} + \varphi_{1\max}) / t_1 = 4,45$ tooth pairs are simultaneously in gears mesh. A prototype of semi-rolling bevel precessional gear with straight teeth is shown in figure 5.

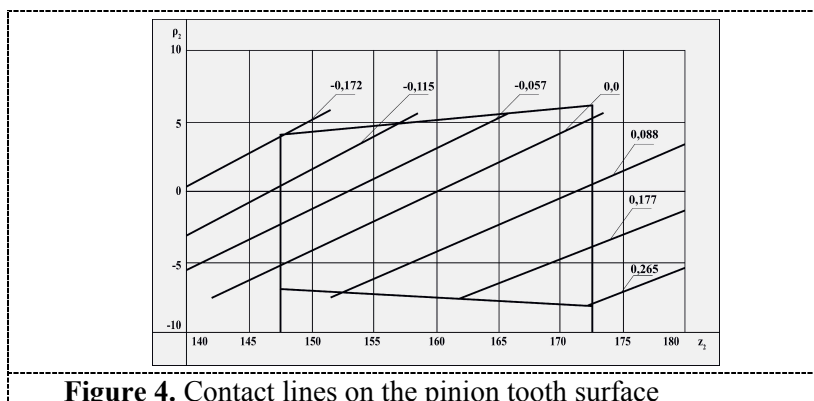


Figure 4. Contact lines on the pinion tooth surface

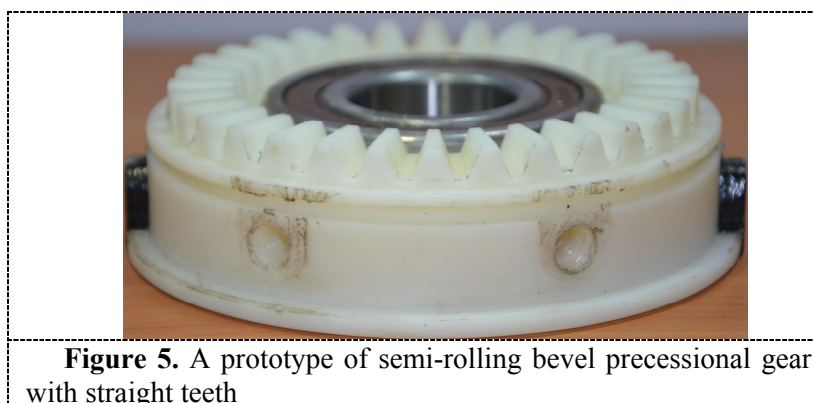


Figure 5. A prototype of semi-rolling bevel precessional gear with straight teeth

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