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Recognition of one class of quadrics from 3D point clouds

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Abstract

Within cyber physical production systems 3D vision as a source of information from real-world provides enormous possibilities. While the hardware of contemporary 3D scanners is characterized by high speed along with high resolution and accuracy, there is a lack of real-time on-line data processing algorithms that would give certain elements of intelligence to the sensory system. Critical elements of data processing software are efficient, real-time applicable methods for fully automatic recognition of high level geometric primitives from point cloud (surface segmentation and fitting). This paper presents a method for recognition of one class of quadrics from 3D point clouds, in particular for recognition of cylinders, elliptical cylinders and ellipsoids. The method is based on the properties of scatter matrix during direct least squares fitting of ellipsoids. Presented recognition procedure can be employed for segmentation of regions with G1 or higher continuity, and this is its comparative advantage to similar methods. The applicability of the method is illustrated and experimentally verified using two case studies. First case study refers to a synthesized, and the second to a real-world scanned point cloud.

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Keywords: 3D point cloud; surface recognition; parametric model;

1. Introduction

Smart devices (sensors and actuators) with embedded computational and communication modules are nowadays widely implemented in manufacturing. These devices provide enormous possibilities for control of manufacturing processes through interconnection of physical systems and cyber world within Cyber Physical Systems (CPS) concept. CPS are the systems that integrate physical processes and computing elements through their real-time interaction [1]. They can be regarded as systems of systems [2] and can be implemented at various levels of manufacturing process control, starting from manufacturing resources' elements, their subsystems, machines, up to the manufacturing system as a whole. Cyber Physical Production Systems (CPPS) are foreseen as the highest level of CPS implementation in manufacturing. There are a number of R&D challenges related to CPPS [3], and one of them is the fusion of real world manufacturing systems and their virtual representation (Fig. 1.). CPPS require real-time interconnection between shop floor and its virtual representation through automatic data acquisition/information retrieval from real world and feedback from cyber system. Timely delivery of big data from real-world, as well as timely and effective extraction and interpretation of information are crucial for implementation of this concept.

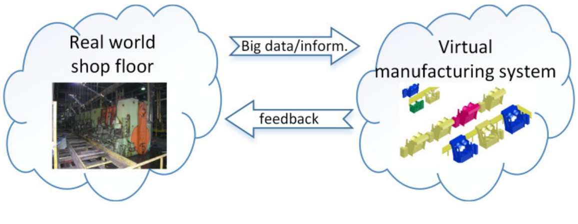

Fig. 1. Fusion of real world manufacturing systems and its virtual representation.

Retrieval (extraction and interpretation) of geometric information and its timely representation in virtual system is one of the most important issues within CPPS. Besides, this kind of feedback is significant element of virtualization module within cloud manufacturing concept in which virtual manufacturing resources should be available to all interested parties (consumers, providers, operators…) through virtual enterprise [4]. Contemporary three-dimensional (3D) scanning

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devices are characterized by high speed, resolution, and accuracy [5] and can be effectively employed for acquisition of big data in the form of 3D point clouds. These point clouds contain information about geometric features of objects from the real world. There exist a number of very efficient algorithms for point cloud registration, integration, and meshing [6], and automatic generation of aesthetically sound 3D model in the form of triangular mesh from point cloud is a standard feature of CAD systems. However, retrieval of high level information about geometric features of scanned object in the form of geometric primitives and generation of parametric 3D CAD model from point cloud are still carried out interactively by user and represent an open research area.

Geometric primitives that build objects along with their parameters can be regarded as the most important geometric features of objects. Extraction of information about these features from point cloud, i.e. recognition of geometric primitives from point cloud is carried out through the following steps: 1) segmentation of points that belong to specific geometric primitive, and 2) estimation of parameters of geometric primitive segmented in previous step. For the success of this process, the first step is crucial – once geometric primitive is adequately segmented, the estimation of its parameters (fitting) is straightforward. Implementation of 3D point clouds within CPPS requires effective, real-time applicable segmentation algorithms that are executed automatically.

Planes and rotational surfaces, in particular cylinders, are most frequently met geometric primitives in mechanical engineering. Recognition of planar surfaces from 3D point cloud has attracted most research efforts, and there is a number of techniques based on: 3D Hough transform [7], RANAC (Random Sample Consensus) algorithm [8,9], various region growing algorithms [10,11], wavelet transform [12], etc. Cylinders, on the other hand, belong to the class of natural second order surfaces (quadrics). Retrieval of quadrics' geometric parameters from point clouds has also been the subject of a number of research works. All, so far proposed strategies for segmentation of quadrics from 3D point clouds can be classified into the following two groups [13]: 1) strategies based on edge detection, and 2) strategies based on regions. Edge based strategies assume that there is an abrupt change between two adjacent geometric primitives in the point cloud and they are only applicable for G0 continuous surfaces. On the other hand, region based segmentation is carried out using split and merge approach or using region growing. In region growing techniques, algorithm starts from a chosen seed point (or a seed region) and grows a region around it using different criteria referring to differential geometry features of surfaces such as local surface normal [14], average curvature [15], principal curvatures [16], or bicubic Bézier surface properties [17]. In the most of these methods the seed point is selected manually, since automatic selection of seed point can be a complex task. An interesting approach for segmentation of quadrics from scanned lines that is based on numerically stable least squares fitting of ellipse is presented in [18].

This paper represents an extension of our previous research referring to recognition of elliptical segments from scanned lines [19] and to initial research referring to recognition of quadrics from point clouds [20]. We present a method for recognition of one class of quadrics from unstructured 3D point clouds, along with the guidelines for interpretation of recognized segments in the form suitable as input to CAD system. In particular, we consider second order surfaces that can be represented by ellipsoid equation, such as cylinders, elliptical cylinders, spheres, and ellipsoids. The method is region growing method that exploits properties of scatter matrix calculated during direct least squares fitting of ellipsoids as a region growing criterion. Opposite to edge based techniques, the proposed method is convenient for recognition of adjacent surface segments with G1 or higher continuity. An advantage of proposed method compared to region based techniques is that seed point is selected automatically thus providing fully automatic performance of the recognition process.

The rest of the paper is organized as follows. In Section 2 we present the method for recognition of considered class of quadrics, relevant theoretical background, and the role of the method within the system for retrieval of information about geometric features from 3D point cloud. Section 3 illustrates the applicability of the method using two case studies; first case study considers synthesized point cloud, and the second a point cloud obtained by scanning of a real world object. Finally in Section 4, we provide some concluding remarks and future work guidelines

2. Recognition of one class of quadrics from 3D point clouds

Proposed method for recognition of considered class of quadrics from 3D point clouds represents a part of the system (information machine) for retrieval of information about considered geometric primitives from point cloud. This system has the following two modules: 1) module for recognition (segmentation and fitting) of G1 continuous surface segments, and 2) module for interpretation of extracted information (Fig. 2.). Module for recognition of surfaces segments and estimates surfaces' parameters from 3D point cloud using algorithm that is presented in the sequel. From this module, estimated surfaces' parameters are output in the vector form which is not convenient for automatic input to CAD models within virtual manufacturing system. This information needs to be transformed into one of the suitable, preferably neutral, formats for model representation in virtual manufacturing environment, such as STEP, IGES, UPR [21]. Information interpreter (Fig. 2) carries out required transformation. In our research we have opted to use transformation into Initial Graphics Exchange Specification - IGES format.

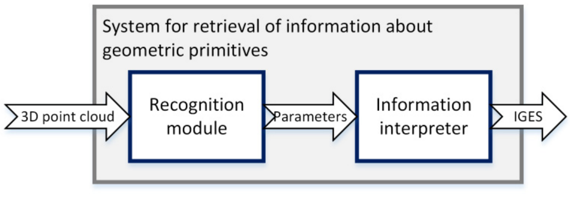

Fig. 2. System for retrieval of information about geometric primitives.

In this section we will first provide some theoretical background regarding direct least squares fitting of ellipsoids, and the method for recognition of considered quadrics from point clouds along with accompanying algorithm will be presented afterwards. Finally we will provide some guidelines for generation of IGES surface representation from estimated segments' parameters.

2.1. Direct least squares fitting of ellipsoids

Ellipsoids represent quadric surfaces which, in general can be described using the following relation:

$$a_1x^2 + a_2y^2 + a_3z^2 + a_4xy + a_5yz + a_6xz + a_7x + a_8y + a_9z + a_{10} = 0 \tag{1}$$

where [x y z] represent coordinates of point on the surface and a_i denote surface parameters. Equation (1) can be written in vector form:

$$\mathbf{x} \cdot \mathbf{a} = 0 \tag{2}$$

where $\mathbf{x}=[x^2\ y^2\ z^2\ xy\ yz\ xz\ x\ y\ z\ 1]$, and $\mathbf{a}=[a_1\ a_2 \ldots\ a_{10}]^T$ denotes the vector of surface parameters. There are 17 different types of quadric surfaces that can be distinguished using the correlation of coefficients a_i [22]. In this paper real ellipsoid and real elliptic cylinders, as well as their special cases – sphere and cylinder, are of interest. Real elliptic cylinder can be observed as a degenerate real ellipsoid with smaller ranks of specific matrices [22], so the same fitting procedure can be used for both types of surfaces.

Recognition method that we present in this paper employs the procedure for direct least square fitting of ellipsoid that was recently developed [23] starting from well known Fitzgibbon's direct least squares fitting of ellipse [24]. The procedure is based on the postulate that the cross section of ellipsoid and arbitrary plane represents an ellipse, and can be briefly presented as follows [23]. Ellipsoid parameters a_i are the solution of the minimization problem:

$$\begin{gathered} min\|\mathbf{Da}\|^2 \\ \text{subject to } \mathbf{a}^T\mathbf{Ma} = 1 \end{gathered} \tag{3}$$

Where matrix $\mathbf{D}$ is design matrix which has dimension N×10 (N is the number of points in the cloud) in the form:

$$\mathbf{D} = \begin{bmatrix} x_i^2 & y_i^2 & z_i^2 & x_iy_i & y_iz_i & x_iz_i & x_i & y_i & z_i & 1 \end{bmatrix}, i = 1, \ldots N \tag{4}$$

and matrix $\mathbf{M}$ has the dimension 10×10 with values that depend on the parameters of arbitrary plane that intersects ellipsoid [23]. Ellipsoidal solution of minimization problem (3) is single positive eigenvalue of [23]:

$$\mathbf{Sa} = \lambda\mathbf{Ma} \tag{5}$$

where λ denotes Lagrange multiplier, and matrix $\mathbf{S}$:

$$\mathbf{S} = \mathbf{D}^T\mathbf{D} \tag{6}$$

is scatter matrix.

When all points are sampled from an exact ellipsoid without noise, matrix $\mathbf{S}$ will be singular and its reciprocal condition number will be close to zero. As scatter of points from estimated ellipsoid is larger, the reciprocal condition number of scatter matrix will have higher value.

2.2. Method for recognition of considered class of quadrics from 3D point cloud

The method for recognition that we present in this paper belongs to the class of region growing techniques. In our approach region growing strategy is based on the value of reciprocal condition number of scatter matrix from (6).

Recognition method can be described by algorithm that is briefly presented in Fig. 3. It starts from automatically selected seed point and expands region around it point by point using developed region growing strategy [20].

In the presented algorithm we automatically choose seed point as the point with minimum value of z coordinate. This choice was made since there is a large possibility that selected seed point will be the border point of surface or surface segment extremum point, and that it will lead to better segmentation results. In our initial research [20] seed point was randomly selected. However, selection of seed point that is utilized in this paper has shown better results.

Developed region growing strategy detects the nearest neighbors of seed point using k-nearest-neighbors (kNN) algorithm [25]. This algorithm selects the data with smallest algebraic distance from selected point. Generally, distance can be calculated using different distance functions (e.g., Euclidean, Lagrange, Manhattan). In this research we employ kNN with Euclidean distance to sort points from point cloud

```
INPUT: x y z – coordinates of N points from point cloud;
do until all points from point cloud are exhausted or until predefined number
of trials is attempted
  select seed point ps as point from point cloud with minimum z
  do until all strategies are exhausted
    sort points from point cloud acceding according to distance from ps
    select pi - next nearest point to ps and temporarily add it to the region
    if for all points in region rcond(S)>threshold
      select new strategy and new seed point ps
    else
      permanently add pi to the region and remove it from point cloud
    end
  end
  do until no more points are added to the region
    compute parameters ai of ellipsoid surface for points in the region
    select points from cloud that belong to surface with predefined threshold;
    remove these points from point cloud and add them to the region
  end
end
OUTPUT: ai=[a1 a2… a10] – parameters of m recognized segments
```

Fig. 3. Algorithm of the method for recognition of considered class of quadrics from 3D point cloud.

in acceding order according to their vicinity to the seed point. Thus sorted points are added one by one to the region using the following procedure.

Point is temporarily added to the region and reciprocal condition number of scatter matrix **S** of all points in the region is calculated. If the value of the reciprocal condition number is smaller than predefined threshold, point is permanently added to the region – the point belongs to ellipsoid since scatter matrix is close to singular. On the other hand, if reciprocal condition number is larger than predefined threshold, scatter of points from ellipsoid is high and the point belongs to another surface.

Since we use Euclidean distance for region growing, detected region will be within the sphere with center in the seed point. The region will be expanded until algorithm finds the point that does not belong to the given ellipsoidal segment according to reciprocal condition number condition; this point is the marker point. After detection of marker point, to carry on region growing, new seed point should be selected. For selection of new seed point we employ three strategies [20]:

Strategy 1: New seed point is the point which is "half way" between marker and the point from the region with largest distance from marker.

Strategy 2: New seed point is the point from the region with largest distance from marker point

Strategy 3: New seed point is in the vicinity of marker point, i.e., the point that was added to the region five cycles before marker point.

These strategies are sequentially employed until no more points can be added to the region. Obtained region contains points from one ellipsoidal surface segment and can be utilized as a basis for segmentation.

During segmentation, algorithm calculates parameters of the surface segment whose points are in the region, and adds to the region all the points from the cloud whose distance from the surface segment is smaller than predefined threshold. This procedure is iteratively applied until no more points are added to the region. In this way one ellipsoidal surface segment is extracted from point cloud.

The whole procedure repeats from the beginning, and new segments are extracted until all points from point cloud are exhausted or until a predefined number of trials are attempted.

In some situations the procedure can lead to over-segmentation [20]. To remove these effects, at the end of segmentation algorithm temporarily merges two segments and calculates ellipsoid surface parameters. If all points from temporarily merged segments belong to estimated surface with predefined threshold, segments are permanently merged, or remain separated otherwise. At the output of the method we obtain m ellipsoidal surface segments and their parameters in the form $\mathbf{a_i}=[a_1\ a_2 \ldots\ a_{10}]^T$, i=1,…*m*.

2.3. Interpretation of estimated segments' parameters

Parameters of surface segments obtained from algorithm presented in Fig. 3 are in vector form and they are not suitable for input into the CAD system. This information needs to be translated into the acceptable format. In this paper we have opted to use IGES neutral format [26] for parametric representation of 3D model. To carry out this translation, we first have to determine the type of quadric segment. This is done according to the properties of coefficient matrix and classification matrix which are obtained from segment parameters vector [27]. In addition, there is a need to represent segments in canonical coordinates [27], and to calculate their cross section curves. Once this preprocessing is carried out, IGES file can be automatically created according to the IGES specification [26].

3. Experimental verification

Presented method is experimentally verified using two case studies. First case study refers to a synthesized point cloud, and the second to an example of a scanned real world surface.

3.1. Case study 1: Synthesized point cloud

To illustrate the applicability of the presented method, we have synthesized a very simple point cloud that contains three

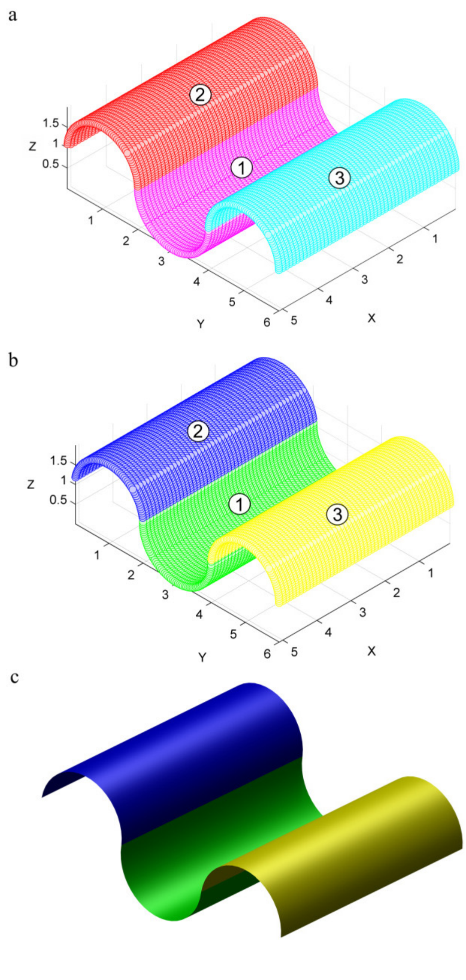

Fig. 4. Case study 1: (a)synthesized point cloud; (b)results of G1 continuous segments' recognition; (c)IGES representation of recognized segments.

Table 1. Synthesized and estimated parameters of surface from Fig. 4; surface equation: $a_1x^2+a_2y^2+a_3z^2+a_4xy+a_5yz+a_6xz+a_7x+a_8y+a_9z-1=0$.

Synthesized surface parameters (Fig. 4a) [$a_i \times 10^3$], i= 1, …9									
Segment 1	[0	-0.111	-0.111	0	0	0	0	0.667	0.222]
Segment 2	[0	-1	-1	0	0	0	0	2	2]
Segment 3	[0	-0.04	-0.04	0	0	0	0	0.4	0.08]
Estimated surface parameters (Fig. 4b) [a_i], i= 1, …9									
Segment 1	[0.0	-0.111	-0.111	0.0	0.0	0.0	0.0	0.667	0.222]
Segment 2	[0.0	-1.000	-1.000	0.0	0.0	0.0	0.0	2.0	2.000]
Segment 3	[0.0	-0.040	-0.040	0.0	0.0	0.0	0.0	0.40	0.080]

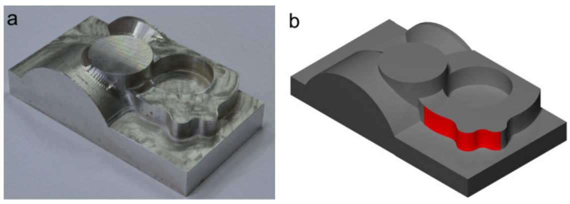

Fig. 5. Case study 2: (a)Photograph of test part used for real world verification; (b) CAD model of test part with marked surface of interest.

G1 continuous cylindrical segments (Fig. 4a). Parameters of the synthesized segments are presented in the first part of Table 1. To the point cloud we have added white noise with signal to noise ratio of 80dB.

The presented method was able to adequately recognize G1 continuous cylindrical segments as illustrated in Fig. 4b and Table 1. From Table 1 it can be observed that for the presented number of decimal places the values of estimated and synthesized parameters are almost equal. Maximal relative error between synthesized and estimated parameters when calculated with higher number of decimal places is below 0.35%. During segmentation the reciprocal condition number threshold was set to 10^{-16}, and the threshold for checking whether a point belongs to the surface was 0.0005.

According to the properties of coefficient matrix and classification matrix calculated from parameters (Table 1), all three recognized segments belong to the class of real elliptic cylinders. Segments' parameters from Table 1 define real elliptic cylinders in global coordinate system. After transformation of these parameters into canonical coordinates, generation of graphical transformation matrix between these two coordinate systems, and calculation of cross-section curves, we have created IGES representation of recognized segments (Fig. 4c).

3.2. Case study 2: Real-world point cloud

Second case study considers a real-world part shown in Fig. 5a. This part represents a typical benchmark test used in a number of studies referring to recognition of geometric primitives from point clouds [12,13]. The part was machined on 3-axis horizontal machining center and it is characterized by very poor quality, as can be observed from Fig. 5a. Part was scanned using ATOS Compact Scan 3D scanning device by GOM mbh [28], and unstructured point cloud was obtained. In this case study we have analyzed surface that is marked on 3D model from Fig. 5b. This surface consists of 5 G1 continuous cylindrical segments.

The presented procedure was applied to the selected surface and segmentation results are shown in Fig. 6a, whereas Table 2 presents estimated parameters. We have applied the same thresholds as in the first case study, i.e. 10^{-16} for reciprocal condition number, and 0.0005 for detection of points that belong to the surface.

Based on surface parameters from Table 2 it can be found that segments marked 1-4 in Fig. 6a represent real elliptic cylinders, while segment 5 belongs to the class of elliptic paraboloids [27]. Starting form estimated parameters recognized surfaces are presented in IGES format (Fig. 6b) that is suitable for direct input into the CAD system.

Table 2. Estimated parameters of surface from Fig. 6; surface equation: $a_1x^2+a_2y^2+a_3z^2+a_4xy+a_5yz+a_6xz+a_7x+a_8y+a_9z-1=0$.

Estimated surface parameters (Fig. 6a) [$a_i \times 10^2$], i= 1, …9									
Segment 1	[0.0	-0.165	-0.181	0.0	0.0	0.0	0.0	-2.814	-8.120]
Segment 2	[0.0	-0.350	-0.340	0.0	0.0	0.0	0.0	-5.615	-10.468]
Segment 3	[0.0	-0.231	-0.230	0.0	0.0	0.0	0.0	-6.932	-6.889]
Segment 4	[0.0	-0.179	-0.241	0.0	0.0	0.0	0.0	-7.832	-4.352]
Segment 5	[0.0	-0.038	-0.270	0.0	0.0	0.0	0.0	-2.305	-8.606]

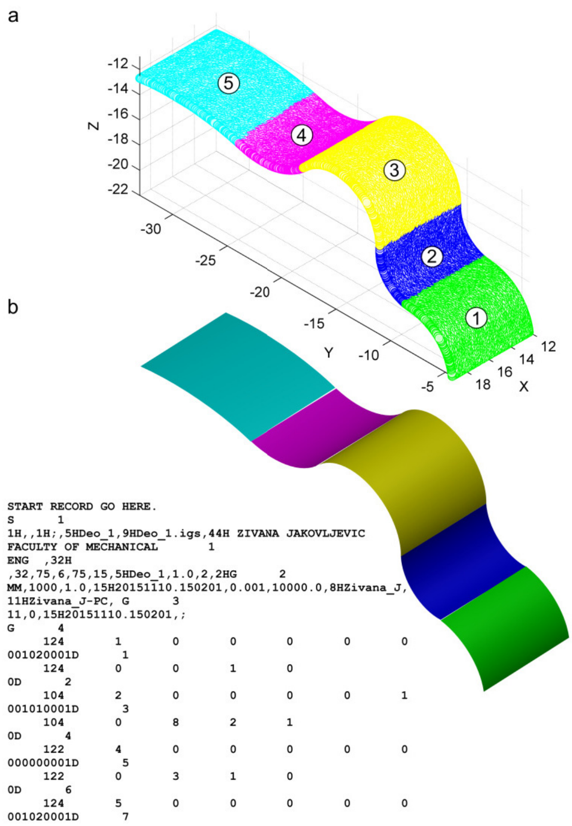

Fig. 6. Case study 2: (a) results of segments' recognition; (b)IGES representation of recognized segments.

4. Conclusion

In this paper we have presented a method for recognition of a class of quadrics from unstructured point clouds. The method is based on properties of scatter matrix during direct least squares fitting of ellipsoids and it can be applied for recognition of all quadrics whose cross section with arbitrary plane is in elliptic form. This class of quadrics includes ellipsoids, elliptic cylinders, spheres, and cylinders.

The method belongs to the class of region growing methods. It uses the value of reciprocal condition number of scatter matrix as region growing criterion, and region expanding strategy is based on kNN algorithm with Euclidean distance. Comparative advantage of this method to edge based recognition techniques is that it can be effectively applied for segmentation of surfaces with G1 (or higher) continuous segments. Another, very important characteristic of the method is that seed point is selected automatically, and that the application of the procedure does not require human involvement. This property is significant within CPPS in which fully automatic retrieval of information from real world system is necessary. Besides, we have shown that recognition results can be interpreted in the form suitable for input into virtual (cyber) model of the manufacturing system. Performances of the method are illustrated using two case studies considering synthesized and real world point cloud.

Our future research efforts will consider methods for automatic recognition of other types of quadric surfaces, as well as higher order surfaces.

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