

## THE ANALYSIS OF IMPACT OF INTENSITY OF CONTACT LOAD AND ANGULAR SHAFT SPEED ON THE HEAT GENERATION WITHIN RADIAL BALL BEARING

by

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*This paper considers the factors that influence the heat generation within ball bearing. Various lubrication regimes are taken into consideration and a mathematical model for determination of the coefficient of heat generation is set up. Due to the complexity of mathematical tools, and in order to perform better and easier analysis of the considered phenomenon, the application that integrates the mathematical models of load distribution and heat generation was developed. The impact of the contact load and angular shaft speed on the level of heat generated in radial ball bearing was analyzed.*

Key words: heat generation, radial ball bearing, contact load, angular speed

### Introduction

Rolling bearings are high precision machine elements that have the support function for shafts and axles, enabling the relative movement of working parts and their connection with the fixed elements of the system, but also the reduction of friction in the contact of movable and immovable elements. Friction causes wear of working surfaces and heat generation, which are generally undesirable phenomena within the mechanical systems. Heat generation in bearings is manifested by the power losses of rotating parts. Because of the inevitable rise in temperature, it adversely affects the geometrical characteristics of the bearings due to the appearance of dilatation. Heat generation has an adverse effect on the properties of lubricant, its efficiency of work and rheological characteristics.

The heat in bearings is mainly generated due to friction between rolling bodies and raceways, balls penetration throughout lubricant and friction in contact between the rings and seals. The level of heat generated in contact of rolling bodies with bearing rings depends on the load on each rolling body, bearing geometrical characteristics and the way of lubrication. The aim of this paper is to analyze the relationships between these factors and to consider their impact on heat generation. For the purpose of effective analysis, the appropriate application was developed and presented in the paper.

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### Review of literature

One of the most comprehensive reviews of the problems of rolling bearings was given by Harris in [1, 2], his attention being focused on bearing geometrical characteristics, kinematic analysis, contact stress and strain, load distribution, heat generation, friction, lubrication, fatigue, *etc.* He analyzed the mechanisms of heat transfer and the causes of heat generation in rolling bearings establishing mathematical relations that can be used to determine the factors of heat generation.

In the field of internal load distribution, the results of Ristivojević and Mitrović [3] are significant. Effect of load distribution on the rolling bearing performance: radial stiffness, static load capacity, and service life, has been examined, and concluded that the rolling bearing contact loads do not have constant values, but are depend on: (1) the character and intensity of the external load, (2) the elastic properties of the material of bearing elements, (3) geometry of coupled parts, (4) the total number of rolling bodies in the bearing, (5) the internal radial clearance, (6) the deviation of the working surfaces shape and other factors. In the paper [4], Mitrović *et al.* considered the influence of the bearing operating temperature on its performance parameters, concluding that an increase in operating temperature leads to an increase of the bearing radial stiffness, reduction of the equivalent von Mises stresses, and reduction of the strength of the material as well. Therefore, it could not be discussed about the increased bearing capacity.

Misković *et al.* [5] considered the influence of grease contamination on the value of internal radial clearance. In accordance with the experimental research they had carried out, they defined the mathematical relation for changes in internal radial clearance in dependence on the coefficient of linear expansion, the working temperature, the diameter of the rings, and the weight of contaminating particles in grease.

Among other things, Tomović [6-9] investigated the distribution of external load on the rolling elements of rolling bearings. He has developed his own mathematical model of load distribution and considered the impact of bearing geometry and the total number of rolling bodies on the number of bodies that take an active role in load transfer.

Depending on the parameters of bearing micro and macro geometry, its working characteristics and lubricating fluid properties, lubrication regime will differ, and therefore the tangential stresses. Elasto-hydrodynamic lubrication, as the most complex and common form of lubrication, was the subject of interest of many researchers, [10-12]. In this form of lubrication, lubricant loses Newtonian properties and behaves like a solid body because its viscosity increases significantly with increasing of the pressure. The most commonly applied relations that establish the dependence of the viscosity of pressure are given by Barus and Roelands (according to [1]), while Trachman and Cheng [13] use the experimentally determined limiting shear stress in order to take into account the influence of non-Newtonian behavior of lubricants on the value of tangential stress due to friction. Based on their empirical results, Schipper found the approximate value of the ratio between limiting shear stress and average contact pressure (according to [1]). One of the causes of heat generation within the oil lubricated rolling bearing is a penetration of rolling bodies through a lubricant. The resistance of lubricant depends on the ratio between the volume of oil in a bearing free part and the volume of a bearing free part. Parker [14] empirically determined the ratio on the basis of the obtained results.

### Contact loads within rolling bearing

External load is transferred from one ring of rolling bearing to another via rolling bodies. Based on Hertzian contact theory, it is possible to establish the relationship between

corresponding contact load and contact deformation for each rolling body according to the following expression [2]:

$$Q = K_c \sqrt{\delta_u^3} \quad (1)$$

where  $Q$  [N] is the contact load,  $K_c$  [Nmm<sup>-3/2</sup>] – the effective coefficient of contact stiffness, and  $\delta_u$  [mm] – the overall contact deformation, which is the sum of deformations on both raceways. The level of the corresponding contact load of rolling bodies depends on the external load distribution within the rolling bearing and can be calculated based on a mathematical model of load distribution proposed by Tomović [6-8]. The model of Tomović provides initial data for the calculation of heat generation in rolling bearing.

### Heat generation within the ball bearing with a radial contact

Heat in the ball bearing is generated due to:

- friction between rolling bodies and raceways ( $H_{BRC}$ ),
- balls penetration throughout the lubricant – hydraulic resistance in the lubricant ( $H_{fdrag}$ ),
- sliding between the cage and the rim of rings ( $H_{CRL}$ ),
- friction between the surfaces of rolling bodies and the cage ( $H_{CPB}$ ), and
- friction between the seals and bearing rings ( $H_{CRS}$ ) [1].

The estimation of the amount of heat generated in the bearing is made on the basis of total factor of heat generation  $H_{tot}$ , which represents the sum of individual factors. This paper takes into account only the first two factors, because, according to [1], a very little heat is generated by friction between the cage and the rim of rings and by friction between the rolling bodies and the cage. Although the biggest factor of heat generation occurs between the seals and rings, it was not the subject of this paper, because only some of the bearings contain the seals.

### Heat generation due to friction between rolling bodies and raceways

If we introduce a presumption of negligibility of gyroscopic movement of the balls, which is justified at higher levels of external loads and lower speeds, only the coefficient of heat generation in the direction of the y-axis exists. It is determined from the expression for the contact between the individual ball and raceway [1]:

$$H_{nij} = a_{nj} b_{nj} \cdot 10^{-3} \int_{-1-\sqrt{1-q_{nj}^2}}^{+1+\sqrt{1-q_{nj}^2}} \int_{-1-\sqrt{1-q_{nj}^2}}^{+1+\sqrt{1-q_{nj}^2}} v_{nj} \tau_{nj} dt_{nj} dq_{nj} \quad [\text{W}], \quad n = i, o \quad (2)$$

where  $a$  and  $b$  [mm] are longer and shorter semi-axis of the contact ellipse,  $q = x/a$ ,  $v$  [mms<sup>-1</sup>] is the sliding velocity between the contact bodies, and  $\tau$  [MPa] – tangential stress due to friction; index “i” means inner, and “o” outer raceway, while index “j” marks a particular active rolling body. The overall coefficient  $H_{BRC}$  is obtained as the sum of the coefficients  $H_{nj}$  for contact of each active ball with both raceways.

The Hertzian contact theory gives the expressions for semi-axes of contact ellipse [2]:

$$a = a^* \sqrt[3]{\left[ \frac{3Q}{2\Sigma\rho} \left( \frac{1-\xi_I^2}{E_I} + \frac{1-\xi_{II}^2}{E_{II}} \right) \right]^2} \quad (3)$$

$$b = b^* \sqrt[3]{\left[ \frac{3Q}{2\Sigma\rho} \left( \frac{1-\xi_1^2}{E_1} + \frac{1-\xi_2^2}{E_2} \right) \right]^2} \quad (4)$$

where dimensionless semi-axes of the ellipse  $a^*$  and  $b^*$ , depend on the geometry of coupled parts.

Using the kinematic analysis of ball bearing, Harris [1] defined the expressions for determination of the sliding velocity between the balls and raceways  $v_{ny}$ . Applying his expressions to the ball bearing with radial contact, it could be obtained [15]:

$$v_{ny} = M_n \omega \left[ \frac{1}{2} \frac{\sqrt{R_n^2 - a_n^2 q_n^2} - \sqrt{R_n^2 - a_n^2} + \sqrt{\left(\frac{D_b}{2}\right)^2 - a_n^2}}{D_b} \right] [\text{mms}^{-1}], \quad n = i, o \quad (5)$$

where  $\omega$  [ $s^{-1}$ ] is the angular speed of the shaft,  $D_b$  [mm] – the ball diameter,  $R$  [mm] – the radius of the deformed ring groove, and  $M$  [mm] – an auxiliary value which has the following form:

$$M_i = -\frac{D_m + D_b}{2} \quad (6)$$

$$M_o = \frac{D_m - D_b}{2} \quad (7)$$

while  $D_m$  [mm] is an average cage diameter.

According to [1], Hertz defined the radius of the deformed ring groove as:

$$R_n = \frac{2r_n D_b}{2r_n + D_b} [\text{mm}], \quad n = i, o \quad (8)$$

where  $r_n$  [mm] presents a nominal radius of the observed ring groove.

The values of tangential stress due to friction  $\tau$  depend on the lubrication regime, which is determined on the basis of the minimum oil film thickness and roughness of the contact surfaces by means of parameter  $\lambda$ :

$$\lambda = \frac{h_{\min}}{\sqrt{R_{qr}^2 + R_{qb}^2}} \quad (9)$$

where  $R_{qr}$  and  $R_{qb}$  [mm] are mean-square roughness of raceways and rolling bodies, and  $h_{\min}$  [mm] is a minimum lubricant thickness [1]. There are:

- complete or elasto-hydrodynamic lubrication ( $\lambda > 3$ ),
- boundary lubrication ( $\lambda < 1$ ), and
- mixed lubrication ( $\lambda = 1-3$ ) [1].

The roughness depends on many factors, but primarily on the process and quality of surface treatment. Its precise values could be determined only by the direct measurement. However, based on previous experience in the production of rolling bearings, the approximate roughness of rolling bodies and raceways is given in some literature sources, [10, 16].

### Complete lubrication

There is a complete separation of the contact surfaces in elasto-hydrodynamic lubrication, because the height of the oil film is sufficient to overlay the micro roughness of rolling bodies and raceways. However, the film thickness is not constant in the contact zone. The reason for this is a pressure distribution according to which the pressure is rising sharply at the exit of lubricant from the contact zone, where it is the thinnest.

The film thickness (a thin layer of lubricant)  $h$  in a certain point of elliptical contact surface is given as a function of a minimal film thickness  $h_{\min}$  according to the following expression [10]:

$$h = h_{\min} + \frac{x^2}{2R_x} + \frac{y^2}{2R_y} + d(x, y) \quad (10)$$

where  $d(x, y)$  is the elastic deformation, and  $R_x$  and  $R_y$  are the reduced radius of curvature .

Elastic deformation  $d(x, y)$  could not be determined due to the complexity of the expression which defined it. Therefore, the assumption that the oil film thickness is constant in the contact area and equal to the thickness in its center  $h_c$ , given by the expression as suggested by Hamrock and Dowson [17], is introduced:

$$h_c = 2.69 \bar{U}^{-0.67} \bar{G}^{-0.53} \bar{W}^{-0.067} (1 - 0.61 \cdot e^{-0.73a/b}) R_x \quad [\text{m}] \quad (11)$$

where  $\bar{U}$ ,  $\bar{G}$ , and  $\bar{W}$  are parameters of velocity, material and load, and can be calculated on the basis of the expressions set out in [17].

Elasto-hydrodynamic lubrication is characterized by a large angular speed of the inner ring of the bearing and high contact load. In cases of high contact pressures, lubricant loses Newtonian properties, and behaves like a solid body.

Trachman and Cheng [13] have taken into account non-Newtonian behavior of lubricant by use of the limiting shear stress  $\tau_{\text{lim}}$  and defined the following expression for the surface tangential stress due to friction between the rolling bodies and raceways:

$$\tau_f = \left( \frac{h}{\eta_0 v} + \tau_{\text{lim}}^{-1} \right)^{-1} \quad [\text{MPa}] \quad (12)$$

where  $\eta_0$  [MPa·s] is the viscosity of the lubricant at atmospheric pressure.

Limiting shear stress  $\tau_{\text{lim}}$  (the maximum stress at a constant pressure) could be accurately determined only by fully testing of rolling bearing, but it can be taken as a value approximately equal to one tenth of average value of the contact pressure  $p$  at the contact surface, [1].

### Boundary lubrication

Boundary lubrication occurs when the lubricant fills the micro roughness (asperities) of the contact bodies, so that the overall contact load is transferred over the peaks of asperities. In that case, lubricant does not participate in load transfer. Surface tangential stress due to the mutual sliding of peaks is determined by the expression:

$$\tau_d = \pm \frac{3\mu_a Q}{2\pi ab} \sqrt{1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2} \quad [\text{MPa}] \quad (13)$$

where  $\mu_a$  represents the coefficient of friction between the asperities [1]. Distribution of tangential stress due to friction at the contact surface depends on the distribution of the sliding velocity [1]. Therefore, sign “+” or “-” will occur in (13), depending on the direction of the sliding velocity.

#### Mixed lubrication

Oil film thickness in mixed lubrication is not large enough to prevent a penetration of peaks of asperities through fluid film and the mutual contact of the peaks. Therefore, both asperities and lubricant participate in load transfer. According to Harris and Kotzalas [1], the total surface tangential stresses due to friction are determined from the expression:

$$\tau = \frac{A_c}{A_0} \tau_d + \left(1 - \frac{A_c}{A_0}\right) \tau_f \quad [\text{MPa}] \quad (14)$$

where  $A_0$  [mm<sup>2</sup>] is overall contact area and  $A_c$  [mm<sup>2</sup>] – a part of the contact area exposed to mutual friction of asperities. Harris and Kotzalas [1] analyzed the relationship between these values and found that it depends on the parameters of micro geometry of contact surfaces and the oil film thickness. He, formulated the expressions for determining the factors that influence the mentioned relationship. However, the expressions are relatively complex and it is necessary to measure the spectral moments of profile for determining the precise values. Therefore, the following expression for establishing the ratio between the mentioned areas based on the examination of the phenomenon is proposed [15]:

$$\frac{A_c}{A_0} = (15.3\lambda^4 - 163.5\lambda^3 + 646.2\lambda^2 - 1121.3\lambda + 723.3)10^{-2} \quad (15)$$

#### Heat generation due to hydraulic friction resistance in the lubricant

In the oil lubricated rolling bearing, besides the other sources of heat generation, heat is generated due to penetration of the balls into lubricant. An appropriate factor of heat generation is given by:

$$H_{\text{fdrag}} = \frac{D_m \omega_m F_v Z}{2} \cdot 10^{-3} \quad [\text{W}] \quad (16)$$

where  $D_m$  [mm] is the average diameter of the cage,  $\omega_m$  [s<sup>-1</sup>] – the angular speed of the cage,  $F_v$  [N] – the force of viscous resistance of lubricant, and  $Z$  – the total number of balls [1].

Neglecting the sliding between contact surfaces, the following expression can be applied:

$$\omega_m = \frac{D_m - D_b}{2D_m} \omega \quad (17)$$

Harris and Kotzalas approximated the force of viscous resistance of lubricant by the expression:

$$F_v = \frac{c_v \pi \rho_c D_b^2 (D_m \omega_m)^{1.95}}{32g} \quad [\text{N}] \quad (18)$$

where  $\rho_c$  [gmm<sup>-3</sup>] is the effective density of oil,  $D_b$  [mm] – the ball diameter,  $D_m$  [mm] – the average diameter of the cage,  $g$  [mms<sup>-2</sup>] – gravitational acceleration and  $c_v$  – the coefficient of resistance.

The effective density of oil presents the ratio between the volume of oil in the bearing free part and the volume of the bearing free part. According to Parker [14], it could be determined as:

$$\rho_c = XCAV \cdot \rho_f = \frac{\rho_f W_f^{0.37}}{\omega D_m^{1.7}} 10^5 \quad [\text{gmm}^{-3}] \quad (19)$$

where  $\rho_f$  [ $\text{gmm}^{-3}$ ] is the oil density,  $W_f$  [ $\text{cm}^3\text{min}^{-1}$ ] – the oil flow through the bearing,  $\omega$  [ $\text{min}^{-1}$ ] – the angular speed of the shaft, and  $D_m$  [mm] – the average diameter of the cage.

Oil flow through the bearing depends on lubricant characteristics, the direction of its flow, the bearing geometry and the angular speed. The XCAV coefficient represents a percentage share of the free part of the bearing filled with oil and the volume of the bearing free part. Parker [14] has empirically developed the expression for its determination.

*The mathematical model for calculating the coefficient of heat generation*

In accordance with the expression (2) and the previously given expressions for determining the contact surfaces, sliding speed and shear stresses due to friction, the expressions for the coefficient of heat generation due to friction between the individual ball and raceway in different modes of lubrication are developed [15]:

– for complete lubrication:

$$H_{\text{Eny}} = \frac{a_n b_n}{500} \eta_0 Q M_n^2 \omega^2 \int_{-1}^{+1} \frac{\left( \frac{1 - \sqrt{R_n^2 - a_n^2 q_n^2} + T_{E2}}{D_b} \right)^2 \sqrt{1 - q_n^2}}{h_{\text{cn}} Q + \frac{T_{E1} M_n \omega}{\left( \frac{1 - \sqrt{R_n^2 - a_n^2 q_n^2} + T_{E2}}{D_b} \right)^{-1}}} dq_n \quad [\text{W}] \quad (20)$$

$$T_{E1} = 10 a_n b_n \pi \eta_0 \quad [\text{Ns}] \quad (21)$$

$$T_{E2} = \sqrt{\left( \frac{D_b}{2} \right)^2 - a_n^2} - \sqrt{R_n^2 - a_n^2} \quad [\text{mm}] \quad (22)$$

Lengths are in [mm],  $\eta_0$  in [ $\text{MPa} \cdot \text{s}$ ],  $Q$  in [N],  $M_n$  in [mm],  $\omega$  in [ $\text{s}^{-1}$ ] and  $h_{\text{cn}}$  in [mm]. As there is no general solution of the integral (20), it is handled numerically;

– for boundary lubrication

$$H_{\text{Bny}} = \frac{\mu_a Q \omega T_B}{1000} \quad [\text{W}] \quad (23)$$

where  $T_B$  is the coefficient that depends on the bearing geometry, bearing material and the corresponding contact load of the ball (lengths are in [mm],  $Q$  in [N], and  $\omega$  in [ $\text{s}^{-1}$ ]):

$$T_B = \left\{ \frac{M_n}{D_b} \left\{ \frac{D_b}{2} + \sqrt{R_n^2 - a_n^2} \left[ 1 - \frac{3(R_n^2 + 2a_n^2)}{16a_n^2} \right] - \sqrt{\left(\frac{D_b}{2}\right)^2 - a_n^2} + \frac{3R_n^2 \left[ \arctan\left(\frac{a_n}{\sqrt{R_n^2 - a_n^2}}\right) \right] \frac{\pi}{180} (R_n^2 - 4a_n^2)}{16a_n^3} \right\} \right\} \text{ [mm]} \quad (24)$$

– for mixed lubrication

$$H_{Mny} = \frac{A_c}{A_0} H_{Bny} + \left( 1 - \frac{A_c}{A_0} \right) H_{Eny} \quad (25)$$

The overall coefficient of heat generation within the ball bearing with a radial contact represents the sum of the coefficients of heat generation among all active balls and both raceways  $H_{BRC}$  plus the coefficient of heat generation due to hydraulic resistance of lubricant  $H_{fdrag}$ .

The previous analysis has shown that the mathematical model of heat generation in bearings is quite complex. In order to make the process of analysis of heat generation within the

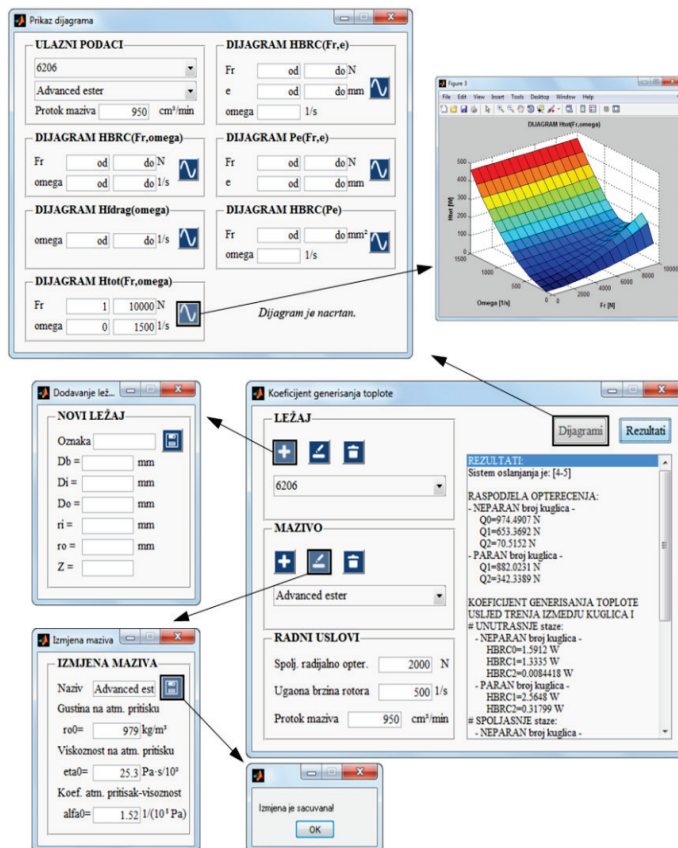


Figure 1. Application for heat generation analysis

radial ball bearing more efficient, an application written in MATLAB was developed. Its working environment looks as shown in fig. 1. The application has unified the previously described mathematical models for calculating the load distribution and the heat generation within radial ball bearing. Thus, the input data for the analysis are the geometrical characteristics of the rolling bearing, the flow and properties of the lubricant, the angular speed of the shaft and external radial load, while at the output we get information on the distribution of load and coefficients of heat generation.

The application contains a database with geometrical characteristics of bearings and lubricant properties, which is filled with the available data for 16 radial ball bearings and 11 lubricants. There is a possibility of simple database updates, as entering new bearing



data, and modifying and deleting existing bearings and lubricants. Also, graphical representation of mutual interdependence of individual factors with different 2-D and 3-D diagrams is enabled. Accordingly, the application shown in fig. 1 has allowed for the simplification of the procedure of calculating the required sizes and conducting analyses based on consideration of interdependence of: working conditions (external radial load and the shaft angular speed), the coefficient of heat generation, load distribution, bearing geometrical characteristics (primarily internal radial clearance), the number of active balls and the value of the contact surface.

**Results and discussion**

The results of a particular use of the developed application are shown in tab. 1 as distribution of the values of the coefficient of heat generation in the contact between the individual active balls and raceways for 6206 bearing with internal radial clearance  $e = 0.02$  mm and external load  $F_r = 1000$  N.

The analysis of the results presented in tab. 1 shows that much more heat is generated in the contact between the ball and inner raceway than in the contact between the ball and outer raceway. Based on experimental research [5], it is concluded that the temperature of the inner ring is higher than the temperature of the outer ring during the operation of bearing, which is in correlation with the previously presented conclusion. Figure 2 shows graphically the dependence of the overall coefficient of heat generation due to friction between the rolling bodies and raceways  $H_{BRC}$  of external radial load  $F_r$  and the shaft angular speed  $\omega$  for 6206 bearing with zero internal radial clearance when bearing is lubricated with advanced ester.

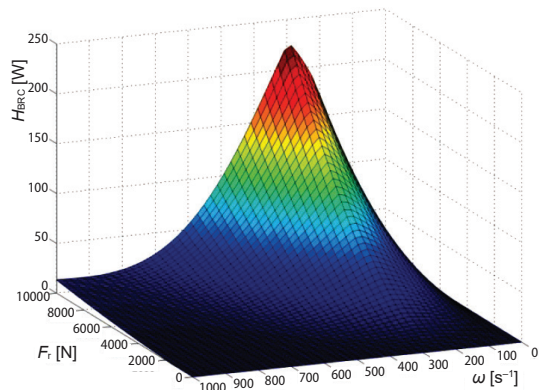
**Table 1. The values of the coefficient of heat generation in the contact between the individual active balls and raceways for 6206 bearing with internal radial clearance  $e = 0.02$  mm loaded by force  $F_r = 1000$  N**

	$\omega$ [s <sup>-1</sup> ]	Odd number of active balls			Even number of active balls			$H_{BRC}$ [W]
		$j = 0$	$j = 1$	$H_{BRCN}$ [W]	$j = 1$	$j = 2$	$H_{BRCP}$ [W]	
$Q_j$	100 boundary	582.57	272.46	1.8092	512.03	37.69	1.8918	1.8505
$H_{BRCj}$		0.91757	0.51392		1.478	0.01889		
$H_{BRCoj}$		0.24183	0.13588		0.38981	0.005011		
$Q_j$	500 mixed	582.57	272.46	0.75336	512.03	37.69	0.83097	0.79216
$H_{BRCj}$		0.51904	0.19033		0.78093	$2.04 \cdot 10^{-3}$		
$H_{BRCoj}$		0.031885	0.012107		0.047571	$0.42 \cdot 10^{-3}$		
$Q_j$	1500 complete	582.57	272.46	$4.22 \cdot 10^{-3}$	512.03	37.69	$4.59 \cdot 10^{-3}$	$4.40 \cdot 10^{-3}$
$H_{BRCj}$		$2.85 \cdot 10^{-3}$	$1.17 \cdot 10^{-3}$		$4.35 \cdot 10^{-3}$	$1.92 \cdot 10^{-5}$		
$H_{BRCoj}$		$0.14 \cdot 10^{-3}$	$5.86 \cdot 10^{-5}$		$0.22 \cdot 10^{-3}$	$9.75 \cdot 10^{-7}$		

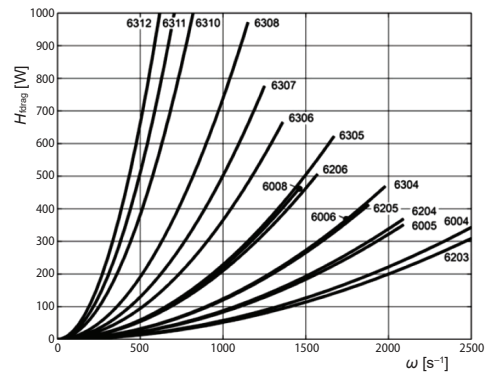
$H_{BRCN}$  – coefficient of heat generation due to friction between the raceways and odd number of balls  
 $H_{BRCP}$  – coefficient of heat generation due to friction between the raceways and even number of balls  
 $H_{BRC}$  – total coefficient of heat generation due to friction between the raceways and balls for concrete relying system  
 Coefficients  $H_{BRCn1}$  and  $H_{BRCn2}$  are doubled because of the symmetry of the problem, while the values of coefficient  $H_{BRCn0}$  are not.

The area of complete lubrication occurs for higher values of angular speed. Based on table 1 and fig. 2, it is evident that complete lubrication reduces the coefficient of heat generation due to friction between the rolling bodies and raceways. The coefficient of heat generation due to hydraulic resistance of lubricant  $H_{fdrag}$  is given by the expression (16). In the case of lubricating with advanced ester oil lubricant with flow  $W_f = 950$  cm<sup>3</sup>/min,

fig. 3 shows the dependence of the coefficient of heat generation due to hydraulic resistance of lubricant  $H_{\text{fdrag}}$  of shaft angular speed  $\omega$  for 16 bearings with radial contact.



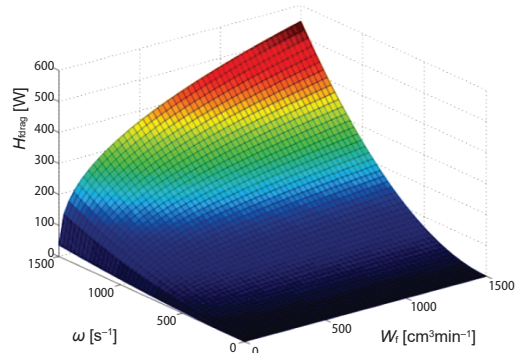
**Figure 2.** The dependence of the coefficient  $H_{\text{BRC}}$  of external radial load  $F_r$  and the shaft angular speed  $\omega$  for 6206 bearing with zero internal clearance when lubricating with advanced ester



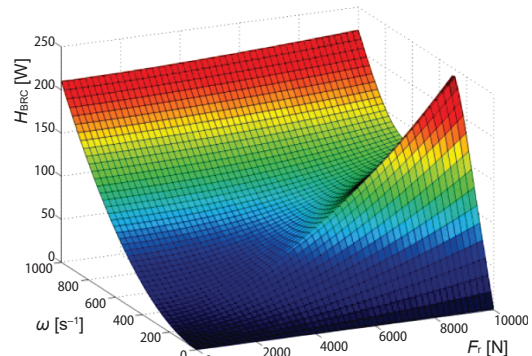
**Figure 3.** The dependence of the coefficient  $H_{\text{fdrag}}$  of shaft angular speed  $\omega$  for 16 bearings with radial contact lubricated with advanced ester with flow  $W_f = 950 \text{ cm}^3/\text{min}$

In case of lubrication of the bearing 6206 with advanced ester, fig. 4 shows the dependence of the coefficient of heat generation due to lubricants hydraulic resistance  $H_{\text{fdrag}}$  of shaft angular speed  $\omega$  and the oil flow  $W_f$ .

Comparison of fig. 2 and fig. 4 indicates that the amount of heat generated by friction between the rolling bodies and raceways is higher than heat generated by balls penetration throughout lubricant if bearings run at lower rpm and higher external radial load. However, the situation is the opposite if the shaft is rotated at a higher speed and higher flow rates of oil. Therefore, it is important to make a suitable selection of lubricants and lubrication mode. This conclusion is supported by the diagram shown in fig. 5. It shows the dependence of the coefficient of total heat generation  $H_{\text{tot}}$  of external radial load  $F_r$  and the shaft angular speed  $\omega$  at bearing 6206 when lubricating with advanced ester oil with flow  $W_f = 950 \text{ cm}^3/\text{min}$ . The dependence of coefficient  $H_{\text{BRC}}$  of external radial load  $F_r$  for several ball bearings lubricated with advanced ester at shaft angular speed  $\omega = 200 \text{ s}^{-1}$  is shown in fig. 6(a), while the same



**Figure 4.** The dependence of the coefficient  $H_{\text{fdrag}}$  of the shaft angular speed  $\omega$  and the oil flow  $W_f$  for 6206 bearing when lubricating with advanced ester oil



**Figure 5.** The dependence of the coefficient  $H_{\text{tot}}$  of shaft angular speed  $\omega$  and external load  $F_r$  for 6206 bearing lubricated with advanced ester with flow  $W_f = 950 \text{ cm}^3/\text{min}$

dependence at  $\omega = 1500 \text{ s}^{-1}$  is shown in fig. 6(b). It can be seen that the coefficient  $H_{\text{BRC}}$  always increases with load  $F_r$  increment. Such a trend is not always clearly visible in figs. 2 and 5 because the values of the coefficient  $H_{\text{BRC}}$  in the zone of complete lubrication (higher shaft speeds) are significantly lower than the values from the zone of incomplete lubrication (lower shaft speeds), as can be seen by comparing figs. 6(a) and 6(b).

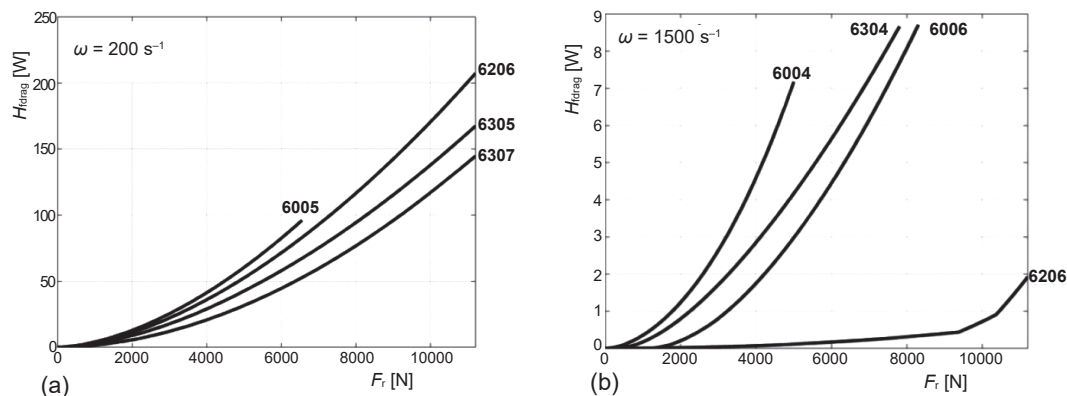


Figure 6. The dependence of the coefficient  $H_{\text{BRC}}$  of external radial load  $F_r$  for several radial ball bearings when lubricating with advanced ester at shaft speed: (a)  $\omega=200 \text{ s}^{-1}$ , (b)  $\omega=1500 \text{ s}^{-1}$

### Conclusions

The paper discusses the factors that influence heat generation within the ball bearing with a radial contact with different modes of lubrication. Appropriate mathematical model to determine the coefficient of heat generation was created. The mathematical tools for calculation of heat generated within the bearing are characterized by a very high complexity. Therefore, the developed computer application, whose basic elements were presented in this paper, allows more efficient, more accurate and easier analysis of the phenomenon of heat generation in ball bearings. There are several ways for heat generation within rolling bearing: by friction between the rolling bodies and raceways, due to hydraulic resistance of lubricants, friction between the rolling bodies and cage, friction between the cage and rim of rings and friction between the seals and rings. The paper discusses the first two of these five causes, which are considered to be dominant.

Analyses made in this paper have shown that the amount of heat generated by friction between the balls and inner raceway is much higher than the amount of heat generated by friction between the balls and outer raceway. For higher flows of lubricant and higher shaft angular speeds, heat generated due to penetration of balls through lubricant is higher than heat generated due to friction between the active balls and raceways. The situation is reverse for the lower shaft angular speeds and greater values of the external radial load.

### Nomenclature

$A_c/A_0$ – part of the total contact area exposed to friction of asperities, [–]	$H_{\text{BRC}}$ – coefficient of heat generation due to friction between balls and raceways, [W]
$a, b$ – semi-axis of contact ellipse, [mm]	$H_B$ – coefficient $H_{\text{BRC}}$ at boundary, complete and mixed lubrication, [W]
$D_b$ – ball diameter, [mm]	$H_E$ – coefficient $H_{\text{BRC}}$ at boundary, complete and mixed lubrication, [W]
$D_m$ – average diameter of cage, [mm]	$H_M$ – coefficient $H_{\text{BRC}}$ at boundary, complete and mixed lubrication, [W]
$E^m$ – modulus of elasticity, [MPa]	
$e$ – internal radial clearance, [mm]	
$h_c$ – film thickness in ellipse center, [mm]	

$H_{\text{fdrag}}$	– coefficient of heat generation due to hydraulic resistance of lubricant, [W]	$\mu_a$	– friction coefficient between the peaks, [–]
$Q$	– contact load, [N]	$\xi_a$	– Poisson's ratio, [–]
$R$	– radius of the deformed groove, [mm]	$\Sigma\rho$	– the sum of the radius of curvature, [mm <sup>-1</sup> ]
$R_x, R_y$	– reduced radii of curvature, [mm]	$\tau_d, \tau_f$	– tangential stress due to friction at boundary and complete lubrication, [MPa]
$v$	– sliding speed, [mms <sup>-1</sup> ]	$\omega, \omega_m$	– angular speed of the shaft and cage, [s <sup>-1</sup> ]
$x, y$	– distance in the direction of the main and auxiliary contact axis of the ellipse, [mm]		
		<b>Subscripts</b>	
<b>Greek symbols</b>		I, II	– designation of contact body (ball or ring)
$\eta_0$	– fluid viscosity at atm pressure, [MPa·s]	i, o	– inner and outer raceway
$\lambda$	– coefficient of lubrication, [–]	n	– n = i, o

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