The fractional PID controllers tuned by genetic algorithms for expansion turbine in the cryogenic air separation process

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Abstract
This paper deals with the design of a new algorithm of PID control based on fractional calculus (FC) in production of technical gases, i.e. in a cryogenic air separation process. Production of low pressure liquid air was first introduced by P.L. Kapitsa and involved expansion in a gas turbine. For application in the synthesis of the control law, for the input temperature and flow of air to the expansion turbine, it is necessary to determine the appropriate differential equations of the cryogenic process of mixing of two gaseous airflows at different temperatures before entrance to the expansion turbine. Thereafter, the model is linearized and decoupled and consequently classical PID and fractional order $PI^{\beta}D^{\alpha}$ controllers are taken to assess the quality of the proposed technique. A set of optimal parameters of these controllers are achieved through the genetic algorithm optimization procedure by minimizing a cost function. Our design method focuses on minimizing performance criterion which involves IAE, overshoot, as well as settling time. A time-domain simulation was used to identify the performance of $PI^{\beta}D^{\alpha}$ controller with respect to a traditional optimized PID controller.

Keywords: technical gases, cryogenic liquid, fractional PID control, FOPID optimal tuning, genetic algorithms.

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Cryogenic air separation process is used to produce large quantities of purified oxygen, nitrogen or argon for the steel, chemical, food processing, semiconductor and health care industries [1,2]. A cryogenic air separation process is operated at extremely low temperatures (–170 to –190 °C) to separate air components according to their different boiling temperatures. It takes place in air separation units (ASUs) which present cryogenic distillation systems. Due to a high demand for these commodity materials, the ASU has become a crucial technology in many processes, including the next generation power plants. Cryogenic air separation process is an energy intensive process that consumes a tremendous amount of electrical energy. Therefore, ASU must automatically and rapidly respond to the changing product demands from customers. Process designers have increasingly utilized mass and energy integration, substituted less efficient unit operations by more efficient ones, accelerated the development of machinery and equipment used in the process, generated the alternatives to cryogenic production of gaseous products, and set the foundation for implementation of advanced control strategies. As expected, there is significant economic interest in reducing the operating costs of ASUs through advanced process control technology. So far, the dominating control practice in ASU processes has been to adapt traditional regulatory controllers to maintain good performance. A number of studies of the process control and optimization of the cryogenic air separation process have been published. For example, in [3] air separation control technologies are discussed and model predictive control is considered as the current state-of-the-art control technology in the air separation industry. Authors in [4,5] investigated the control structure selection and linear model predictive control tuning for an air separation plant.

Here, we focus our research on improvement of implementation of traditional PID controllers. It is known that, due to its functional simplicity and performance robustness, PID controllers have been widely used in the process industries [6,7]. However, in the recent years, the emergence of effective methods of solving differentiation and integration of non-integer order equations makes fractional-order systems more and more attractive for the control systems community. Fractional calculus (FC) has existed for over three centuries and the fractional integral-differential operators are generalization of integration and derivation to non-integer order (fractional) operators [8–10]. The increasing number of studies related to the application of fractional controllers in many areas of science and

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engineering is remarkable, where special fractional-order systems are of interest for both modelling and controller design purposes [11–13]. In some of these works, it is verified that the fractional-order controllers can have better disturbance rejection ratios and less sensitivity to plant parameter variations compared to the traditional controllers. According to the development of fractional order calculus application in the recent years, the researchers are trying to replace the common PID controller use by the use of fractional controller [11,14–16]. Frequency domain approach in using fractional order PID controllers was also studied [17]. So, the fractional-order PIDs (FOPID) are becoming an important research topic since they result into more tuning parameters allowing robust performances to be attained. Nevertheless, it is well known that the additional degrees of freedom, *i.e.*, varying orders of integration and differentiation, are accompanied with some complexity in the synthesis, even if fractional operators allow a compact representation of such high order controllers which means that only few parameters need to be adjusted. Further research activities run towards defining new effective tuning techniques for non-integer order controllers. Generally, two tuning methods of $PI^{\beta}D^{\alpha}$ controllers are distinguishable, analytic and heuristic [18–20].

In this paper, we propose and elaborate new optimal algorithms of fractional PID control based on genetic algorithms (GA) in the production of technical gases, *i.e.*, in the cryogenic air separation process. GAs have received much interest in recent years [21–23], the basic operating principles of GA being based on the principles of natural evolution. GA is a stochastic global adaptive search optimization technique based on the mechanisms of natural selection. GA can solve non-linear multi-objective optimization problems, requires little knowledge of the problem itself, and does not require that the search space is differentiable or continuous. GA does not suffer from the basic setback of traditional optimization methods such as getting stuck at local minima. In this regard, a GA is used to find out optimal settings for a fractional $PI^{\beta}D^{\alpha}$ controller in order to fulfil different design specifications for the closed-loop system, taking advantage of the fractional orders, α and β. We propose time-domain criterion which involves integral absolute error (IAE), overshoot, as well as settling time. This will be done through a fitness function to achieve rise in the performance indices. Both PID and $PI^{\beta}D^{\alpha}$ controllers, where the gains are optimized by genetic algorithm, are applied to a cryogenic process of mixing of two gaseous air flows at different temperatures before entrance of expansion turbine in the cryogenic air separation process. The quality of the system response is verified by a simulation study.

MODEL OF THE CRYOGENIC PROCESS OF MIXING OF TWO GASEOUS AIR FLOWS BEFORE ENTRANCE OF EXPANSION TURBINE

Cryogenics is the science and technology dealing with temperatures less than about 120 K, although this historical summary does not adhere strictly to 120K definition. The techniques used to produce cryogenic temperatures differ in several ways from those dealing with conventional refrigeration. Also, liquid air has been cooled to very low temperatures (cryogenic temperatures) so that it has condensed to a pale blue mobile liquid. To protect it from room temperature, it must be kept in a vacuum flask. In practice, these two areas often overlap and the boundary between conventional and cryogenic refrigeration is often indistinct. Significant reductions in temperature often have very pronounced effects on the properties of materials and behaviour of the systems. A new way of technical production of liquid air has been obtained by C. Linde at the end of the nineteenth century [1–3]. Production of low pressure liquid air was first introduced in 1938. by Russian academician P.L. Kapitsa, and includes the production of liquor air at pressure, p_2, of 6–7 bar and its expansion in a gas turbine. So, the expansion turbine in the liquid air production, used for expansion of air from thermodynamic state $\mathrm{P}\left(p_\mathrm{p},T_\mathrm{p}\right)$ to state $\mathrm{K}\left(p_\mathrm{k},T_\mathrm{k}\right)$, lowering air temperature from T_p to T_k and the pressure from p_p to p_k, Fig. 1. Expansion of cold air after the equipment is started, creates a waste heat due to exchange of heat with the environment during this work. The amount of air that expands in the gas turbine, does not exceed 25% of the amount of usable air [24]. The air from the compressed state 1, is cooled down to the state 2 by turbo-compressor, Fig. 1b. The compressed air at pressure p_2 is led to the reverse heat exchanger where it is cooled to the state 3. Part of the air from the middle of heat exchanger state 3* and part at the state 3, which constitute m_e(kg/kg) of compressed air, are led to the expansion turbine where, after expansion, the state 8 at pressure p_1 is obtained. Because of the losses and other non-reversible processes, the expansion does not follow the adiabatic line to state 8_{ad}, but to state 8, which is shifted to the right. The place for removal of air at state 4 is elected to be at state 8, at the end of expansion, which is in the area near the upper limiting curve (in the TS diagram $x = 1.0$), by 1–3 K above the temperature of saturated steam. The basic devices of the plant are: TK – turbo-compressor, H – air refrigerator, RR – reverse heat exchanger, ET – expansion turbine, RK – heat exchanger, *i.e.*, air condenser, and PV – damping valve.

Liquid air quantity, m_{TV}, can be determined on the basis of the heat balance∗ [25]:

∗ The list of used symbols is given in -Nomenclature.

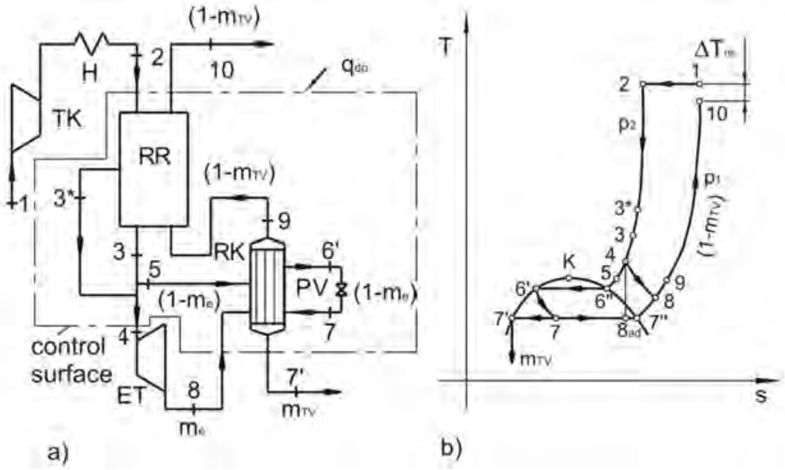

Figure 1. Schematic diagram of the liquid air flow within the plant (a) and TS diagram of the process (b).

$$h_2 = m_{TV} h_7^{'} + (1 - m_{TV}) i_{10} + m_e (h_4 - h_8) - q_{do} \text{ (kJ/kg)} \quad (1)$$

where q_{do} (kJ/kg) is the heat from the environment brought by 1 kg of air, m_{TV} (kg/kg) is the mass of the liquid air and m_e (kg/kg) is the mass of the air which expands in the expansion turbine. In the ideal case, when $q_{do} \approx 0$ and $\Delta T_{nr} = T_1 - T_{10} \approx 0$, the liquid air mass is:

$$m_{TV} = \frac{h_{10} - h_2}{h_{10} - h_7^{'}} + m_e \frac{h_4 - h_8}{h_{10} - h_7^{'}} \le \frac{h_1 - h_2}{h_{10} - h_7^{'}} + m_e \frac{h_4 - h_8}{h_{10} - h_7^{'}} \quad (2)$$

The main advantage of Kapitsa's procedure, compared to the other procedures for liquid air production [2], is that even at a low p_2 pressure it still does not have to spend excessive work for production of 1 kg of liquid air. Since the turbine is capable of much greater flows than the reciprocating compressor, the procedure is adapted for large plants such as are met in practice. For performing expansion of the part of the air after recuperation, in the Factory of technical gasses in Bor, two expansion turbines are built-in, one which is permanently in the process and the other is hot standby. The energy obtained in the expansion turbine during the process is used for driving the fan which absorbs atmospheric air, irrespective of the air flow in the gas turbine. The fan compresses the air and in this way prevents an unlimited growth of the number of turns of the turbine; the compressed air is let to the atmosphere which is not rational from the energy point of view. For the purpose of the synthesis of the control of the input air temperature and air flow through the expansion turbine, it is necessary to determine the corresponding linearized differential equations of the part of the cryogenic process of mixing two streams of gaseous air of different temperatures at the entrance to the expansion turbine. Figure 2 presents diagram of the process and symbolic-functional scheme with the relevant variables nominal value of gaseous air flow at the entrance to the expansion turbine $G_{56N} = 7600 \text{ m}_N^3/\text{h}$, nominal value of gaseous air temperature at the entrance to the expansion turbine $T_{5N} = 124$ K, nominal value of temperature of gaseous air in the heat exchanger, $T_{1N} = 153$ K, nominal value of temperature of air at the end of the cold heat exchanger $T_{3N} = 101$ K, nominal value of the position of control valve TV946A $Y_{AN} = 14.7$ mm, nominal value of the position of control valve TV946B $Y_{BN} = 30.2$ mm. Mathematical modeling of this process is based on the following assumptions.

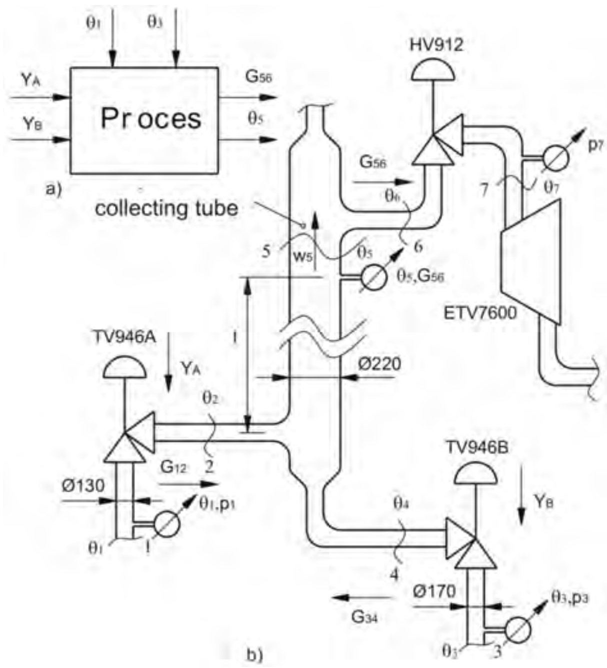

Figure 2. Diagram of the cryogenic process (a)and symbolic-functional scheme (b).

A1. Airflow field is homogeneous in the pipeline. Continuity equation for the air flowing through the pipeline has the following form:

$$\frac{\mathrm{d}M_{\mathrm{sc}}(t)}{\mathrm{d}t}=G_{12}(t)+G_{34}(t)-G_{56}(t) \quad (3)$$

A2. Air temperature field is homogeneous in the pipeline. Based on this assumption, the heat balance equation is:

$$\frac{\mathrm{d}\left(M_{sc}(t)h_5(t)\right)}{\mathrm{d}t}=G_{12}(t)h_2(t)+ \\ +G_{34}(t)h_4(t)-G_{56}(t)h_6(t)+Q_{\mathrm{o}} \quad (4)$$

A3. Temperature increment through the pipe wall is negligibly small. Since the pipeline is located in a cold block which is insulated from the environment by perlite material, it can be assumed that $Q_{\mathrm{o}}\approx 0$.

A4. Enthalpies of the air in the given sections can be expressed as functions of appropriate temperatures:

$$h_2(t)=c_p\theta_2(t)\approx c_p\theta_1(t); \\ h_4(t)=c_p\theta_4(t)\approx c_p\theta_3(t); \\ h_6(t)=c_p\theta_6(t)\approx c_p\theta_5(t) \quad (5)$$

A5. Average air velocity in section 5 is approximately constant, $w_5(t)=w_{5\mathrm{N}}=w_5\approx \mathrm{const.}$.

A6. Static characteristics of the control valves are linear.

A7. Control-valve HV912 is opened.

A8. Mass flow rate through the control valves depends only on the linear displacement of control valves:

$$G_{12}(t)=k_{\mathrm{vA}}Y_{\mathrm{A}}(t);\quad G_{34}(t)=k_{\mathrm{vB}}Y_{\mathrm{B}}(t) \quad (6)$$

By adopting relative perturbations of the relevant values, the following choice is made:

$$\Delta G_{56}(t)=G_{56}(t)-G_{56\mathrm{N}}=x_{01}(t) \\ \Delta\theta_5(t)=\theta_5(t)-\theta_{5\mathrm{N}}=x_{02}(t) \\ \Delta Y_{\mathrm{A}}(t)=Y_{\mathrm{A}}(t)-Y_{\mathrm{AN}}=y_{\mathrm{A}}(t)=u_1(t); \\ \Delta Y_{\mathrm{B}}(t)=Y_{\mathrm{B}}(t)-Y_{\mathrm{BN}}=y_{\mathrm{B}}(t)=u_2(t); \\ \Delta\theta_1(t)=\theta_1(t)-\theta_{1\mathrm{N}}=z_1(t); \\ \Delta\theta_3(t)=\theta_3(t)-\theta_{3\mathrm{N}}=z_2(t) \quad (7)$$

Therefore, it is possible to obtain a system of differential equations which presents nonlinear mathematical model as follows:

$$\frac{\mathrm{d}x_{\mathrm{o}1}(t)}{\mathrm{d}t}=f_1\left(x_{\mathrm{o}1}(t),y_{\mathrm{A}}(t),y_{\mathrm{B}}(t)\right) \quad (8)$$

$$\frac{\mathrm{d}x_{\mathrm{o}2}(t)}{\mathrm{d}t}= \\ =f_2\left(x_{\mathrm{o}1}(t),x_{\mathrm{o}2}(t),y_{\mathrm{A}}(t),y_{\mathrm{B}}(t),z_1(t),z_2(t)\right) \quad (9)$$

A9. Deviations of all values are small enough so that function $f_2(\cdot)$ can be replaced by the first two terms of the corresponding Maclaurin series.

After Eqs. (8) and (9) are linearized, one can obtain linear differential equations that describe the cryogenic process of mixing of two gaseous air flows of different temperatures before entrance to the expansion turbine, given as the appropriate equations of the state and output as follows:

$$\dot{x}(t)=\begin{bmatrix}-0.2 & 0\\ 0 & -0.2\end{bmatrix}x(t)+ \\ +\begin{bmatrix}45.736 & 28.07\\ 0.174 & -0.085\end{bmatrix}u(t)+\begin{bmatrix}0 & 0\\ 0.088 & 0.112\end{bmatrix}z(t) \quad (10)$$

$$x_{\mathrm{o}}(t)=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}x(t) \quad (11)$$

or, in condensed form:

$$\dot{x}(t)=Ax(t)+B_u u(t)+B_z z(t),\ x_{\mathrm{o}}(t)=Cx(t) \quad (12)$$

where the corresponding vectors are:

$$u(t)=\left[y_{\mathrm{A}}(t)\ y_{\mathrm{B}}(t)\right]^T,\ z(t)=\left[z_1(t)\ z_2(t)\right]^T,$$

and $A, B_{\mathrm{u}}, B_{\mathrm{Z}}, C$ are matrices of appropriate dimensions. From what has been said above, it is clear that the model represents a MIMO system, where the number of inputs is to equal to that of the outputs, *i.e.*, the system is “square”, therefore, it is possible to apply the decoupling control strategy, whereby each of the inputs can affect only one output. In that way, one may obtain the so-called non-interactive system where the transfer function $W(s)$ of the system is decoupled, *i.e.*, matrix of the system is diagonal and non-singular. To decouple the system, a new input $u(t)$ is introduced by means of a feedback [26]:

$$u(t)=-K_{\mathrm{c}}x(t)+F_{\mathrm{c}}v(t) \quad (13)$$

where are c_i is i-th row of matrix **C** and:

$$N=\left(c_1A^{p_1}B_{\mathrm{u}},c_2A^{p_2}B_{\mathrm{u}},\ldots,c_mA^{p_m}B_{\mathrm{u}}\right)^T,\det N\neq 0 \\ p_i=\begin{cases}\min\left(j,c_iA^jB_u\neq 0\right) \\ n-1,c_iA^jB_u=0,\ \forall j\end{cases},\forall j=0,1,2,\ldots,n-1 \quad (14)$$

Thus, one can obtain:

$$F_c=N^{-1},\ K_c=N^{-1}(c_1A^{p_1+1}\quad c_2A^{p_2+1}\quad \ldots\quad c_mA^{p_m+1})^T \quad (15)$$

Transfer function $W(s)$ of the original system is:

$$W(s)=C\left(sI-A\right)^{-1}B_{\mathrm{u}}=\begin{bmatrix}\frac{45.736}{s+0.2} & \frac{28.07}{s+0.2}\\ \frac{0.174}{s+0.2} & \frac{-0.85}{s+0.2}\end{bmatrix} \tag{16a}$$

and, after applying new control:

$$W(s)=C\left(sI-A+B_{\mathrm{u}}K_{\mathrm{c}}\right)^{-1}B_{\mathrm{u}}F_{\mathrm{c}} \tag{16b}$$

By taking into account the proposed procedure for $F_{\mathrm{c}},K_{\mathrm{c}}$, it follows:

$$\begin{aligned}&c_1=[1\ \ 0],\ c_2=[0\ \ 1],\ p_1=0,\ p_2=0,\\&N=B_{\mathrm{u}},\ F_{\mathrm{c}}=B_{\mathrm{u}}^{-1},\ K_{\mathrm{c}}=B_{\mathrm{u}}^{-1}A\end{aligned} \tag{17}$$

and

$$W(s)=C\left(sI\right)^{-1}=\begin{bmatrix}1/s & 0\\ 0 & 1/s\end{bmatrix} \tag{18}$$

Now, the decoupled system is:

$$\begin{aligned}&\dot{x}_1=v_1,\\&\dot{x}_2=v_2+0.088z_1+0.112z_2\end{aligned} \tag{19}$$

GA-BASED OPTIMAL FRACTIONAL ORDER PID CONTROL

The essence of fractional calculus and FOPID

Fractional calculus (FC) is a mathematical topic with more than 300 years old history, but its application to physics and engineering has been reported only in the recent years. The fractional integral-differential operators are generalization of the integration and derivation to non-integer order (fractional) operators. The applications of FC are very wide nowadays: rheology, viscoelasticity, acoustics, optics, chemical physics, thermodynamics, robotics, control theory of dynamical systems, electrical engineering, and bioengineering [7,8,16]. The main reason for the success of applications of FC is that these new fractional-order models are more accurate than integer-order models and that fractional derivatives provide an excellent instrument for the description of memory and hereditary properties of various materials and processes due to the existence of a "memory" term in their model. There are, today, many different forms of fractional integral operators, ranging from divided-difference types to infinite-sum types, Riemann-Liouville, Grunwald-Letnikov, Caputo's, Weyl's and Erdely-Kober, Jumarie's, etc., definitions of fractional derivative [10,28,29]. Three definitions are generally used for the fractional differintegral. The first one is the GL definition, *i.e.*, Grunwald and Letnikov developed an approach to fractional differentiation based on the following definition:

$$\begin{aligned}&{}_{\mathrm{GL}}D_x^{\alpha}f(x)=\lim_{h\to 0}\frac{\left(\Delta_h^{\alpha}f(x)\right)}{h^{\alpha}},\\&\Delta_h^{\alpha}f(x)=\sum_{0\le|j|<\infty}(-1)^{|j|}\binom{\alpha}{j}f(x-jh),\ h>0\end{aligned} \tag{20}$$

which is the left Grunwald-Letnikov (GL) derivative as a limit of the fractional order backward difference. Similarly, there exists the right one as:

$$\begin{aligned}&{}_{\mathrm{GL}}D_x^{\alpha}f(x)=\lim_{h\to 0}\frac{\left(\Delta_{-h}^{\alpha}f(x)\right)}{h^{\alpha}},\\&\Delta_{-h}^{\alpha}f(x)=\sum_{0\le|j|<\infty}(-1)^{|j|}\binom{\alpha}{j}f(x+jh),\ h<0\end{aligned} \tag{21}$$

As indicated above, the previous definition of GL is valid for $\alpha > 0$ (fractional derivative) and for $\alpha < 0$ (fractional integral) and, commonly, these two notions are grouped into one single operator called differintegral. If $n=t-a/h$ is considered, where a is a real constant which expresses a limit value, one may write:

$${}_{\mathrm{GL}}D_{a,t}^{\alpha}f(t)=\lim_{h\to 0}\frac{1}{h^{\alpha}}\sum_{j=0}^{\left[\frac{t-a}{h}\right]}(-1)^{j}\binom{\alpha}{j}f(t-jh) \tag{22}$$

where t means the integer part of x, and a and t (in subscript) are the bounds of the operation for ${}_{\mathrm{GL}}D_{a,t}^{\alpha}f(t)$. The left Riemann-Liouville (RL) and the *right* RL fractional integral of the order α for function $f(t)$, for $\alpha,a\in R$, can be expressed as follows:

$${}_{\mathrm{RL}}I_a^{\alpha}f(t)\equiv{}_{\mathrm{RL}}D_a^{-\alpha}f(t)=\frac{1}{\Gamma(\alpha)}\int_a^t(t-\tau)^{\alpha-1}f(\tau)\,\mathrm{d}\tau \tag{23}$$

$${}_{\mathrm{RL}}I_b^{\alpha}f(t)\equiv{}_{\mathrm{RL}}D_b^{-\alpha}f(t)=\frac{1}{\Gamma(\alpha)}\int_t^b(\tau-t)^{\alpha-1}f(\tau)\,\mathrm{d}\tau \tag{24}$$

where $\alpha>0, n-1<\alpha<n$ and $\Gamma(.)$ is the well-known Euler's gamma function. Furthermore, the left RL fractional derivative is defined as:

$${}_{\mathrm{RL}}D_{a,t}^{\alpha}f(t)=\frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_a^t(t-\tau)^{n-\alpha-1}f(\tau)\,\mathrm{d}\tau \tag{25}$$

and the right RL fractional derivative is:

$${}_{\mathrm{RL}}D_{t,b}^{\alpha}f(t)=\frac{(-1)^n}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_t^b(\tau-t)^{n-\alpha-1}f(\tau)\,\mathrm{d}\tau \tag{26}$$

where $n-1\le\alpha<n$; a and b are terminal points of interval $[a,b]$, which can also be $-\infty,\infty$. Also, the Caputo fractional derivatives are defined as follows. The left Caputo fractional derivative is:

$$ {}_{\mathrm{C}}D_{a,t}^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau)\,\mathrm{d}\tau \quad (27) $$

and the right Caputo fractional derivative is:

$$ {}_{\mathrm{C}}D_{t,b}^{\alpha} f(t) = \frac{(-1)^n}{\Gamma(n-\alpha)} \int_t^b (\tau-t)^{n-\alpha-1} f^{(n)}(\tau)\,\mathrm{d}\tau \quad (28) $$

where $f^{(n)}(\tau) = d^n f(\tau)/\mathrm{d}\tau^n$ and $n-1 \le \alpha < n \in \mathbb{Z}^+$. The initial conditions of fractional differential equations with the Caputo derivative have a clear physical meaning and the Caputo derivative is extensively used in real applications. The Caputo and Riemann-Liouville formulations coincide when the initial conditions are zero [7].

Background of FOPID

Thanks to the widespread industrial use of PID controllers, even a small improvement in PID features, achieved by using $PI^{\beta}D^{\alpha}$, could have a relevant impact. Recently published results, [12,15,27] indicate that the use of a fractional-order PID controller can improve both the stability and performance robustness of feedback control systems. In his book [7], Podlubny proposed a generalization of the PID controller, namely fractional PID ($PI^{\beta}D^{\alpha}$), where α and β are the order of integration and derivation, respectively, that can be real numbers. One of the most important advantages of the $PI^{\beta}D^{\alpha}$ controller is better control of the dynamical systems which are described by fractional order mathematical models. Another advantage lies in the fact that the $PI^{\beta}D^{\alpha}$ controllers are less sensitive to changes of parameters of a controlled system [12]. This is due to the two extra degrees of freedom to better adjust the dynamical properties of a fractional order control system. However, in theory, the $PI^{\beta}D^{\alpha}$ itself is an infinite dimensional linear filter due to the fractional order in the differentiator or integrator. This also implies that the tuning of the controller can be much more complex. In order to address this problem, different methods for the design of a fractional order PID (FOPID) controller have been proposed in the literature. Some of these techniques are based on an extension of the classical PID control theory. In this paper, a fractional order PID controller ($PI^{\beta}D^{\alpha}$) is used to control the process of production of technical gases as follows**:**

$$ u(t) = K_{\mathrm{p}} e(t) + K_{\mathrm{d}\,0} D_t^{\alpha} e(t) + K_{i\,0} D_t^{-\beta} e(t) \quad (29) $$

The continuous transfer function of the controller is obtained through Laplace transform of $PI^{\beta}D^{\alpha}$:

$$ G_{\mathrm{FOPID}}(s) = \frac{K_p s^{\beta} + K_i + K_d s^{\beta+\alpha}}{s^{\beta}},\ (\alpha,\beta > 0) \quad (30) $$

The controller parameters are: proportional gain K_{p}, derivative gain K_{d}, integral gain K_i, as well as non-integer order of derivative α and integrator β, Eq. (28). Unlike conventional PID controllers, there is no systematic and rigor design or tuning method for $PI^{\beta}D^{\alpha}$ controllers. For practical digital realization, the derivative part has to be complemented by the first order filter:

$$ G_{\mathrm{FOPID}}(s) = K_p \left(1 + \frac{1}{s^{\beta} T_i} + \frac{T_d s^{\alpha}}{\frac{T_d}{N} s + 1} \right) \quad (31) $$

Several methods have been proposed for tuning $PI^{\beta}D^{\alpha}$ controllers [16,18–20] by many contemporary researchers. Besides, for the most applications load disturbances are typically low frequency signals and their attenuation is a very important characteristic of a controller. It is shown in [6] that by maximizing integral gain K_i, the effect of load disturbance at the output will be minimum. It is observed that is difficult, for the general adjustment of fractional PID parameters, to satisfy the overall performance at the same time. The design of a fractional PID controller could be treated as a multi-objective optimization problem, which is to compromise the rapidity, stability and accuracy of system control. Some works use performance indices as the objective functions as follows: integral of the absolute value of the error (*IAE*), mean of the squared error (*MSE*), integral of time multiplied by absolute error (*ITAE*), integral of absolute magnitude of the error (*IAE*) and integral of the squared error (*ISE*):

$$ IAE = \int |e(t)|\,\mathrm{d}t,\ MSE = \frac{1}{T}\int (e(t))^2\,\mathrm{d}t, $$
$$ ITAE = \int t|e(t)|\,\mathrm{d}t,\ ISE = \int e(t)^2\,\mathrm{d}t,\ ITSE = \int te(t)^2\,\mathrm{d}t \quad (32) $$

As a mathematical means for optimization, GA can naturally be applied to the optimal-tuning of fractional order PID controllers.

Optimal tuning of FOPID by using GA

Here, we propose using genetic algorithms (GA) for determining the optimal parameters of fractional order PID controllers, [22]. Recently, GA has been recognized as an effective and efficient technique to solve optimization problems [23]. GA is a search technique that manipulates the coding representation of a parameter set to search a near optimal solution through cooperation and competition among the potential solutions. This algorithm is highly relevant for the industrial application, because this algorithm is capable of handling problems with constraints, objectives and dynamic components. GA uses such natural evolution to get the global optimization. Therefore, this paper describes the application of GA to fine-tuning of the parameters for fractional PID controllers. In real coding implementation, each chromosome is encoded as a vector of real numbers, of the same lengths as the solution vector. According to control objectives, five parameters K_p, K_d, K_i, α and β of a fractional PID controller are required to be designed in these settings. In this study, a next-to-optimality criterion is introduced which involves, besides steady state error *e*, *i.e.*, *IAE*, also the overshoot P_o, and settling time T_s:

$$J = |P_0| + T_s + \int |e| dt \rightarrow \min \tag{33}$$

Fitness function is designed as:

$$f_g = J_{max} + J_{min} - J_g \tag{34}$$

where J_{max} and J_{min} are the largest and the smallest values of *J*, respectively, observed thus far, and J_g is value of the criterion for the current population. All the GA parameters are arranged as follows:

– population size: *N* = 100;

– crossover probability: $p_c = 0.75$;

– mutation probability: $p_m = p_{m0} \min(1\ l/g)$, $p_{m0} = 0.1$

– initial mutation probability, *l* = 25

- generation threshold, *g*, current number of generation;

– generation gap, *gr* = 0.25.

Here, as selection method the remainder stochastic sampling with replacement is used. In our case, stopping conditions for GA are: the GA stops when the maximum number of generation (2.5*N*) has been reached or the first 50% of individuals reaches approximately the same value of the fitness function.

SIMULATIONS AND DISCUSSION

To demonstrate the feasibility of the proposed approach to the control of a cryogenic process of mixing of two gaseous air flows at different temperatures before entrance of expansion turbine, the system shown in Fig. 1 is used for illustration. Both the fractional PID and conventional PID controllers are designed based on the proposed GA. The Crone approximation of second order [30] was used for the calculation of fractional derivatives and integrals. Here, in order to obtain step response, the simulation model has been developed by using Simulink Library of MATLAB and a special toolbox for non-integer control. In Table 1 the optimal parameters of the FOPID as well as classical PID controller using GA are presented.

Table 1. The optimal parameters of the fractional PID controller and the conventional PID controller based on the proposed GA

Controller		K_p	K_i	K_d	β	α	*J*
PID	1.	15	1	0	1	1	0.81
	2.	15	7	0	1	1	19.15
FOPID	1.	13	3	11	0.034	0.073	0.24
	2.	14	8	11	0.98	0.069	13.22

In our case, each individual vector has the FOPID parameters (five parameters) where for the purpose of reducing the optimization time, the ranges of FOPID parameters are selected as:

$$K_p \in [0,20], K_i \in [0,20], K_d \in [0,20], \\ \alpha \in (0,1], \beta \in [0,1] \tag{35}$$

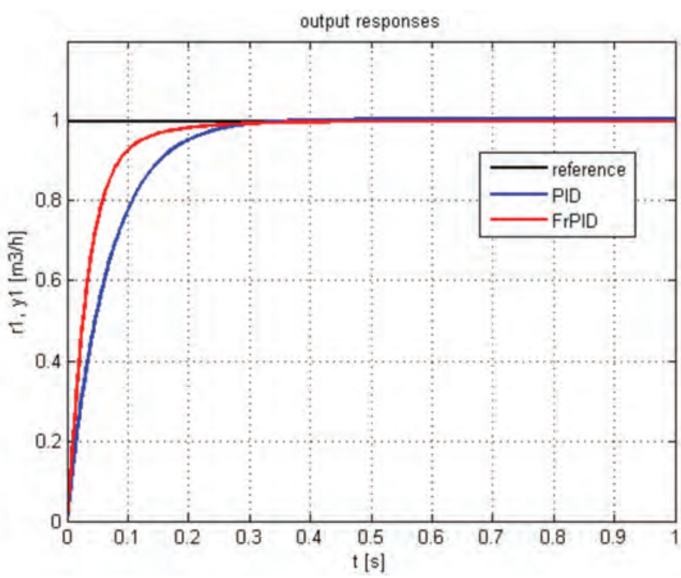

Figure 3. The step responses of the $x_{i1}(t)(m_N^3/h)$ *gas's air flow at the entrance to the expansion turbine using the optimized FOPID and conventional PID controller.*

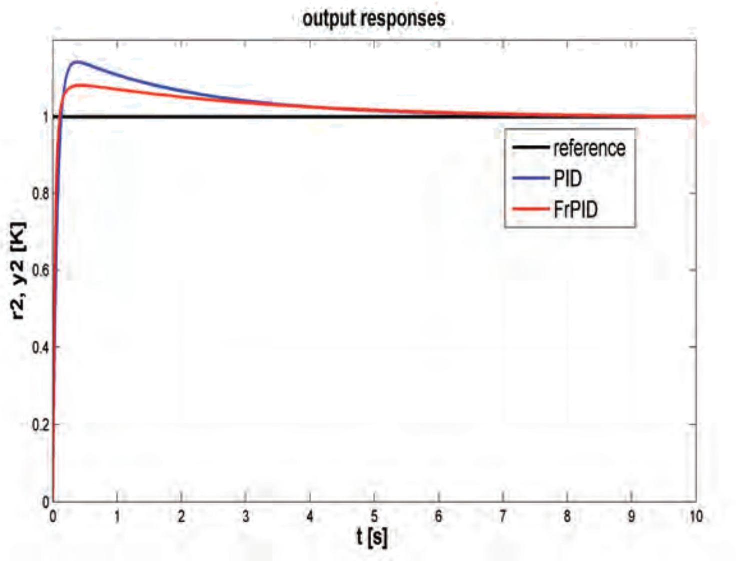

Figure 4. The step responses of the $x_{i2}(t)[K]$ – gas's air temperature at the entrance to the expansion turbine using the optimized FOPID and conventional PID controller.

As can be seen in Fig. 3, the step response of $y_1(t) = x_{i1}(t)(m_N^3/h)$ – gaseous air flow at the entrance to the expansion turbine using the optimized FOPID and conventional PID controllers has a better transient response in the case of using FOPID. Also, we obtained with FOPID controller that overshoot is 0.006% and rise time is 0.184; on the other hand, with classical PID we have 0.43% overshoot and rise time is 0.248. In view of [19], disturbances affect only $y_2(t) = x_{i2}(t)$, where the disturbances are $z_1 = 10$ K, $z_2 = 10$ K. Also, Fig. 4 represents the step response of $y_2(t) = x_{i2}(t)[K]$ – gas's air temperature at the entrance to the expansion turbine, obtained by applying the optimized FOPID and conventional PID controllers. In a similar way, we obtained that with robust FOPID controller overshoot was 8.19% and rise time was 4.520; on the other hand, with robust classical PID overshoot is 14.25% and rise time is 4.609.

Test of degree of robustness of the proposed FO PID controller

An efficient controller is the one that is still stable, even if a disturbance signal is applied to the plant. Therefore, to establish the effectiveness for a controller, the robustness should be examined.

Particularly, after the optimal values of the FOPID controller have been obtained, the degree of robustness of the proposed FOPID controller with optimal values should be tested. The next types of disturbances are applied in turn to test the robustness of the FOPID controller:

– disturbances $z_1 = z_2 = -10$ K.

As can be observed in Fig. 5, by applying the proposed disturbances $z_1 = -10$ K and $z_2 = -10$ K, the optimized FOPID and conventional PID controllers show significant of degree of robustness, but the step response of $x_{i2}(t)$ has better transient characteristic when using FOPID controller than when using the conventional PID controller.

CONCLUSION

The proposed genetic algorithm for the multi-objective optimization design of a fractional PID controller, as well as of a classical PID controller, has been applied to the control of a cryogenic process of mixing of two gaseous air flows at different temperatures before entrance to the expansion turbine. This method allows the optimal design of all major parameters of the fractional PID controller thus enhancing the flexibility and capability of the fractional PID controller. In simulations the step responses of these two optimal controllers are compared. It was shown that FOPID controller improves transient response and provides a better robustness than the conventional *PID,* particularly in disturbance rejection.

Nomenclature

p – pressure, [bar]
T, θ – temperature, [K or °C]
h – specific enthalpy, [kJ kg^{-1}]
t – time, $[s]$
x – state variable
x_o – output variable
u – control variable
z – disturbance variable
s – complex operator
q_{do} – heat brought from the environment 1 kg air [kJ/kg]
Q_o – heat exchanged with the surroundings [kW]
M_{sc} – mass of gaseous air in the tube [kg]

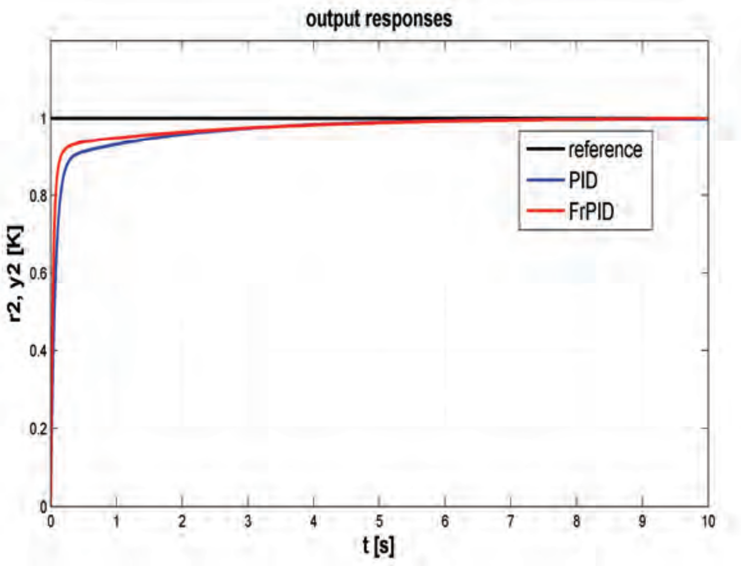

Figure 5. The step responses of the $x_{i2}(t)[K]$ – gas's air temperature at the entrance to the expansion turbine using the optimized FOPID and conventional PID controller under in case of disturbances $z_1 = z_2 = -10$ K.

m_{TV} – mass of liquid air
m_e – mass of air which expands in expansion turbine
$h_{(\cdot)}$ – specific enthalpy of gaseous air [kJ/kg]
$G_{(\cdot)}$ – mass flow rate of gaseous air $\left[m_N^3/h\right]$, $\left[kg/h\right]$
$Y_{(\ )}$ – position of control valve TV946(.) [mm]
$y_{(\ )}$ – relative variation of position of the position control valve $TV946(.)$ $[mm]$
$g(t)$ – relative variation of mass flow rate from the nominal value of gaseous air $\left[m_N^3/h\right]$.

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IZVOD

PID KONTROLERI NECELOBROJNOG REDA PODEŠENI GENETSKIM ALGORITMIMA ZA UPRAVLJANJE EKSPANZIONE TURBINE U PROCESU SEPARACIJE UTEČNJENOG VAZDUHA

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(Naučni rad)

Ovaj rad se bavi realizacijom jednog novog algoritma PID upravljanja zasnovanog na računu necelobrojnog reda (fractional calculus) u proizvodnji tehničkih gasova, odnosno u procesu separacije utečnjenog vazduha. Proizvodnja utečnjenog vazduha niskog pritiska je po prvi put bila uvedena od strane Kapice gde se ekspanzija odvijala u gasnoj turbini.Za primenu u sintezi upravljanja ulazne temperature i protoka vazduha u ekspanzionoj turbini, potrebno je odrediti odgovarajuće diferencijalne jednačine kriogenog procesa mešanja dva gasa na različitim temperaturama na ulazu u ekspanzionu turbinu. Pri tome, odgovarajući model je linearizovan i dekuplovan gde su primenjeni istovremeno klasični PID kao i $PI^{\beta}D^{\alpha}$ kontroleri necelobrojnog reda da bi se procenio kvalitet predloženog novog upravljanja datim procesom. Skup optimalnih parametara datih kontrolera se postiže primenom optimizacione procedure bazirane na genetskim algoritmima minimizovanjem odgovarajućeg kriterijuma optimalnosti. Naš metod se fokusira u okviru kriterijuma optimalnosti na smanjenje preskoka, vreme smirenja i minimizaciju integralne greske. Simulacije sprovedene u vremenskom domenu pokazuju bolje performance optimalnog $PI^{\beta}D^{\alpha}$ kontrolera u odnosu na klasični optimalni PID kontroler.

Ključne reči: Tehnički gasovi • Utečnjeni gas • PID upravljanje necelobrojnog reda • FOPID optimalno podešavanje • Genetski algoritmi