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Multi-Objective Fuzzy Optimization of Sizing and Location of Piezoelectric Actuators and Sensors for Vibration Control Based on the Particle Swarm Optimization Technique (Part 2: Numerical Analysis)

Nemanja Zorić[1)]
Aleksandar Simonović[1)]
Slobodan Stupar[1)]
Miroslav Jovanović[2)]
Nebojša Lukić[2)]
Slobodan Ilić[2)]

This is the second part of a two-paper research of the multi-objective fuzzy optimization of sizing and location of the collocated piezoelectric actuators and sensors on the thin-walled composite beam. The first paper is focussed on the theoretical model of a optimization techique for optimal sizing and location of piezoelectric actuators and sensors on a smart structure for active vibration control. In this part, the numerical analysis of the presented optimization technique will be presented for the cantilever composite beam with and without limitations in degrees of controllability (DCs) for residual modes.

Key words: **piezoelectric element, actuator, vibration measurement, composite materials, beam, fuzzy logic, multiobjective optimization, numerical analysis.**

Introduction

IN Part 1 [1] of this paper the multi-objective fuzzy optimization technique of placement and sizing of collocated piezoelectric actuators and sensors on a composite beam for maximum active vibration control effectiveness is developed. The main idea of this optimization technique is to transform both objective functions and constraints into pseudogoal functions through a process of fuzzification converting a multi-objective optimization problem into a single-objective one using the fuzzy decision principle proposed by Bellman and Zadeh [2] and finding an optimal configuration with the help of the Particle swarm optimization [3] (PSO) algorithm.

This Part 2 presents the numerical analysis of the presented optimization technique. The numerical analysis is provided on a smart cantilever composite beam modeled by using the finite element method based on the third-order shear deformation theory [4, 5]. The numerical analysis also discusses the effects of a number of actuators and the limitation of degrees of controllability (DC) of residual modes on the DC of controlled modes. The distribution of DC among actuators is discussed to ensure its uniform distribution.

Numerical analysis

In this example, a cantilever laminated beam is considered. The length of the beam is 0.6 m, and its width is 0.03 m. The beam is made of seven Graphite-Epoxy (Carbon-Fibre Reinforced) layers. The thickness of each layer is 0.5 mm and the orientations are ($90^o/90^o/0^o/0^o/0^o/0^o/90^o/90^o$). The piezoelectric patches are made of lead-zirconate-titanate (PZT). Their thicknesses are 0.2 mm. The material properties of Graphite-Epoxy and PZT are given in Table 1,

Table 1. Material properties of Graphite-Epoxy and PZT.

Material properties	Graphite-Epoxy	PZT
Modulus of elasticity, E_1 (GPa)	174	63
Modulus of elasticity, E_2 (GPa)	10.3	63
Shear modulus, G_{13} (GPa)	7.17	24.6
Shear modulus, G_{23} (GPa)	6.21	24.6
Poisson's ratio, ν_{12}	0.25	0.28
Density, ρ (kg/m^3)	1389.23	7600
Piezoelectric constant, e_{31} (C/m^2)	/	10.62
Permittivity constant, k_{33} (F/m)	/	0.1555×10^{-7}

The beam is discretized in 60 elements. The first five modes are considered as controlled modes, and the next five modes are used to reduce the spillover effect. As mentioned earlier, mounting piezoelectric sensor/actuator pairs cause changes in the original dynamic properties and the mass of the parent structure. Table 2 shows the natural frequencies of the first ten modes of the parent beam and the maximum change of natural frequencies after mounting the piezoelectric S/A pairs.

1) University of Belgrade, Faculty of Mechanical Engineering, Kraljice Marije 16, 11020 Belgrade 35, SERBIA
2) Technical Test Center, Vojvode Stepe 445, 11000 Belgrade, SERBIA

Table 2. Natural frequencies of the parent beam, the maximum value of natural frequencies after mounting S/A pairs and the maximum change of natural frequencies after mounting S/A pairs.

Mode	Natural frequency of the parent beam (Hz)	Maximum value of natural frequencies after mounting S/A pairs (Hz)	Maximum NF change (%)
1	6.437	9.189	42.755
2	40.347	50.545	25.277
3	113.014	137.879	22.002
4	221.573	267.361	20.665
5	366.513	439.328	19.867
6	547.942	653.635	19.289
7	766.030	910.748	18.892
8	1020.962	1209.595	18.476
9	1313.008	1549.875	18.040
10	1642.389	1931.187	17.584

The mass of the beam without S/A pairs is 0.0875 kg, and the mass of the beam which is fully covered by S/A pairs is 0.1422 kg, resulting with the maximum change of the beam mass 62.56%.

In the optimization problem, the number of S/A pairs varies from one to four. Two examples for every number of S/A pairs are done: one without a limit of the spillover effect, and the other one where the spill-over effect is considered to be less than 2% with a tolerance of the upper limit of $\overline{\beta}_{Ri} = 1.1$. The changes of first five natural frequencies are fewer than 10%, and the mass change is less than 15% with a tolerance of the upper limit of $\overline{\beta}_{fi} = \overline{\beta}_{m} = 1.15$. In order to search efficiently for the optimal sizing and location of S/A pairs, the cognition and the social learning factor in the PSO algorithm are set as $c_1 = c_2 = 1.5$, while the inertia weight is linearly varied from 1 to 0.5. The number of population is 30 particles, and the number of iterations is 50.

Single S/A pair

For unconstrained DC for residual modes, a solution is to having an S/A pair with a length of 160 mm and the distance from the root of the beam is 120 mm. In the case of constrained DC for residual modes (less than 2%), the S/A pair has a length of 20 mm, and the distance from the root of the beam is 220 mm. Figures 1 and 2 present the location and the length of a single S/A pair and DC for the first ten modes for unconstrained and constrained DC for residual modes, respectively.

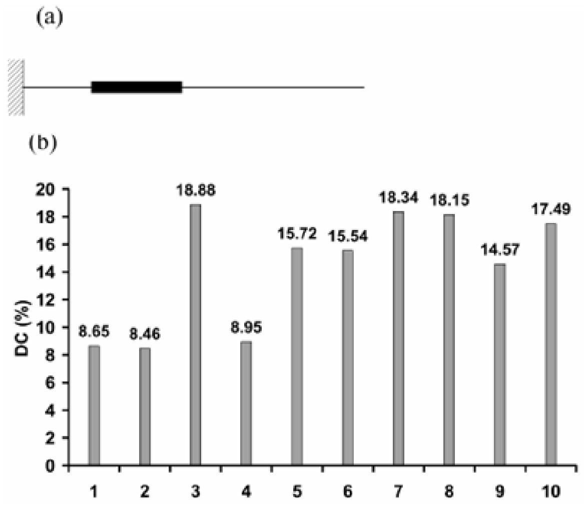

Figure 1. Single S/A pair, unconstrained DC for residual modes: (a) location and sizing of the S/A pair, (b) DC for the first ten modes

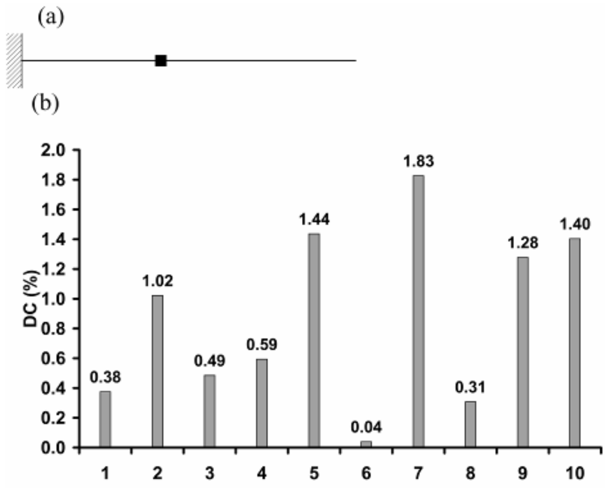

Figure 2. Single S/A pair, DC for residual modes less than 2%: (a) location and sizing Of the S/A pair, (b) DC for the first ten modes

Multiple S/A pairs

Figures 3-8 present the location and the length of a single S/A pair, DC for the first ten modes and their distribution among the actuators for unconstrained (Figures 3, 5, 7) and constrained (Figures 4, 6, 8) DC for residual modes, in the case of two, three and four S/A pairs, respectively. Table 3 presents the location and the length of two, three and four S/A pairs for unconstrained and constrained DC for residual modes.

Table 3. Optimal location and sizing of S/A pairs for unconstrained and constrained DC for residual modes

Number of S/A pairs	Unconstrained DC for res. modes		DC for res. modes less than 2%	
	Location (mm)	Length (mm)	Location (mm)	Length (mm)
2	0	90	10	30
	490	50	230	10
3	0	70	10	30
	190	50	240	10
	480	40	380	10
4	0	60	10	30
	80	10	110	10
	170	40	360	10
	450	50	510	10

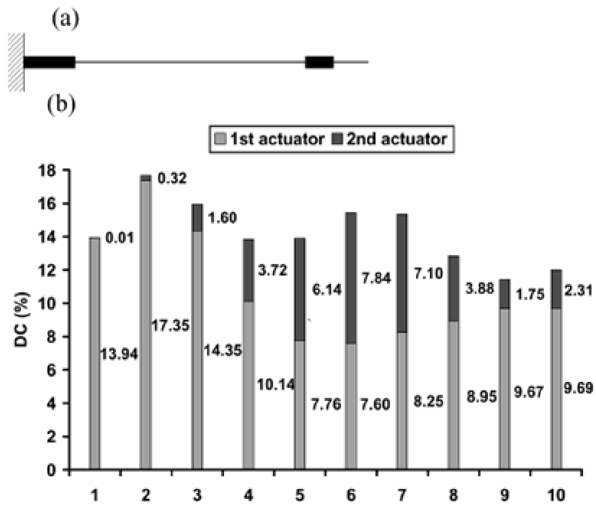

Figure 3. Two S/A pair, unconstrained DC for residual modes: (a) location and sizing of S/A pairs, (b) DC for the first ten modes

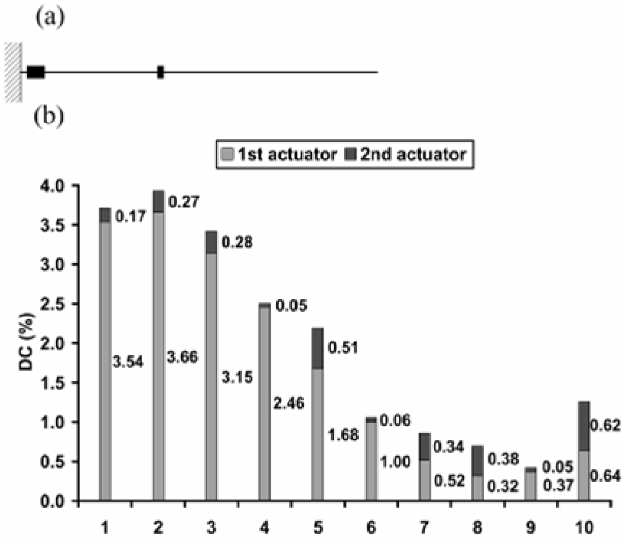

Figure 4. Two S/A pairs, DC for residual modes less than 2%: (a) location and sizing of S/A pairs, (b) d DC for the first ten modes

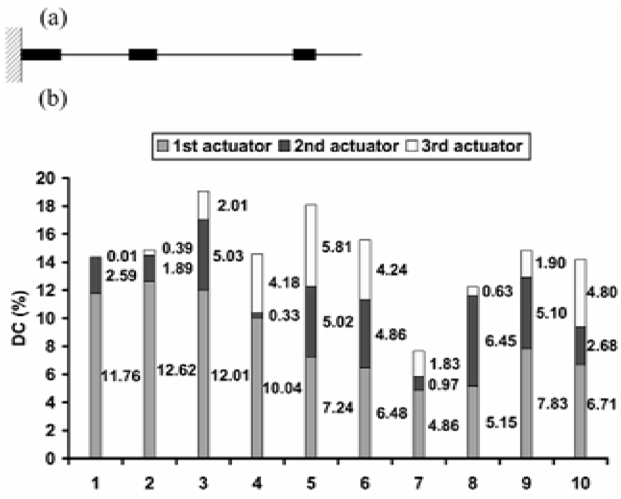

Figure 5. Three S/A pair, unconstrained DC for residual modes: (a) location and sizing of S/A pairs, (b) DC for the first ten modes

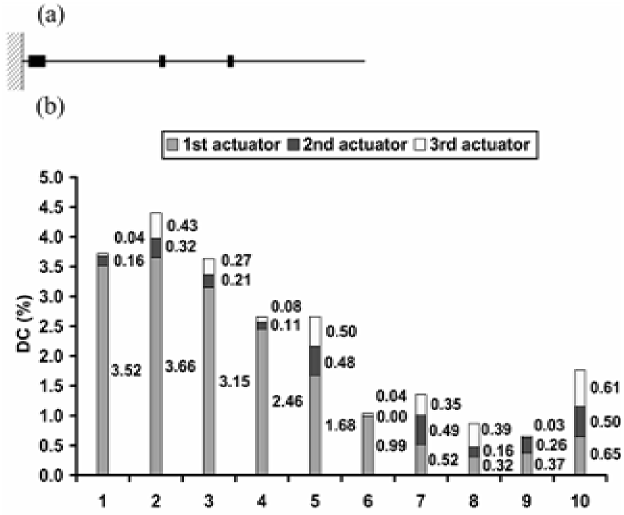

Figure 6. Three S/A pairs, DC for residual modes less than 2%: (a) location and sizing of S/A pairs, (b) DC for the first ten modes

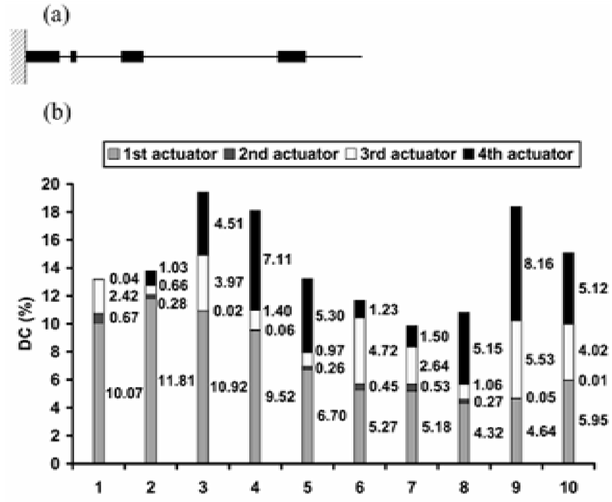

Figure 7. Four S/A pair, unconstrained DC for residual modes: (a) location and sizing of S/A pairs, (b) DC for the first ten modes

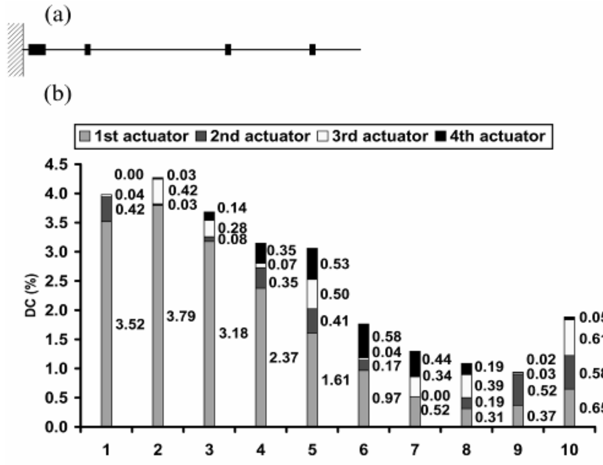

Figure 8. Four S/A pairs, DC for residual modes less than 2%: (a) location and sizing of S/A pairs, (b) DC for the first ten modes

Discussion

Comparing DC in the case of unconstrained DC for residual modes among a number of S/A pairs (Fig.9 (a)), it is clear that the increase of the number of S/A pairs leads to the increase of DC for all modes, globally. Fig.1(b) shows that, in the case of single S/A pairs, only two modes have good DC, but the other three have low DC. By increasing the number of S/A pairs, DC for all modes are increased, but DC for residual modes for the unconstrained cases are very high, even higher than DC for some controlled modes (Figures 1(b), 3(b), 5(b) and 7(b)).

Keeping DC for residual modes below 2% leads to a decrease of the DC for controlled modes. For a single S/A pair, DC for controlled modes are lower than DC for residual modes (Fig.2(b)). In this case, increasing the number of S/A pairs leads to an increase of DC for controlled modes (Fig.9(b)).

Considering the DC of each actuator particularly, it can be seen that the distribution of DC among actuators is not uniform. In the case of four S/A pairs, almost all controllability for the first two modes is carried out by the first actuator (Fig.7(b)). In the case of failure or damage of this actuator, this might have negative consequences. In this case, DC for the first mode will fall out form 13.21% to 3.14% and for the second mode from 13.78% to 1.97%. Also, the degree of controllability of the second actuator is very low for all controlled modes. Taking into account Figures 3-7, the first actuator has good controllability for all controlled modes, but the actuators placed near the top of the beam have good controllability for the last three controlled modes, but for the first two, controllability is very low. Due to that, simultaneous DC increasing of a certain mode and keeping DC more uniform among all actuators for the same mode is very hard. Instead of that, the objective can be uniform distribution among a few actuators for controlled modes.

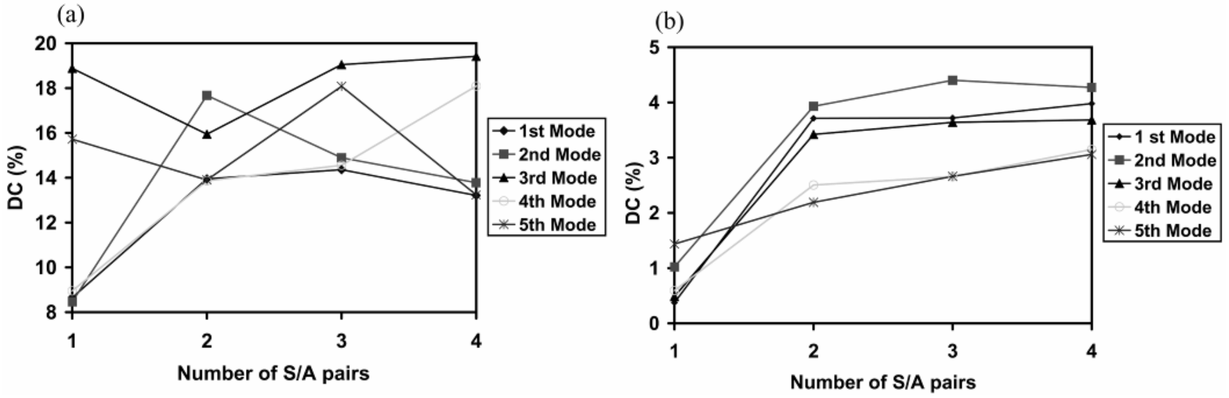

Figure 9. DC for controlled modes versus the number of S/A pairs: (a) for unconstrained DC for residual modes, (b) for constrained DC for residual modes

Considering the case with four S/A pairs, the goal is to achieve a uniform distribution of DC for three actuators with the largest DC values. After fuzzification, the new objective functions are

$$\mu_{\mathrm{Dis}\,i}(\mathbf{p}) = \frac{\min\{\max3\{DC_{\mathrm{C}\,ij}(\mathbf{p})\}\}}{\overline{DC}_{\mathrm{C}\,i}}, \quad j = 1{,}4 \tag{1}$$

where

$$\overline{DC}_{\mathrm{C}i} = \frac{DC_{\mathrm{C}\,i}(\mathbf{p}) - \min\{DC_{\mathrm{C}ij}(\mathbf{p})\}}{3} \tag{2}$$

and $\max3\{DC_{\mathrm{C}ij}(\mathbf{p})\}$ denotes the set of values of the first three highest DC of actuators for the i-th mode. The membership function for the distribution of DC among actuators is presented in Fig.10. This mean that, in an ideal case $\left(\mu_{\mathrm{Dis}i}(\mathbf{p}) = 1\right)$, all three actuators will have the same value for DC which is $\overline{DC}_{\mathrm{C}i}$.

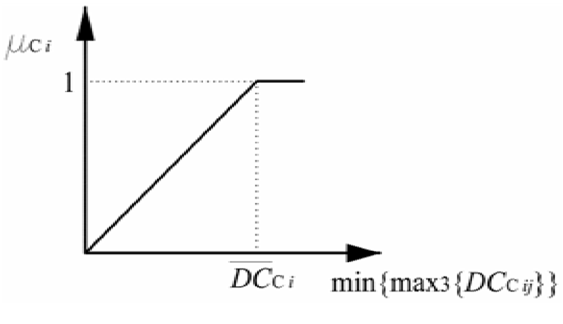

Figure 10. Membership function for the distribution of DC among actuators

Considering Eq. (1), the optimization problem becomes

$$\mu_{\mathrm{Dis}\,i}(\mathbf{p}) = \frac{\min\{\max3\{DC_{\mathrm{C}\,ij}(\mathbf{p})\}\}}{\overline{DC}_{\mathrm{C}\,i}} \tag{3}$$

Where

$$\mu_{\mathrm{D}} = \min\left\{\begin{array}{l} \min\limits_{i=1,N_{\mathrm{C}}} \mu_{\mathrm{C}\,i}\,,\ \min\limits_{i=1,N_{\mathrm{C}}} \mu_{\mathrm{Dis}\,i}\,, \\ \min\limits_{j=1,N_{\mathrm{fmods}}} \mu_{\mathrm{f}\,j}\,,\ \mu_{\mathrm{m}}\,,\ \min\limits_{k=1,N_{\mathrm{R}}} \mu_{\mathrm{R}\,k} \end{array}\right\} \tag{4}$$

The location and length of S/A pairs are presented in Table 4 and Fig.11(a). Fig.11(b) presents the distribution of DC among actuators.

Table 4. Location and length of four S/A pairs in the case of a uniform distribution of DC among actuators

No of S/A pair	Location (mm)	Length (mm)
1	0	40
2	50	30
3	210	50
4	440	40

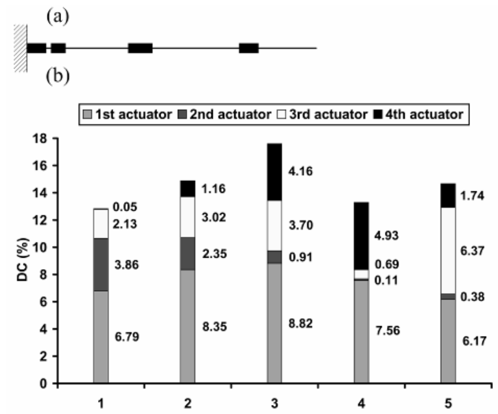

Figure 11. Four S/A pair, uniform distribution of DC among actuators: (a) location and sizing of S/A pairs, (b) DC for the first five modes

Conclusion

The maximization of DC for the first five modes and a variation of the number of S/A pairs in the range from one to four have shown that increasing the number of S/A pairs results in better control effectiveness for residual modes, in the case of unconstrained and constrained DC. The obtained solutions have a non-uniform distribution of DC among actuators. Particularly, in the case of four S/A pairs, almost all controllability for the first two modes is carried out by the first actuator. Also, the degree of controllability of the second actuator is very low for all controlled modes. Eventually, failure or damage of the first actuator might have negative consequences for the control effectiveness.

For achieving more uniform distribution in the case of four S/A pairs, the corresponding objective functions based on the fuzzy set theory are constructed. Involving these objective functions in the optimization problem, the obtained results show more uniform distribution of DC among actuators. Thus, the total percentage of DC of the 2nd, 3rd and 4th actuator is increased from 3.14% to 6.04% for the first mode, and from 1.97% to 6.53% for the second mode without drastic changes in the control performance of modes. The degree of controllability of the second actuator is also increased (from 0.67% to 3.86% for the first mode and from 0.28% to 2.35% for the second mode).

Although this work deals with optimization problems in the case of a laminated beam, taking into account all the advantages presented here, the considered optimization technique can be also studied for more complex structures.

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References

[1] ZORIĆ,N., SIMONOVIĆ,A., STUPAR,S., JOVANOVIĆ,M., LUKIĆ,N.: *Multi-Objective Fuzzy Optimization of Sizing and Location of Piezoelectric Actuators and Sensors for Vibration Control Based on the Particle Swarm Optimization Technique (Part 1: Theoretical Model)*, Scientific Technical Review, ISSN 1820-0206, 2014, Vol.64, No.2, pp.21-26,

[2] BELLMAN,R.E., ZADEH,L.A.: *Decision-making in fuzzy environment*, Management Science, 1970, Vol.17, pp.141-164,

[3] KENNEDY,J., EVERHART,R.C.: *Particle swarm optimization*, Proceedings of the IEEE international conference on neural networks, 1995, Vol.4, pp.1942-1948.

[4] HEYLIGER,N.D., REDDY,N.: *A higher order beam finite elements for bending and vibration problem*, Journal of Sound and Vibration, 1985, Vol.126, pp.309-326.

[5] ZORIĆ,N., SIMONOVIĆ,A., MITROVIĆ,Z., STUPAR,S.: *Optimal vibration control of smart composite beams with optimal size and location of piezoelectric sensing and actuation*, Journal of Intelligent Material Systems and Structures, 2013, Vol.24, pp.499-526.

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Višeciljna fazi optimizacija veličine i položaja piezoelektričnih aktuatora i senzora za upravljanje vibracijama bazirana na optimizaciji rojem čestica (deo 2: numerička analiza)

Ovo je drugi deo istraživanja višeciljne fazi optimizacije veličine i položaja piezoelektričnih aktuatora i senzora na tankozidnoj kompozitnoj gredi. Prvi rad je fokusiran na teorijskom modelu optimizacione tehnike za optimalno dimenzionisanje i postavljanje piezoelektičnih aktuatora i senzora na pametnu strukturu za aktivno upravljanje vibracijama. U ovom delu, biće prikazana numerička analiza predstavljene optimizacione tehnike za kompozitnu konzolu sa i bez ograničenja u pogledu stepena upravljivosti za rezidualne modove.

Ključne reči: **piezoelektrični element, aktuator, merenje vibracija, kompozitni materijali, greda, fazi logika, višekriterijumska optimizacija, numerička analiza.**

Мультицелевой этап оптимизации нескольких размеров и позиций пьезоэлектрических приводов и датчиков для управления вибрациями на основе оптимизации роя частиц (часть 2: численный анализ)

Это вторая часть научно-исследовательского мультицелевого этапа оптимизации размера и положения пьезоэлектрических приводов и датчиков на тонкостенной композитной балке. Первая часть работы сосредоточена на теоретическом методе оптимизации модели для выбора оптимального размера и размещения пьезоэлектрических приводов и датчиков в смарт-структуру для активного контроля вибрациями. В этой части работы будет представлен численный анализ представленых методов оптимизации для композитной консоли с ограничением и без каких-либо ограничений по степени контроля для остаточных режимов.

Ключевые слова: **пьезоэлектрический элемент, привод, измерение вибрации, композитные материалы, балка, нечёткая логика, многокритериальная оптимизация, численный анализ.**

Optimisation multi objective "fuzzy" de la taille et de la location des actuaires piézoélectriques et des capteurs pour le contrôle des vibrations basées sur l'optimisation par essaim des particules (seconde partie: analyse numérique)

C'est la seconde partie de la recherche sur l'optimisation "fuzzy" multi objective de la taille et de la location des actuaires piézoélectriques et des capteurs sur la poutre composite aux parois minces. La première partie de ce travail a été centrée sur le modèle théorétique de la technique d'optimisation pour le dimensionnement optimale et la location des actuaires piézoélectriques et des capteurs sur la structure intelligente pour le contrôle actif des vibrations. Dans la seconde partie de ce travail on a présenté l'analyse numérique de la technique d'optimisation pour la console composite avec ou sans limite à l'égard du degré de contrôlabilité pour les modes résiduels.

Mots clés: **élément piézoélectrique, actuaire, mesurage des vibrations, matériaux composites, poutre, logique « fuzzy », optimisation multicritère, analyse numérique.**