Fixed Point Theory and Applications
a SpringerOpen Journal

RESEARCH Open Access

Some coupled coincidence and common fixed point results for a hybrid pair of mappings in 0-complete partial metric spaces

Wei Long[1*], Satish Shukla[2], Stojan Radenović[3] and Slobodan Radojević[3]

*Correspondence: hopelw@126.com
[1]College of Mathematics and Information Science, Jiangxi Normal University, Nanchang, Jiangxi 330022, People's Republic of China
Full list of author information is available at the end of the article

Abstract
In this paper we extend some coupled coincidence and common fixed point theorems for a hybrid pair of mappings obtained by Abbas *et al.* (Fixed Point Theory Appl. 2012:4, 2012, doi:10.1186/1687-1812-2012-4) from the complete metric space to 0-complete partial metric spaces. An example showing that this extension is proper is given.
MSC: 47H10; 54H25

Keywords: set-valued mapping; hybrid pair of mappings; coupled fixed point; coupled coincidence point; partial metric space

1 Introduction

Let A be any nonempty subset of a metric space (X,d). For $x \in X$, define

$$d(x,A) = \inf\{d(x,y) : y \in A\}.$$

Let $CB(X)$ denote the set of all nonempty closed bounded subset of X. For $A, B \in CB(X)$, define

$$\delta(A,B) = \sup\{d(x,B) : x \in A\},$$
$$H(A,B) = \max\{\delta(A,B), \delta(B,A)\}.$$

Then H is a metric on $CB(X)$ and is called a Hausdorff metric.

Nadler [1] generalized the Banach contraction mapping principle to set-valued functions and proved the following fixed point theorem.

Theorem 1 *Let (X,d) be a complete metric space and let T be a mapping from X into $CB(X)$ such that for all $x, y \in X$,*

$$H(Tx,Ty) \le \lambda d(x,y),$$

where $0 \le \lambda < 1$. Then T has a fixed point.

Later, an interesting and rich fixed point theory was developed. On the other hand, Matthews [2] introduced the notion of a partial metric space as a part of the study of denotational semantics of dataflow networks, with the interesting property 'non-zero self-distance' in space. He showed that the Banach contraction mapping theorem can be generalized to the partial metric context for applications in program verification. Subsequently, several authors (see, *e.g.*, [3–22]) derived fixed point theorems in partial metric spaces. Romaguera [17] introduced the notion of 0-Cauchy sequence, 0-complete partial metric spaces and proved some characterizations of partial metric spaces in terms of completeness and 0-completeness. Recently, Aydi *et al.* [9] introduced the notion of a partial Hausdorff metric and extended the Nadler's theorem in partial metric spaces.

Bhaskar and Lakshmikantham [23] introduced the concept of a coupled fixed point and established some coupled fixed point theorems in partially ordered sets. As an application, they studied the existence and uniqueness of a solution for a periodic boundary value problem associated with a first-order ordinary differential equation. Recently Abbas *et al.* [24] extended these concepts to set-valued mappings and obtained coupled coincidence points and coupled common fixed point theorems involving a hybrid pair of single-valued and multi-valued maps satisfying generalized contractive conditions in the framework of a complete metric space (see also [25, 26]). The study of a coincidence point and common fixed points of a hybrid pair of mappings in Banach spaces and metric spaces is interesting and well developed. For applications of hybrid fixed point theory, we refer to [27–30].

In this paper, we extend and generalize the results of Abbas *et al.* [24] and Aydi *et al.* [9] for a hybrid pair of mappings in 0-complete partial metric spaces. Also, some new results are obtained. An example is included to support our results.

2 Preliminaries

Consistent with [2, 8, 9, 16, 17, 19], the following definitions and results will be needed in the sequel.

Definition 1 A partial metric on a nonempty set X is a function $p : X \times X \to \mathbb{R}^+$ ($\mathbb{R}^+$ stands for nonnegative reals) such that for all $x, y, z \in X$,

(P1) $x = y \Leftrightarrow p(x,x) = p(x,y) = p(y,y)$,

(P2) $p(x,x) \leq p(x,y)$,

(P3) $p(x,y) = p(y,x)$,

(P4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$.

A partial metric space is a pair (X,p) such that X is a nonempty set and p is a partial metric on X.

It is clear that if $p(x,y) = 0$, then from (P1) and (P2) $x = y$. But if $x = y$, $p(x,y)$ may not be 0. Also, every metric space is a partial metric space, with zero self-distance.

Example 1 If $p : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is defined by $p(x,y) = \max\{x,y\}$, for all $x, y \in \mathbb{R}^+$, then $(\mathbb{R}^+, p)$ is a partial metric space.

Some more examples of a partial metric space can be seen in [2, 9, 16].

Each partial metric on X generates a T_0 topology τ_p on X which has as a base the family of open p-balls $\{B_p(x,\epsilon) : x \in X, \epsilon > 0\}$, where $B_p(x,\epsilon) = \{y \in X : p(x,y) < p(x,x) + \epsilon\}$ for all $x \in X$ and $\epsilon > 0$.

Theorem 2 [2] *For each partial metric* $p : X \times X \to \mathbb{R}^+$, *the pair* (X,d), *where* $d(x,y) = 2p(x,y) - p(x,x) - p(y,y)$ *for all* $x,y \in X$, *is a metric space*.

Here (X,d) is called an induced metric space and d is an induced metric. In further discussion, unless specified otherwise, (X,d) will represent an induced metric space.

Let (X,p) be a partial metric space.

(1) A sequence $\{x_n\}$ in (X,p) converges to a point $x \in X$ if and only if $p(x,x) = \lim_{n\to\infty} p(x_n,x)$.
(2) A sequence $\{x_n\}$ in (X,p) is called a Cauchy sequence if there exists (and is finite) $\lim_{n,m\to\infty} p(x_n,x_m)$.
(3) (X,p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges with respect to τ_p to a point $x \in X$ such that $p(x,x) = \lim_{n,m\to\infty} p(x_n,x_m)$.
(4) A sequence $\{x_n\}$ in (X,p) is called 0-Cauchy sequence if $\lim_{n,m\to\infty} p(x_n,x_m) = 0$. The space (X,p) is said to be 0-complete if every 0-Cauchy sequence in X converges with respect to τ_p to a point $x \in X$ such that $p(x,x) = 0$.

Lemma 1 [2, 17, 19] *Let* (X,p) *be a partial metric space and* $\{x_n\}$ *be any sequence in* X.

(i) $\{x_n\}$ *is a Cauchy sequence in* (X,p) *if and only if it is a Cauchy sequence in the metric space* (X,d).
(ii) (X,p) *is complete if and only if the metric space* (X,d) *is complete*. *Furthermore*, $\lim_{n\to\infty} d(x_n,x) = 0$ *if and only if* $p(x,x) = \lim_{n\to\infty} p(x_n,x) = \lim_{n,m\to\infty} p(x_n,x_m)$.
(iii) *Every* 0-*Cauchy sequence in* (X,p) *is Cauchy in* (X,d).
(iv) *If* (X,p) *is complete, then it is* 0-*complete*.

The converse assertions of (iii) and (iv) do not hold. Indeed, the partial metric space $(\mathbb{Q}\cap[0,\infty),p)$, where $\mathbb{Q}$ denotes the set of rational numbers and the partial metric p is given by $p(x,y) = \max\{x,y\}$ for all $x,y \in X$, provides an easy example of a 0-complete partial metric space which is not complete. It is easy to see that every closed subset of a 0-complete partial metric space is 0-complete.

Let (X,p) be a partial metric space. Let $CB^p(X)$ be the family of all nonempty, closed and bounded subsets of the partial metric space (X,p) induced by the partial metric p. Note that closedness is taken from (X,τ_p) (τ_p is the topology induced by p) and boundedness is given as follows: A is a bounded subset in (X,p) if there exist $x_0 \in X$ and $M \geq 0$ such that for all $a \in A$, we have $a \in B_p(x_0,M)$, that is, $p(x_0,a) < p(a,a) + M$.

For $A,B \in CB^p(X)$ and $x \in X$, define

$$p(x,A) = \inf\bigl\{p(x,a) : a \in A\bigr\}, \qquad \delta_p(A,B) = \sup\bigl\{p(a,B) : a \in A\bigr\}.$$

Lemma 2 [8] *Let* (X,p) *be a partial metric space*, $A \subset X$. *Then* $a \in \overline{A}$ *if and only if* $p(a,A) = p(a,a)$.

Proposition 1 [9] *Let* (X,p) *be a partial metric space*. *For any* $A,B,C \in CB^p(X)$, *we have the following*:

(i) $\delta_p(A,A) = \sup\{p(a,a) : a \in A\}$;
(ii) $\delta_p(A,A) \leq \delta_p(A,B)$;
(iii) $\delta_p(A,A) = 0$ *implies that* $A \subseteq B$;
(iv) $\delta_p(A,B) \leq \delta_p(A,C) + \delta_p(C,B) - \inf_{c\in C} p(c,c)$.

Let (X,p) be a partial metric space. For $A,B \in CB^p(X)$, define

$$H_p(A,B) = \max\{\delta_p(A,B), \delta_p(B,A)\}.$$

Proposition 2 [9] *Let (X,p) be a partial metric space. For $A,B,C \in CB^p(X)$, we have*

(h1) $H_p(A,A) \leq H_p(A,B)$;
(h2) $H_p(A,B) = H_p(B,A)$;
(h3) $H_p(A,B) \leq H_p(A,C) + H_p(C,B) - \inf_{c\in C} p(c,c)$.

Corollary 1 [9] *Let (X,p) be a partial metric space. For $A,B \in CB^p(X)$, the following holds*:

$$H_p(A,B) = 0 \quad \textit{implies that} \quad A = B.$$

In view of Proposition 2 and Corollary 1, we call the mapping $H_p : CB^p(X) \times CB^p(X) \to [0,\infty)$ a partial Hausdorff metric induced by p.

Lemma 3 [9] *Let (X,p) be a partial metric space, $A,B \in CB^p(X)$ and $h > 1$. For any $a \in A$, there exists $b = b(a) \in B$ such that $p(a,b) \leq hH_p(A,B)$.*

The following lemma is crucial for the proof of our main result and its proof is similar to Lemma 3.

Lemma 4 *Let (X,p) be a partial metric space and $A,B \in CB^p(X)$, $a \in A$. Let $\epsilon > 0$ be arbitrary, then there exists $b = b(a) \in B$ such that*

$$p(a,b) \leq H_p(A,B) + \epsilon.$$

Definition 2 [24] Let X be a nonempty set, $F : X \times X \to 2^X$ (collection of all nonempty subsets of X) and $g : X \to X$. An element $(x,y) \in X \times X$ is called

(i) a coupled fixed point of F if $x \in F(x,y)$ and $y \in F(y,x)$;
(ii) a coupled coincidence point of the hybrid pair $\{F,g\}$ if $gx \in F(x,y)$ and $gy \in F(y,x)$;
(iii) a coupled point of coincidence if there exists $(u,v) \in X \times X$ such that $x = gu \in F(u,v)$ and $y = gv \in F(v,u)$;
(iv) a coupled common fixed point of the hybrid pair $\{F,g\}$ if $x = gx \in F(x,y)$ and $y = gy \in F(y,x)$.

Definition 3 Let X be a nonempty set, let $F : X \times X \to 2^X$ and $g : X \to X$ be two mappings. The hybrid pair $\{F,g\}$ is called weakly compatible if $gF(x,y) \subseteq F(gx,gy)$ and $gF(y,x) \subseteq F(gy,gx)$ whenever (x,y) is a coupled coincidence point of the hybrid pair $\{F,g\}$.

Now we can state our main results.

3 Main results

The following result extends and generalizes the main result of [24] in partial metric spaces.

Theorem 3 *Let (X,p) be a 0-complete partial metric space, let $F : X \times X \to CB^p(X)$ and $g : X \to X$ be mappings satisfying*

$$\begin{aligned} H_p\bigl(F(x,y),F(u,v)\bigr) \le{}& a_1p(gx,gu) + a_2p(gy,gv) + a_3p\bigl(F(x,y),gx\bigr) \\ &+ a_4p\bigl(F(x,y),gu\bigr) + a_5p\bigl(F(u,v),gx\bigr) \\ &+ a_6p\bigl(F(u,v),gu\bigr) \end{aligned} \tag{1}$$

for all $x,y,u,v \in X$, where a_i are nonnegative reals such that $\sum_{i=1}^{6} a_i < 1$. If $F(X \times X) \subseteq g(X)$ and $g(X)$ is a closed subset of X, then F and g have a coupled point of coincidence $(w_c,z_c) \in X \times X$ and $p(w_c,w_c) = p(z_c,z_c) = 0$.

Proof Let $x_0,y_0 \in X$ be arbitrary, then $F(x_0,y_0),F(y_0,x_0) \in CB^p(X)$. As $F(X \times X) \subseteq g(X)$, we can choose $gx_1 \in F(x_0,y_0)$ and $gy_1 \in F(y_0,x_0)$ for some $x_1,y_1 \in X$. Again, as $F(x_1,y_1),F(y_1,x_1) \in CB^p(X)$ and $F(X \times X) \subseteq g(X)$, so by Lemma 4, for any $\epsilon > 0$, there exist $gx_2 \in F(x_1,y_1)$ and $gy_2 \in F(y_1,x_1)$ such that

$$\begin{aligned} &p(gx_1,gx_2) \le H_p\bigl(F(x_0,y_0),F(x_1,y_1)\bigr) + \epsilon, \\ &p(gy_1,gy_2) \le H_p\bigl(F(y_0,x_0),F(y_1,x_1)\bigr) + \epsilon. \end{aligned}$$

Continuing this process, we obtain two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\begin{aligned} &gx_{n+1} \in F(x_n,y_n) \quad \text{and} \quad gy_n \in F(y_n,x_n), \\ &p(gx_n,gx_{n+1}) \le H_p\bigl(F(x_{n-1},y_{n-1}),F(x_n,y_n)\bigr) + \epsilon^n, \\ &p(gy_n,gy_{n+1}) \le H_p\bigl(F(y_{n-1},x_{n-1}),F(y_n,x_n)\bigr) + \epsilon^n. \end{aligned}$$

From the above inequalities and (1), we obtain

$$\begin{aligned} p(gx_n,gx_{n+1}) \le{}& H_p\bigl(F(x_{n-1},y_{n-1}),F(x_n,y_n)\bigr) + \epsilon^n \\ \le{}& a_1p(gx_{n-1},gx_n) + a_2p(gy_{n-1},gy_n) \\ &+ a_3p\bigl(F(x_{n-1},y_{n-1}),gx_{n-1}\bigr) + a_4p\bigl(F(x_{n-1},y_{n-1}),gx_n\bigr) \\ &+ a_5p\bigl(F(x_n,y_n),gx_{n-1}\bigr) + a_6p\bigl(F(x_n,y_n),gx_n\bigr) + \epsilon^n \\ \le{}& a_1p(gx_{n-1},gx_n) + a_2p(gy_{n-1},gy_n) + a_3p(gx_n,gx_{n-1}) \\ &+ a_4p(gx_n,gx_n) + a_5p(gx_{n+1},gx_{n-1}) + a_6p(gx_{n+1},gx_n) + \epsilon^n \\ \le{}& a_1p(gx_{n-1},gx_n) + a_2p(gy_{n-1},gy_n) + a_3p(gx_n,gx_{n-1}) \\ &+ a_4p(gx_n,gx_n) + a_5p(gx_{n+1},gx_n) + a_5p(gx_n,gx_{n-1}) \\ &- a_5p(gx_n,gx_n) + a_6p(gx_{n+1},gx_n) + \epsilon^n, \end{aligned}$$

that is,

$$\begin{aligned} (1 - a_5 - a_6)p(gx_n,gx_{n+1}) \le{}& (a_1 + a_3 + a_5)p(gx_n,gx_{n-1}) + a_2p(gy_{n-1},gy_n) \\ &+ (a_4 - a_5)p(gx_n,gx_n) + \epsilon^n. \end{aligned} \tag{2}$$

Interchanging the roles of x_n and x_{n+1} and using the symmetries of p and H_p, we obtain

$$\begin{aligned}(1-a_4-a_3)p(gx_n,gx_{n+1}) \le{}& (a_1+a_6+a_4)p(gx_n,gx_{n-1}) + a_2p(gy_{n-1},gy_n)\\ &+ (a_5-a_4)p(gx_n,gx_n) + \epsilon^n. \end{aligned} \tag{3}$$

It follows from (2) and (3) that

$$\begin{aligned}(2-a_3-a_4-a_5-a_6)p(gx_n,gx_{n+1}) \le{}& (2a_1+a_3+a_4+a_5+a_6)p(gx_n,gx_{n-1})\\ &+ 2a_2p(gy_{n-1},gy_n) + 2\epsilon^n. \end{aligned} \tag{4}$$

Similarly, it can be obtained that

$$\begin{aligned}(2-a_3-a_4-a_5-a_6)p(gy_n,gy_{n+1}) \le{}& (2a_1+a_3+a_4+a_5+a_6)p(gy_n,gy_{n-1})\\ &+ 2a_2p(gx_{n-1},gx_n) + 2\epsilon^n. \end{aligned} \tag{5}$$

For simplicity, set $p_n = p(gx_n,gx_{n+1}) + p(gy_n,gy_{n+1})$, then it follows from (4) and (5) that

$$(2-a_3-a_4-a_5-a_6)p_n \le (2a_1+2a_2+a_3+a_4+a_5+a_6)p_{n-1} + 4\epsilon^n,$$

that is,

$$p_n \le \frac{2a_1+2a_2+a_3+a_4+a_5+a_6}{2-a_3-a_4-a_5-a_6}p_{n-1} + \frac{4\epsilon^n}{2-a_3-a_4-a_5-a_6}. \tag{6}$$

As $\epsilon > 0$ was arbitrary, choose $\epsilon = \frac{2a_1+2a_2+a_3+a_4+a_5+a_6}{2-a_3-a_4-a_5-a_6}$; also, as $\sum_{i=1}^6 a_i < 1$, we have $\epsilon < 1$. Therefore, from (6) we have

$$p_n \le \epsilon p_{n-1} + \frac{4\epsilon^n}{1+a_1+a_2}.$$

From a successive application of the above inequality, we obtain

$$\begin{aligned}
p_n &\le \epsilon p_{n-1} + \frac{4\epsilon^n}{1+a_1+a_2},\\
p_n &\le \epsilon\left[\epsilon p_{n-2} + \frac{4\epsilon^{n-1}}{1+a_1+a_2}\right] + \frac{4\epsilon^n}{1+a_1+a_2},\\
p_n &\le \epsilon^2 p_{n-2} + \frac{8\epsilon^n}{1+a_1+a_2},\\
&\vdots\\
p_n &\le \epsilon^n p_0 + \frac{4n\epsilon^n}{1+a_1+a_2}.
\end{aligned} \tag{7}$$

For $m,n \in \mathbb{N}$ with $m > n$, using (7) we obtain

$$\begin{aligned}p(gx_n,gx_m) + p(gy_n,gy_m) \le{}& p(gx_n,gx_{n+1}) + p(gy_n,gy_{n+1}) + p(gx_{n+1},gx_{n+2})\\ &+ p(gy_{n+1},gy_{n+2}) + \cdots + p(gx_{m-1},gx_m) + p(gy_{m-1},gy_m)\end{aligned}$$

$$\begin{aligned}
&= p_n + p_{n+1} + \cdots + p_{m-1} \\
&\leq \epsilon^n p_0 + \frac{4n\epsilon^n}{1 + a_1 + a_2} + \epsilon^{n+1} p_0 + \frac{4(n+1)\epsilon^{n+1}}{1 + a_1 + a_2} \\
&\quad + \cdots + \epsilon^{m-1} p_0 + \frac{4(m-1)\epsilon^{m-1}}{1 + a_1 + a_2} \\
&= p_0 \sum_{i=0}^{m-n-1} \epsilon^{n+i} + \frac{4}{1 + a_1 + a_2} \sum_{i=0}^{m-n-1} (n+i)\epsilon^{n+i}.
\end{aligned}$$

As $\epsilon < 1$, it follows from the above inequality that

$$\lim_{n,m\to\infty} p(gx_n, gx_m) = \lim_{n,m\to\infty} p(gy_n, gy_m) = 0.$$

So, $\{gx_n\}$ and $\{gy_n\}$ are 0-Cauchy sequences in $g(X)$; therefore, by the closedness of $g(X)$, there exists $w, z \in X$ such that

$$\lim_{n\to\infty} p(gx_n, gw) = \lim_{n,m\to\infty} p(gx_n, gx_m) = p(gw, gw) = 0, \tag{8}$$

$$\lim_{n\to\infty} p(gy_n, gz) = \lim_{n,m\to\infty} p(gy_n, gy_m) = p(gz, gz) = 0. \tag{9}$$

Using (1) we obtain

$$\begin{aligned}
p\bigl(F(w,z), gw\bigr) &\leq p\bigl(F(w,z), gx_{n+1}\bigr) + p(gx_{n+1}, gw) \\
&\leq H_p\bigl(F(w,z), F(x_n, y_n)\bigr) + p(gx_{n+1}, gw) \\
&\leq a_1 p(gw, gx_n) + a_2 p(gz, gy_n) + a_3 p\bigl(F(w,z), gw\bigr) \\
&\quad + a_4 p\bigl(F(w,z), gx_n\bigr) + a_5 p\bigl(F(x_n, y_n), gw\bigr) \\
&\quad + a_6 p\bigl(F(x_n, y_n), gx_n\bigr) + p(gx_{n+1}, gw),
\end{aligned}$$

that is,

$$\begin{aligned}
(1 - a_3 - a_4) p\bigl(F(w,z), gw\bigr) &\leq a_1 p(gw, gx_n) + a_2 p(gz, gy_n) + a_4 p(gw, gx_n) \\
&\quad + a_5 p(gx_{n+1}, gw) + a_6 p(gx_{n+1}, gx_n) + p(gx_{n+1}, gw) \\
&= (a_1 + a_4) p(gw, gx_n) + a_2 p(gz, gy_n) \\
&\quad + (1 + a_5) p(gx_{n+1}, gw) + a_6 p(gx_{n+1}, gx_n).
\end{aligned}$$

Using (8) and (9) and the fact that $1 - a_3 - a_4 > 0$ in the above inequality, we obtain

$$p\bigl(F(w,z), gw\bigr) = p(gw, gw) = 0.$$

Therefore, by Lemma 2, $gw \in F(w,z)$. Similarly, $gz \in F(z,w)$. Thus (w,z) is a coupled coincidence point and $(gw, gz) = (w_c, z_c)$ (say) is a point of coincidence of the mappings F and g with $p(gw, gw) = p(gz, gz) = p(w_c, w_c) = p(z_c, z_c) = 0$. □

The following is a coupled fixed point result for a set-valued mapping and it can be obtained by taking $g = I_X$ (that is an identity mapping of X) in the above theorem.

Corollary 2 *Let (X,p) be a 0-complete partial metric space, let $F: X \times X \to CB^p(X)$ be a mapping satisfying*

$$H_p\big(F(x,y),F(u,v)\big) \leq a_1 p(x,u) + a_2 p(y,v) + a_3 p\big(F(x,y),x\big) + a_4 p\big(F(x,y),u\big) + a_5 p\big(F(u,v),x\big) + a_6 p\big(F(u,v),u\big)$$

for all $x,y,u,v \in X$, where a_i are nonnegative reals such that $\sum_{i=1}^{6} a_i < 1$. Then F has a coupled fixed point $(w,z) \in X \times X$ and $p(w,w) = p(z,z) = 0$.

With suitable values of control constants in Theorem 3, one can obtain the following corollaries.

Corollary 3 *Let (X,p) be a 0-complete partial metric space, let $F: X \times X \to CB^p(X)$ and $g: X \to X$ be mappings satisfying*

$$H_p\big(F(x,y),F(u,v)\big) \leq a_1 p(gx,gu) + a_2 p(gy,gv) \tag{10}$$

for all $x,y,u,v \in X$, where a_1 and a_2 are nonnegative reals such that $a_1 + a_2 < 1$. If $F(X \times X) \subseteq g(X)$ and $g(X)$ is a closed subset of X, then F and g have a coupled point of coincidence $(w_c, z_c) \in X \times X$ and $p(w_c, w_c) = p(z_c, z_c) = 0$.

Corollary 4 *Let (X,p) be a 0-complete partial metric space, let $F: X \times X \to CB^p(X)$ and $g: X \to X$ be mappings satisfying*

$$H_p\big(F(x,y),F(u,v)\big) \leq a_1 p\big(F(x,y),gx\big) + a_2 p\big(F(x,y),gu\big) + a_3 p\big(F(u,v),gx\big) + a_4 p\big(F(u,v),gu\big)$$

for all $x,y,u,v \in X$, where a_i are nonnegative reals such that $\sum_{i=1}^{4} a_i < 1$. If $F(X \times X) \subseteq g(X)$ and $g(X)$ is a closed subset of X, then F and g have a coupled point of coincidence $(w_c, z_c) \in X \times X$ and $p(w_c, w_c) = p(z_c, z_c) = 0$.

The following example illustrates the case when the results in partial metric spaces are applicable while the same results in usual metric spaces are not.

Example 2 Let $X = [0,1] \cap \mathbb{Q}$, and let $p: X \times X \to \mathbb{R}^+$ be defined by

$$p(x,y) = |x-y| + \max\{x,y\} \quad \text{for all } x,y \in X.$$

Then the metric induced by p is given by $d(x,y) = 3|x-y|$ for all $x,y \in X$ and (X,d) is not complete, therefore (X,p) is not complete. Now, it is easy to see that (X,p) is a 0-complete partial metric space and every singleton subset of X is closed with respect to p. Define $F: X \times X \to CB^p(X)$ and $g: X \to X$ by

$$F(x,y) = \begin{cases} \{0\} & \text{if } x = y = 1; \\ \{0, \frac{x+y}{8}\} & \text{otherwise} \end{cases} \quad \text{and} \quad gx = x \quad \text{for all } x \in X.$$

We shall show that F and g satisfy all the conditions of Corollary 3, with $a_1 = a_2 = \alpha \in [\frac{1}{4}, \frac{1}{2})$, while the metric versions of Corollary 3 are not applicable. We consider the following cases.

Case (i) If $x, y, u, v \in X \setminus \{1\}$ and $x + y \neq u + v$, then suppose $u + v < x + y$, so

$$\begin{aligned}
&H_p\bigl(F(x,y), F(u,v)\bigr) \\
&\quad = H_p\left(\left\{0, \frac{x+y}{8}\right\}, \left\{0, \frac{u+v}{8}\right\}\right) \\
&\quad = \max\left\{\sup_{a\in\{0,\frac{x+y}{8}\}} p\left(a, \left\{0, \frac{u+v}{8}\right\}\right), \sup_{a\in\{0,\frac{u+v}{8}\}} p\left(a, \left\{0, \frac{x+y}{8}\right\}\right)\right\} \\
&\quad = \max\left\{p\left(\frac{x+y}{8}, \left\{0, \frac{u+v}{8}\right\}\right), p\left(\frac{u+v}{8}, \left\{0, \frac{x+y}{8}\right\}\right)\right\} \\
&\quad = \max\left\{\min\left\{\frac{x+y}{4}, \frac{1}{8}|x-u+y-v| + \frac{1}{8}\max\{x+y, u+v\}\right\},\right. \\
&\qquad \left.\min\left\{\frac{u+v}{4}, \frac{1}{8}|x-u+y-v| + \frac{1}{8}\max\{x+y, u+v\}\right\}\right\} \\
&\quad = \max\left\{\frac{1}{8}|x-u+y-v| + \frac{x+y}{8}, \min\left\{\frac{u+v}{4}, \frac{1}{8}|x-u+y-v| + \frac{x+y}{8}\right\}\right\} \\
&\quad = \frac{1}{8}|x-u+y-v| + \frac{x+y}{8} \leq \frac{1}{8}\bigl[|x-u| + |y-v|\bigr] + \frac{x+y}{8} \\
&\quad \leq \alpha\bigl[p(gx, gu) + p(gy, gv)\bigr],
\end{aligned}$$

where $\frac{1}{8} \leq \alpha$. Similarly, we obtain the same result for $u + v > x + y$.

Case (ii) If $x, y, u, v \in X \setminus \{1\}$ and $x + y = u + v$, then

$$\begin{aligned}
H_p\bigl(F(x,y), F(u,v)\bigr) &= H_p\left(\left\{0, \frac{x+y}{8}\right\}, \left\{0, \frac{x+y}{8}\right\}\right) \\
&= \sup_{a\in\{0,\frac{x+y}{8}\}} p(a,a) \\
&= \frac{x+y}{8} \leq \frac{\max\{x,u\} + \max\{y,v\}}{8} \\
&\leq \alpha\bigl[p(gx, gu) + p(gy, gv)\bigr],
\end{aligned}$$

where $\frac{1}{8} \leq \alpha$. Similarly, if any one of x, y, u, v is equal to 1, then we obtain the same result.

Case (iii) If any one of (x, y), (u, v) is equal to $(1, 1)$, for example, let $(u, v) = (1, 1)$ and $(x, y) \neq (1, 1)$, then we have

$$\begin{aligned}
H_p\bigl(F(x,y), F(u,v)\bigr) &= H_p\left(\left\{0, \frac{x+y}{8}\right\}, \{0\}\right) \\
&= \max\left\{\sup_{a\in\{0,\frac{x+y}{8}\}} p\bigl(a, \{0\}\bigr), \sup_{a\in\{0\}} p\left(a, \left\{0, \frac{x+y}{8}\right\}\right)\right\} \\
&= \max\left\{\frac{x+y}{4}, 0\right\} = \frac{x+y}{4} \\
&\leq \alpha\bigl[p(gx, gu) + p(gy, gv)\bigr],
\end{aligned}$$

where $\frac{1}{4} \leq \alpha$. Similarly, the condition (10) is satisfied for $a_1 = a_2 = \alpha \in [\frac{1}{4}, \frac{1}{2})$ in all possible cases and $0 = g0 \in F(0,0)$, that is, $(0,0)$ is a coupled coincidence point of F and g (here it is the unique common fixed point of F and g).

Note that, the metric spaces (X,ρ) and (X,d) (where ρ is usual and d is metric induced by p) are not complete, therefore metric versions of Corollary 3 are not applicable. Also, this example shows that F and g do not satisfy the metric versions of inequality (10). Indeed, if H_ρ is the Hausdorff metric induced by the usual metric ρ, then for $x = y = u = 1$, $v = \frac{9}{10}$, we have

$$H_\rho\big(F(x,y),F(u,v)\big) = H_\rho\left(\{0\}, \left\{0, \frac{19}{80}\right\}\right) = \frac{19}{80}$$

and

$$a_1\rho(gx,gu) + a_2\rho(gy,gv) = \frac{1}{10}a_2.$$

Therefore, we cannot find the nonnegative reals a_1, a_2 such that

$$H_\rho\big(F(x,y),F(u,v)\big) \leq a_1\rho(gx,gu) + a_2\rho(gy,gv)$$

for all $x,y,u,v \in X$ with $a_1 + a_2 < 1$. So, F is not a contraction (in view of contraction condition (10)) with respect to the usual metric ρ. Similarly, one can see that F is not a contraction with respect to the induced metric d.

The following theorem provides a sufficient condition for the uniqueness of a coupled point of coincidence and a common fixed point of the hybrid pair $\{F,g\}$.

Theorem 4 *Let (X,p) be a 0-complete partial metric space, let $F : X \times X \to CB^p(X)$ and $g : X \to X$ be mappings such that all the conditions of Theorem 3 are satisfied and, for any coupled coincidence point (w,z) of F and g, we have $F(w,z) = \{gw\}$ and $F(z,w) = \{gz\}$, then F and g have a unique coupled point of coincidence. Suppose in addition that the hybrid pair $\{F,g\}$ is weakly compatible, then F and g have a unique coupled common fixed point.*

Proof The existence of a coupled coincidence point (w,z) and a point of coincidence (w_c,z_c) follows from Theorem 3. Suppose that, for any coupled coincidence point (w,z) of F and g, we have $F(w,z) = \{gw\} = \{w_c\}$ and $F(z,w) = \{gz\} = \{z_c\}$. We shall show that the coupled point of coincidence is unique. Let (w',z') be another coupled coincidence point and (w'_c,z'_c) be the coupled point of coincidence of F and g, that is, $w'_c = gw' \in F(w',z')$, $z'_c = gz' \in F(z',w')$ and $F(w',z') = \{gw'\} = \{w'_c\}$, $F(z',w') = \{gz'\} = \{z'_c\}$.

Using (1), we obtain

$$\begin{aligned} p\big(gw,gw'\big) &= H_p\big(\{gw\},\{gw'\}\big) \\ &= H_p\big(F(w,z),F\big(w',z'\big)\big) \\ &\leq a_1p\big(gw,gw'\big) + a_2p\big(gz,gz'\big) + a_3p\big(F(w,z),gw\big) + a_4p\big(F(w,z),gw'\big) \\ &\quad + a_5p\big(F\big(w',z'\big),gw\big) + a_6p\big(F\big(w',z'\big),gw'\big) \end{aligned}$$

$$\begin{aligned} &= a_1p(gw,gw') + a_2p(gz,gz') + a_3p(gw,gw) + a_4p(gw,gw') \\ &\quad + a_5p(gw',gw) + a_6p(gw',gw'). \end{aligned} \tag{11}$$

Again, using (1) we obtain

$$\begin{aligned} p(gz,gz') &= H_p(\{gz\},\{gz'\}) \\ &= H_p(F(z,w),F(z',w')) \\ &\leq a_1p(gz,gz') + a_2p(gw,gw') + a_3p(F(z,w),gz) + a_4p(F(z,w),gz') \\ &\quad + a_5p(F(z',w'),gz) + a_6p(F(z',w'),gz') \\ &= a_1p(gz,gz') + a_2p(gw,gw') + a_3p(gz,gz) + a_4p(gz,gz') \\ &\quad + a_5p(gz',gz) + a_6p(gz',gz'). \end{aligned} \tag{12}$$

It follows from (11) and (12) that

$$\begin{aligned} p(gw,gw') + p(gz,gz') &\leq (a_1+a_2+a_4+a_5)p(gw,gw') + a_3p(gw,gw) \\ &\quad + a_6p(gw',gw') + (a_1+a_2+a_4+a_5)p(gz,gz') \\ &\quad + a_3p(gz,gz) + a_6p(gz',gz') \\ &= (a_1+a_2+a_3+a_4+a_5+a_6)\big[p(gw,gw') + p(gz,gz')\big]. \end{aligned}$$

As $\sum_{i=1}^{6} a_i < 1$, it follows from the above inequality that $p(gw,gw') + p(gz,gz') = 0$, that is, $p(gw,gw') = p(gz,gz') = 0$, so $w_c = gw = gw' = w'_c$ and $z_c = gz = gz' = z'_c$. Therefore, a coupled point of coincidence, that is, (w_c,z_c), of F and g is unique.

Suppose that F and g are weakly compatible, then we have

$$g\{w_c\} = gF(w,z) \subseteq F(gw,gz) \quad \text{that is} \quad \{gw_c\} \subseteq F(w_c,z_c) \quad \text{and}$$
$$g\{z_c\} = gF(z,w) \subseteq F(gz,gw) \quad \text{that is} \quad \{gz_c\} \subseteq F(z_c,w_c).$$

Therefore, (gw_c,gz_c) is another coupled point of coincidence of F and g, and by uniqueness we have $w_c = gw_c \in F(w_c,z_c)$ and $z_c = gz_c \in F(z_c,w_c)$. Thus (z_c,w_c) is the unique coupled common fixed point of F and g. □

The following theorem is a new result for a hybrid pair of mappings in partial metric as well as in metric spaces.

Theorem 5 *Let (X,p) be a 0-complete partial metric space, let $F : X \times X \to CB^p(X)$ and $g : X \to X$ be mappings satisfying*

$$\begin{aligned} H_p(F(x,y),F(u,v)) &\leq a_1p(F(y,x),gy) + a_2p(F(y,x),gv) + a_3p(F(v,u),gy) \\ &\quad + a_4p(F(v,u),gv) \end{aligned} \tag{13}$$

for all $x,y,u,v \in X$, where a_i are nonnegative reals such that $\sum_{i=1}^{4} a_i < 1$. If $F(X \times X) \subseteq g(X)$ and $g(X)$ is a closed subset of X, then F and g have a coupled point of coincidence $(w_c,z_c) \in X \times X$ and $p(gw_c,gw_c) = p(gz_c,gz_c) = 0$.

Proof By a similar process as used in Theorem 3, we can find two sequences $\{x_n\}$ and $\{y_n\}$ such that

$$\begin{aligned} &gx_{n+1} \in F(x_n, y_n) \quad \text{and} \quad gy_{n+1} \in F(y_n, x_n), \\ &p(gx_n, gx_{n+1}) \leq H_p\bigl(F(x_{n-1}, y_{n-1}), F(x_n, y_n)\bigr) + \epsilon^n, \\ &p(gy_n, gy_{n+1}) \leq H_p\bigl(F(y_{n-1}, x_{n-1}), F(y_n, x_n)\bigr) + \epsilon^n, \end{aligned}$$

where $\epsilon > 0$ is arbitrary.

From the above inequality and (13), we obtain

$$\begin{aligned} p(gx_n, gx_{n+1}) &\leq H_p\bigl(F(x_{n-1}, y_{n-1}), F(x_n, y_n)\bigr) + \epsilon^n \\ &\leq a_1 p\bigl(F(y_{n-1}, x_{n-1}), gy_{n-1}\bigr) + a_2 p\bigl(F(y_{n-1}, x_{n-1}), gy_n\bigr) \\ &\quad + a_3 p\bigl(F(y_n, x_n), gy_{n-1}\bigr) + a_4 p\bigl(F(y_n, x_n), gy_n\bigr) + \epsilon^n \\ &\leq a_1 p(gy_n, gy_{n-1}) + a_2 p(gy_n, gy_n) + a_3 p(gy_{n+1}, gy_{n-1}) \\ &\quad + a_4 p(gy_{n+1}, gy_n) + \epsilon^n, \end{aligned}$$

that is,

$$\begin{aligned} p(gx_n, gx_{n+1}) &\leq (a_1 + a_3) p(gy_n, gy_{n-1}) + (a_3 + a_4) p(gy_{n+1}, gy_n) + \epsilon^n \\ &\quad + (a_2 - a_3) a_2 p(gy_n, gy_n). \end{aligned} \tag{14}$$

Interchanging the roles of x_n and x_{n+1} and using the symmetries of p and H_p, we obtain

$$\begin{aligned} p(gx_n, gx_{n+1}) &\leq (a_4 + a_2) p(gy_n, gy_{n-1}) + (a_2 + a_1) p(gy_{n+1}, gy_n) \\ &\quad + (a_3 - a_2) a_2 p(gy_n, gy_n) + \epsilon^n. \end{aligned} \tag{15}$$

It follows from (14) and (15) that

$$\begin{aligned} 2p(gx_n, gx_{n+1}) &\leq (a_1 + a_2 + a_3 + a_4)\bigl[p(gy_n, gy_{n-1}) \\ &\quad + p(gy_{n+1}, gy_n)\bigr] + 2\epsilon^n. \end{aligned} \tag{16}$$

Similarly, it can be shown that

$$\begin{aligned} 2p(gy_n, gy_{n+1}) &\leq (a_1 + a_2 + a_3 + a_4)\bigl[p(gx_n, gx_{n-1}) \\ &\quad + p(gx_{n+1}, gx_n)\bigr] + 2\epsilon^n. \end{aligned} \tag{17}$$

For simplicity, set $p_n = p(gx_n, gx_{n+1}) + p(gy_n, gy_{n+1})$, then it follows from (16) and (17) that

$$p_n \leq \frac{a_1 + a_2 + a_3 + a_4}{2 - a_1 - a_2 - a_3 - a_4} p_{n-1} + \frac{4\epsilon^n}{2 - a_1 - a_2 - a_3 - a_4}.$$

As $\epsilon > 0$ was arbitrary, choose $\epsilon = \frac{a_1+a_2+a_3+a_4}{2-a_1-a_2-a_3-a_4}$; also, as $\sum_{i=1}^4 a_i < 1$, we have $\epsilon < 1$. Therefore

$$p_n \leq \epsilon p_{n-1} + \frac{4\epsilon^n}{2 - a_1 - a_2 - a_3 - a_4} \leq \epsilon p_{n-1} + 4\epsilon^n.$$

It follows from a successive application of the above inequality that

$$\begin{aligned} p_n &\le \epsilon p_{n-1} + 4\epsilon^n, \\ p_n &\le \epsilon\left[\epsilon p_{n-2} + 4\epsilon^{n-1}\right] + 4\epsilon^n, \\ p_n &\le \epsilon^2 p_{n-2} + 8\epsilon^n, \\ &\vdots \\ p_n &\le \epsilon^n p_0 + 4n\epsilon^n. \end{aligned} \tag{18}$$

For $m, n \in \mathbb{N}$ with $m > n$, using (18) we obtain

$$\begin{aligned} p(gx_n, gx_m) + p(gy_n, gy_m) &\le p(gx_n, gx_{n+1}) + p(gy_n, gy_{n+1}) + p(gx_{n+1}, gx_{n+2}) \\ &\quad + p(gy_{n+1}, gy_{n+2}) + \cdots + p(gx_{m-1}, gx_m) \\ &\quad + p(gy_{m-1}, gy_m) \\ &= p_n + p_{n+1} + \cdots + p_{m-1} \\ &\le \epsilon^n p_0 + 4n\epsilon^n + \epsilon^{n+1} p_0 + 4(n+1)\epsilon^{n+1} + \cdots + \epsilon^{m-1} p_0 \\ &\quad + 4(m-1)\epsilon^{m-1} \\ &= p_0 \sum_{i=0}^{m-n-1} \epsilon^{n+i} + 4 \sum_{i=0}^{m-n-1} (n+i)\epsilon^{n+i}. \end{aligned}$$

As $\epsilon < 1$, it follows from the above inequality that

$$\lim_{n,m\to\infty} p(gx_n, gx_m) = \lim_{n,m\to\infty} p(gy_n, gy_m) = 0.$$

So, $\{gx_n\}$ and $\{gy_n\}$ are 0-Cauchy sequences in $g(X)$, therefore by the closedness of $g(X)$, there exists $w, z \in X$ such that

$$\lim_{n\to\infty} p(gx_n, gw) = \lim_{n,m\to\infty} p(gx_n, gx_m) = p(gw, gw) = 0, \tag{19}$$

$$\lim_{n\to\infty} p(gy_n, gz) = \lim_{n,m\to\infty} p(gy_n, gy_m) = p(gz, gz) = 0. \tag{20}$$

We shall show that $p(F(w,z), gw) = p(gw, gw) = 0$ and $p(F(z,w), gz) = p(gz, gz) = 0$.

For all $n \in \mathbb{N}$, we have

$$\begin{aligned} p\bigl(F(w,z), gw\bigr) &\le p\bigl(F(w,z), gx_{n+1}\bigr) + p(gx_{n+1}, gw) \\ &\le H_p\bigl(F(w,z), F(x_n, y_n)\bigr) + p(gx_{n+1}, gw) \\ &\le a_1 p\bigl(F(z,w), gz\bigr) + a_2 p\bigl(F(z,w), gy_n\bigr) + a_3 p\bigl(F(y_n, x_n), gz\bigr) \\ &\quad + a_4 p\bigl(F(y_n, x_n), gy_n\bigr) + p(gx_{n+1}, gw) \\ &\le (a_1 + a_2) p\bigl(F(z,w), gz\bigr) + a_2 p(gz, gy_n) + a_3 p(gy_{n+1}, gz) \\ &\quad + a_4 p(gy_{n+1}, gy_n) + p(gx_{n+1}, gw). \end{aligned}$$

Using (19) and (20) in the above inequality, we obtain

$$p\bigl(F(w,z),gw\bigr) \le (a_1 + a_2)p\bigl(F(z,w),gz\bigr) < p\bigl(F(z,w),gz\bigr). \tag{21}$$

Again, for all $n \in \mathbb{N}$, we have

$$\begin{aligned} p\bigl(F(z,w),gz\bigr) &\le p\bigl(F(z,w),gy_{n+1}\bigr) + p(gy_{n+1},gz) \\ &\le H_p\bigl(F(z,w),F(y_n,x_n)\bigr) + p(gy_{n+1},gz) \\ &\le a_1p\bigl(F(w,z),gw\bigr) + a_2p\bigl(F(w,z),gx_n\bigr) + a_3p\bigl(F(x_n,y_n),gw\bigr) \\ &\quad + a_4p\bigl(F(x_n,y_n),gx_n\bigr) + p(gy_{n+1},gz) \\ &\le (a_1 + a_2)p\bigl(F(w,z),gw\bigr) + a_2p(gw,gx_n) + a_3p(gx_{n+1},gw) \\ &\quad + a_4p(gx_{n+1},gx_n) + p(gy_{n+1},gz). \end{aligned}$$

Using (20) and (19) in the above inequality, we obtain

$$p\bigl(F(z,w),gz\bigr) \le (a_1 + a_2)p\bigl(F(w,z),gw\bigr) < p\bigl(F(w,z),gw\bigr). \tag{22}$$

Note that if $p(F(w,z),gw) \neq p(gw,gw) = 0$ or $p(F(z,w),gz) \neq p(gz,gz) = 0$, then (21) and (22) give a contradiction. Therefore, we have $p(F(w,z),gw) = p(gw,gw) = 0$ and $p(F(z,w),gz) = p(gz,gz) = 0$, and by Lemma 2, $gw \in F(w,z)$ and $gz \in F(z,w)$. Thus (w,z) is a coupled coincidence point and $(gw,gz) = (w_c,z_c)$ (say) is a point of coincidence of the mappings F and g with $p(gw,gw) = p(gz,gz) = p(w_c,w_c) = p(z_c,z_c) = 0$. □

The following is a coupled fixed point result for a set-valued mapping and can be obtained by taking $g = I_X$ (that is an identity mapping of X) in the above theorem.

Corollary 5 *Let (X,p) be a 0-complete partial metric space, let $F : X \to CB^p(X)$ be a mapping satisfying*

$$\begin{aligned} H_p\bigl(F(x,y),F(u,v)\bigr) &\le a_1p\bigl(F(y,x),y\bigr) + a_2p\bigl(F(y,x),v\bigr) + a_3p\bigl(F(v,u),y\bigr) \\ &\quad + a_4p\bigl(F(v,u),v\bigr) \end{aligned}$$

for all $x,y,u,v \in X$, where a_i are nonnegative reals such that $\sum_{i=1}^4 a_i < 1$. Then F has a coupled fixed point $(w,z) \in X \times X$ and $p(w,w) = p(z,z) = 0$.

Theorem 6 *Let (X,p) be a 0-complete partial metric space, let $F : X \times X \to CB^p(X)$ and $g : X \to X$ be mappings such that all the conditions of Theorem 5 are satisfied, and for any coupled coincidence point (w,z) of F and g, we have $F(w,z) = \{gw\}$ and $F(z,w) = \{gz\}$. Then F and g have a unique coupled point of coincidence. Suppose in addition that the hybrid pair $\{F,g\}$ is weakly compatible, then F and g have a unique coupled common fixed point.*

Proof The proof of this theorem is followed by a similar process as used in Theorem 4. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

Author details
[1]College of Mathematics and Information Science, Jiangxi Normal University, Nanchang, Jiangxi 330022, People's Republic of China. [2]Department of Applied Mathematics, Shri Vaishnav Institute of Technology and Science, Gram Baroli, Sanwer Road, Indore, MP 453331, India. [3]Faculty of Mechanical Engineering, University of Belgrade, Kraljice Marije 16, Beograd, 11120, Serbia.

Acknowledgements
Wei Long acknowledges support from the NSF of China, and the Research Project of Jiangxi Normal University (2012-114).

Received: 3 January 2013 Accepted: 15 May 2013 Published: 4 June 2013

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doi:10.1186/1687-1812-2013-145
Cite this article as: Long et al.: Some coupled coincidence and common fixed point results for a hybrid pair of mappings in 0-complete partial metric spaces. *Fixed Point Theory and Applications* 2013 **2013**:145.