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***Abstract**. Several methods for a calculation of derivatives of eigenvectors with respect to design parameters are described here. These are the finite-difference method, the modal method, a modified modal method, Nelson's method, an improved first-order approximation of eigenvalues and eigenvectors and an iterative method. By combining the other structural reanalysis techniques and one of these sensitivity methods, it is possible to enhance the efficiency and the accuracy of structural optimization techniques for determining the optimum condition of mechanical structure specified by an analyst. The sensitivity approach is based on the prior selection of updating parameters (design variables) in the initial FE model.*

Key words: *eigenvalue and eigenvector sensitivity*

1. INTRODUCTION

A good finite element (FE) or analytical model of a mechanical structure is important for structural integrity analysis. In practice, a high degree of confidence can be placed on such a FE/analytical model when the dynamic response of that model closely resembles experimental data. However, updating the FE model or inetifying the analytical model directly is usually not the main objective of structural vibration analysis because there are many situations when the dynamic response of the mechanical structure does not satisfy the requirement set by the structural analyst (designer). In such situations, the dynamic response of the mechanical structure has to be altered either (i) by controlling the forcing inputs to the structure, or (ii) by changing the dynamic characteristics of the structure. The forcing inputs often results from interaction with the structure's environment and so cannot easily be controlled at will. When this is the case, it is important to be able to alter the structural response by redesigning the dynamic characteristics of the structure. The use of structural reanalysis techniques to obtain the optimum condition of an FE model of a mechanical structure has grown considerably in recent years. The optimal design of structures with frequency constraints is extremely useful in manipulating the dynamic characteristics in a variety of ways. For example, in most low-frequency vibration problems, the response of the structure to dynamic excitation is primarily a function of its fundamental frequency and mode shape. In such cases, the ability to manipulate the selected frequency can significantly improve the performanse of the structure. Similarly, the aeroelastic characteristics of an aircraft wing the governed primarily by its torsional and bending properties, which can best be studied by the lower torsional and bending modes. A number of techniques exist that can be applied to the dynamic reanalysis of mechanical structures. One of the most popular of these is sensitivity analysis which has been developed and applied by several workers to the general eigenvalue problem [1-7] and, more specifically, to applications of structural dynamic modification analysis in references [8-9]. Some of the areas where sensitivity analysis has been applied include (i) system identification, (ii) development of insensitive control systems, (iii) use in gradient-based mathematical programming methods, (iv) approximation of system response to a change in a system parameter, and (v) assessment of design changes on system performance [19]. In this area, both first- and higher-order eigenvalue and eigenvector sensitivities have been investigated with a view to predicting the response of a modified structure from knowledge of its spatial and modal properties in the original, or unmodified, state. The sensitivity analysis of a mechanical structure is based on a Taylor expansion of eigenvalues and eigenvectors of the unmodified structure. Traditionally, a truncated Taylor or matrix power series evaluated at a nominal design point is used to approximate the eigen parameters of modified structures [21,22]. Earlier studies [20] indicated that the computation of the higher-order terms of this series is difficult and time consuming, the effectiveness of this method is limited to small modifications. Even the use of higher-order terms in the local approximation series cannot guarantee convergence for moderate to large perturbations in the structural parameters. The implication of this observation in the context of structural optimization is that severe move limits have to be imposed in line searches to ensure convergence to a feasible design. Very few studies in the literature have addressed the structural dynamic reanalysis problem for moderate to large modifications in the structural parameters. The approach currently in use can be broadly classified into direct and iterative approaches. The objective of most direct approaches is to increase the range of validity of local approximation techniques. Inamura [25] proposed an approximation procedure in which the eigenpair perturbation equations are interpreted as differential equations in terms of the perturbation parameters. A procedure using the eigensensitivity equations was developed by Pritchard and Adelman [26] based on a similar line of approach. The sensitivity method [24] is a prime representative of the updating approach which allows selection of updating parameters but does not require full experimental mode shapes and as such this method seems to be suitable for updating of large models. Also, it is worth noting that model updating methods based on control methods, such as eigenstructure assignment method proposed by Minas and Inman [22,23] are quite promising sinse they can be defined in such a way that they do not require full experimental mode shape matrix. The general perturbation procedure followed in major papers is diagrammatically shown in Fig. 1.

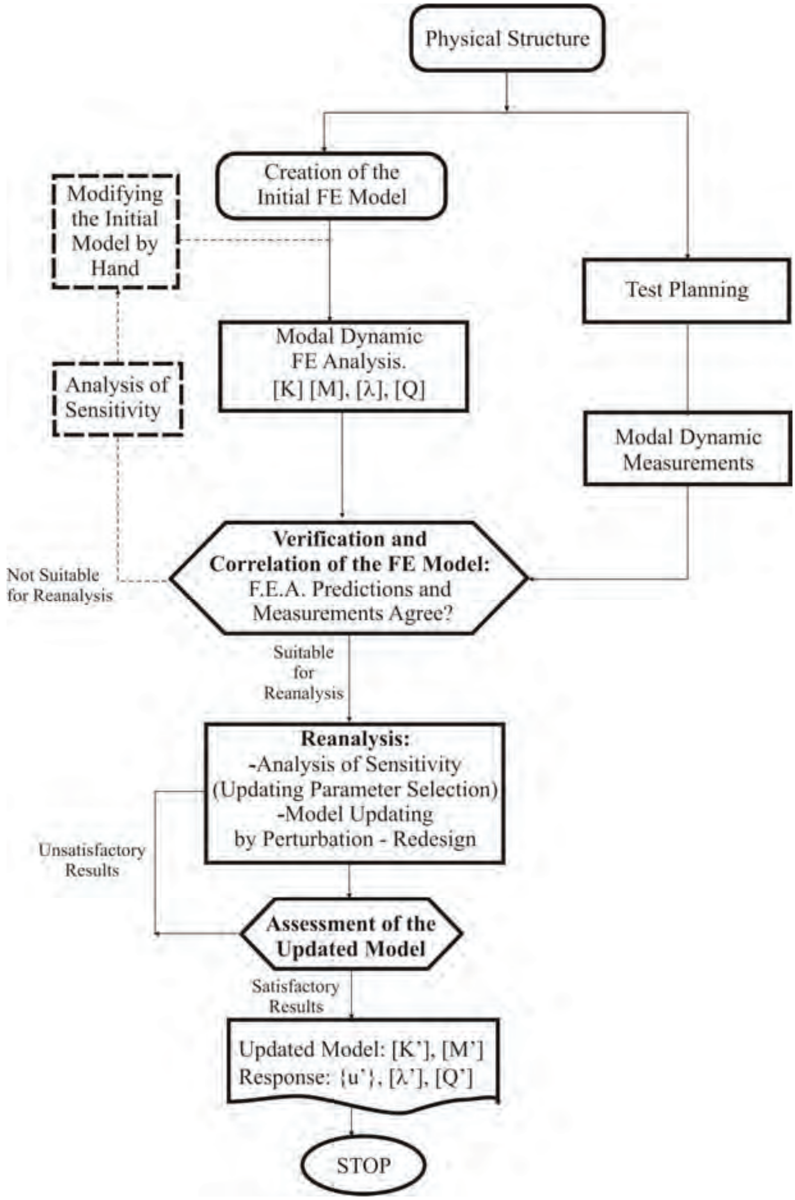

Fig. 1 Flowchart of General Perturbation

2. MODAL SENSITIVITY ANALYSIS. DESIGN SENSITIVITIES. THEORETICAL BACKGROUND. SURVEY

It is becoming widely accepted that sensitivity analysis can be a valuable tool in structural reanalysis where (enough of) the modal properties are known, either through theoretical or experimental analysis. Modal sensitivities are the derivatives of the modal properties of a dynamic system with respect to chosen structural variables. In the modal analysis literature there have been two primary applications. In the first case sensitivity data are used solely as a qualitative indicator of the location and approximate scale of design changes to achieve a desired change in structural properties. The consequences of candidate design changes would then be evaluated using exact methods. The second strategy uses the design sensitivities directly to predict the effect of proposed structural changes. The use of sensitivities in this fashion relies on the Matrix Taylor Series expansion, with the usual implications of convergence and truncation errors. Use only of first order design sensitivities assumes implicitly that the second (and higher) order derivatives are negligible. The use of these second order sensitivities as suitable criteria for the acceptability of first order sensitivities for predictive analysis can be interested in some detail. Sensitivity analysis may be applied to candidate design modifications distributed across a number of degrees of freedom of the structure but is limited in scale.

Modal design sensitivities are the derivatives of the eigensystem of a dynamic system with respect to those variables which are available for modification by the designer. A typical modification would be the change in diameter of a circular section. This would affect both the mass of the section, proportional to the square of the diameter, and its stiffness, which depends on the second moment of area of the section. A change in length would have a mass effect directly proportional to length, but a stiffness change depending on the cube of length. Changing material would similarly affect mass, stiffness and damping. Shape sensitivity analysis of physical systems under dynamic loads may be important from different points of view (i) to understand and model the system's behavior better with respect to shape, (ii) to optimize the physical shapes of the desired systems responses in a prescribed time interval, or (iii) to identify shapes by utilizing the system's measured response in time.

2.1. Problem Statement. Derivation

The matrix form of the equation of undamped motion of an FE model is:

$$\left[M\right]\cdot\left\{\ddot{x}(t)\right\}+\left[K\right]\cdot\left\{x(t)\right\}=\left\{0\right\} \tag{1}$$

The free-vibration natural frequencies and mode shapes of a linear structural system can be computed by solving the above eigenvalue problem

$$[K]\{Q_i\}=\lambda_i[M]\{Q_i\} \tag{2}$$

where $[K],[M]$ are the structural stiffness and mass matrix, respectively. The system matrices are considered to be a general function of the design variables denoted by $\{V\}=\{v_1,v_2,...,v_j,...,v_p\}$, and λ_i and $\{Q_i\}$ are the eigenvalue and the eigenvector of mode i, respectively. Consider the case wherein the design variables are perturbed by $\left\{\Delta V\right\}$. Let $[\Delta K]$ and $[\Delta M]$ be the corresponding perturbation in the stiffness and mass matrices. The perturbed eigenvalue problem can be written as

$$\left([K]+[\Delta K]\right)\left(\{Q_i\}+\{\Delta Q_i\}\right)=\left(\lambda_i+\Delta\lambda_i\right)\left([M]+[\Delta M]\right)\left(\{Q_i\}+\{\Delta Q_i\}\right) \tag{3}$$

where $\Delta\lambda_i$ and $\{\Delta Q\}_i$ are the eigenvalue and eigenvector perturbations, respectively. Equation (2) can be written in a compact form as

$$[K']\{Q_i^{'}\} = \lambda_i^{'}[M']\{Q_i^{'}\} \tag{4}$$

Often it is found that, even for small to moderate perturbations in the stiffness and mass matrices, significant alterations in the modal characteristics of the structure may occur. Hence, an exact reanalysis becomes necessary to compute the perturbed eigenparameters with sufficient accuracy. The objective of approximate reanalysis procedures is the computation of the perturbed eigenparameters using the results of exact analysis for the baseline system without recourse to solving Eq. (3) in its exact form. Typically, the perturbations in the eigenparameters are calculated using first-order sensitivity information as

$$\Delta\lambda_i = \sum_{j=1}^{p}\left(\frac{\partial\lambda_i}{\partial v_j}\right)\Delta v_i \quad \text{and} \quad \{\Delta Q_i\} = \sum_{j=1}^{p}\left(\frac{\partial\{Q_i\}}{\partial v_i}\right)\Delta v_i \tag{5}$$

where $\partial\lambda_i / \partial v_j$ and $\partial\{Q_i\}/\partial v_j$ are the sensitivities of the eigenvalues and eigenvectors with respect to the structural parameters, respectively. The eigenvalue and eigenvector derivatives can be calculated by performing partial differentiation of the equation (2) to an updating structural parameter v_j:

$$([K]-\lambda_i[M])\frac{\partial\{Q_i\}}{\partial v_j} = \left(\lambda_i\frac{\partial[M]}{\partial v_j} + \frac{\partial\lambda_i}{\partial v_j}[M] - \frac{\partial[K]}{\partial v_j}\right)\{Q_i\}. \tag{6}$$

This is an equation for the eigenvector sensitivity. It can seen from Eq. (5) that the computation of the eigenvalue sensitivities involves a simple and straightforward calculation. Left-multiplying with the transpose of the eigenvector gives

$$-\frac{\partial\lambda_i}{\partial v_j}\cdot\{Q_i\}^T\cdot[M]\cdot\{Q_i\} + \{Q_i\}^T\cdot\left(\frac{\partial[K]}{\partial v_j} - \lambda_i\cdot\frac{\partial[M]}{\partial v_j}\right)\cdot\{Q_i\} +$$
$$+\left(\{Q_i\}^T\cdot[K] - \lambda_i\cdot\{Q_i\}^T\cdot[M]\right)\cdot\frac{\partial\{Q_i\}}{\partial v_j} = \{0\}$$

Because $\{Q_i\}^T\cdot[M]\cdot\{Q_i\} = 1$ and $\{Q_i\}^T\cdot[K] - \lambda_i\cdot\{Q_i\}^T\cdot[M] = 0$,

$$\frac{\partial\lambda_i}{\partial v_i} = \{Q_i\}^T\left(\frac{\partial[K]}{\partial v_j} - \lambda_i\frac{\partial[M]}{\partial v_j}\right)\{Q_i\} \tag{7}$$

This is the formula for the eigenvalue sensitivity of the i^{th} mode to the j^{th} design parameter. From this formula, it can be seen that the sensitivity of an eigenvalue to an design parameter can be calculated from the eigenvalue, the corresponding eigenvector, and the sensitivities of the stiffness and mass matrices to the design parameter (variable).

Equations (2-7) have been derived under the assumption that the baseline eigenvectors have been mass normalized.

2.2. Description of the Sensitivity Methods

There mainly exist three categories in the literature: the modal method, the direct method, and the iterative method. Several methods for calculating eigenvector derivatives, $\partial\{Q_i\}/\partial v_j$, are described. Every method, exept the finite-difference method, requires the mass matrix and stiffness matrix derivatives, $\partial[M]/\partial v_j$ and $\partial[K]/\partial v_j$, respectively.

2.2.1 Finite-Difference Method

The most straightforward approach for calculating the derivatives is the finite-difference method. In the finite-difference method, Eq. (2) is solved for $\{Q_i\}=\{Q_i\}_{old}$, the j[th] design variable is perturbed by Δv_j , and a new eigenvector $\{Q_i^{'}\}=\{Q_i\}_{new}$ is obtained by solving Eq. (2) again, where $v_{j,new}=v_{j,old}+\Delta v_j$. The derivative is approximated by the expression

$$\frac{\partial\{Q_i\}}{\partial v_j}=\frac{\{Q_i\}_{new}-\{Q_i\}_{old}}{\Delta v_j} \tag{8}$$

To reduce numerical errors associated with Eq. (8), attention should be paid to the step size Δv_j . An algotithm for determining the optimum step size has been developed to further reduce numerical errors and is described in Ref. [28].

2.2.2 Modal Method

The modal method expresses the derivative of an eigenvector as a series expansion of the system eigenvectors. Because this method is based on the series expansion of the eigenvalues and eigenvectors of the modified (perturbed) system, the efficiency of this method is limited. The approximate derivative is expressed as [34]:

$$\frac{\partial\{Q_i\}}{\partial v_j}=\sum_{k=1}^{N}A_{ijk}\{Q_k\}, \tag{9}$$

where the coefficients A_{ijk} are calculated using

$$A_{ijk}=\frac{\{Q_k\}^T\left(\frac{\partial[K]}{\partial v_j}-\lambda_i\frac{\partial[M]}{\partial v_j}\right)\{Q_i\}}{\lambda_i-\lambda_k}, \quad k\neq i\,. \tag{10}$$

Considering the orthogonality property of the eigenvector, $\{Q_i\}$, $\{Q_i\}^T[M]\{Q_i\}=1$, and partial-differentiating this equation with respect to the updating parameter, v_j, for $k \neq i$, it can be obtained that:

$$2\{Q_i\}^T[M]\frac{\partial\{Q_i\}}{\partial v_j}+\{Q_i\}^T\frac{\partial[M]}{\partial v_j}\{Q_i\}=0 \tag{11}$$

The expression for $\partial\{Q_i\}/\partial v_j$ from Eq. (9) is substituted into Eq. (11), and using the orthogonality condition $\{Q_i\}^T[M]\{Q_i\}=1$, the coefficients A_{ijk} are obtained:

$$A_{ijk}=-\frac{1}{2}\{Q_i\}^T\frac{\partial[M]}{\partial v_j}\{Q_i\}, \quad k=i. \tag{12}$$

2.2.3 Modified Modal Method

The modified modal method uses a pseudostatic solution of Eq (6) as an initial approximation to the mode shape derivative. This is similar in principle to the mode-acceleration method used in transient structural analysis [29]. Equation (6) is solved by neglecting the quantity $\lambda_i[M]\left(\partial\{Q_i\}/\partial v_j\right)$ and obtaining the pseudostatic solution for $\left(\partial\{Q_i\}/\partial v_j\right)_s$, which is

$$\left(\frac{\partial\{Q_i\}}{\partial v_j}\right)_s=[K]^{-1}\cdot\left(\lambda_i\frac{\partial[M]}{\partial v_j}+\frac{\partial\lambda_i}{\partial v_j}[M]-\frac{\partial[K]}{\partial v_j}\right)\cdot\{Q_i\}. \tag{13}$$

This pseudostatic solution is added to Eq. (9) to obtain

$$\frac{\partial\{Q_i\}}{\partial v_j}=\left(\frac{\partial\{Q_i\}}{\partial v_j}\right)_s+\sum_{k=1}^{N}\overline{A}_{ijk}\{Q_k\}, \tag{14}$$

where $\overline{A}_{ijk}$ are coefficients for the modified modal method. To obtain the coefficients $\overline{A}_{ijk}$, Eq. (14) is substituted into Eq. (6), and the result is premultiplied by $\{Q_k\}^T$. When simplified, this result becomes

$$\overline{A}_{ijk}=\frac{\lambda_i\{Q_k\}^T\left(\frac{\partial[K]}{\partial v_j}-\lambda_i\frac{\partial[M]}{\partial v_j}\right)\{Q_i\}}{\lambda_k\cdot(\lambda_i-\lambda_k)}, \quad k\neq i, \tag{15}$$

$$\overline{A}_{ijk}=-\frac{1}{2}\{Q_i\}^T\frac{\partial[M]}{\partial v_j}\{Q_i\}, \quad k=i. \tag{16}$$

The relative convergence of the modified modal method vs the modal method for a given number of eigenvectors can be anticipated by dividing Eq. (15) by Eq. (10):

$$\frac{\overline{A}_{ijk}}{A_{ijk}} = \frac{\lambda_i}{\lambda_k} \tag{17}$$

Assuming that to calculate $\partial\{Q_i\}/\partial v_j$ accurately i modes or more are needed; then for k>i, $\overline{A}_{ijk}$ is smaller than A_{ijk}, and Eq. (14) will converge faster than Eq. (9).

2.2.4 Nelson's Method

Nelson's method (the direct method) obtains an exact solution to Eq. (6). This method expresses the eigenvector derivative in terms of a particular solution $\{\xi_{ij}\}$ and a complementary solution $\{Q_i\}\cdot c_{ij}$ where c_{ij} is an undetermined coefficient. In this case, any solution for equation (6) can be written in the form of [27]:

$$\frac{\partial\{Q_i\}}{\partial v_j} = \{\xi_{ij}\} + \{Q_i\}\cdot c_{ij}\,, \tag{18}$$

The particular solution is found by identifying the component of the eigenvector $\{Q_i\}$ with the largest absolute value and constraining the derivative of that component to zero. Combining equations (18) and (11), it is shown that

$$2\{Q_i\}^T[M]\left(\{\xi_{ij}\} + \{Q_i\}\cdot c_{ij}\right) + \{Q_i\}^T\frac{\partial[M]}{\partial v_j}\{Q_i\} = 0\ . \tag{19}$$

The coefficient c_{ij} can be obtained by the following formula:

$$c_{ij} = -\left(\{Q_i\}^T[M]\{\xi_{ij}\} + \frac{1}{2}\{Q_i\}^T\frac{\partial[M]}{\partial v_j}\{Q_i\}\right) \tag{20}$$

2.2.5 Improved First-Order Approximation of Eigenvalues and Eigenvectors

A method based on reduced basis approximation concepts is presented for improved first-order approximation of eigenvalues and eigenvectors of modified structural dynamic systems [33]. The approximation procedure involves the use of the baseline eigenvector and the first-order approximation term as basic vector for Ritz analysis of the perturbed eigenvalue problem. An assumption is made that the eigenvector of the perturbed system can be approximated in the subspace spanned by $\{Q_i\}$ and $\{\Delta Q_i\}$, which is computed using Eqs. (5-7), i.e., an approximation for the perturbed eigenvector can be written as

$$\{\hat{Q}_i\} = \zeta_1\{Q_i\} + \zeta_1\{\Delta Q_i\} \tag{21}$$

where ζ_1 and ζ_2 are undetermined scalar quantities in the approximate representation of the perturbed eigenvector. The assumption implicit in this proposition is that, even for moderate to large perturbations in the structural parameters, the first-order approximation yields a $\{\Delta Q_i\}$ vector, which usually gives a reasonable indication of the likely change of a baseline eigenvector, although the magnitude or even direction of change may be erroneous. Eq. (21) can be expressed in matrix form as

$$\{\hat{Q}_i\} = [T]\{Z\} \tag{22}$$

where $[T]=[Q_i, \Delta Q_i] \in \Re^{n\times 2}$ and $\{Z\}^T = \{\zeta_1, \zeta_1\} \in \Re^{1\times 2}$.
Substituting equation (22) in to equation (4) and premultiplying by $[T]^T$, the resulting set of equations can be expressed as

$$[K_T]\{Z\} = \lambda [M_T]\{Z\} \tag{23}$$

where

$$[K_T] = [T]^T [K'][T] \in \Re^{2\times 2} \tag{24}$$

and

$$[M_T] = [T]^T [M'][T] \in \Re^{2\times 2}. \tag{25}$$

After matematical transformation, the mass normalized perturbed eigenvector can be written as [33]:

$$\{\hat{Q}_i\} = \frac{1}{\{Z\}^T [M_T]\{Z\}} \cdot \left[\{Q_i\} - \frac{(k_{11} - \hat{\lambda}_i m_{11})}{(k_{12} - \hat{\lambda}_i m_{12})} \cdot \{\Delta Q_i\} \right]. \tag{26}$$

The following inequality relationship can be estabilished as criteria for selection of the best approximation

$$\hat{\lambda}_i^{\min} \le \lambda_i^{rqa0} \le \hat{\lambda}_i^{\max} \tag{27}$$

where λ_i^{rqa0} is the zero order Rayleigh quotient approximation which is defined below as

$$\lambda_i^{rqa0} = \frac{\{Q_i\}^T [K]\{Q_i\}}{\{Q_i\}^T [M]\{Q_i\}}. \tag{28}$$

Hence, criteria for selection of the best approximation are (i) maximum value of $|\zeta_1/\zeta_2|$, (ii) minimum distance from the zero-order Rayleigh quotient λ_i^{rqa0}, (iii) minimum distance from λ_i, (iv) minimum magnitude, (v) minimum distance from the root selected for the previous mode. This approximation procedure could also be interpreted as an improved Rayleigh quotient approximation procedure with one free parameter, i.e., $\left|\zeta_2/\zeta_1\right|$.

2.2.6 Iterative Method for Calculating Eigenvectors Derivatives

The calculation of the eigenvector derivatives involves extensive computational effort. The direct method is one of the most efficient methods that produces exact solutions and does not need eigenvectors more than those whose derivatives are to be computed. But because its amount of computational effort is proportional to the number of eigenvector derivatives required, the application of the method becomes expensive when many eigenvector derivatives are demanded. On the other hand, the truncated modal method has an insuperable efficiency but suffers a serious accuracy problem. To improve the accuracy of the modal method, Wang [35] proposed a modified modal method, which was extended by Liu et al. [36] and Zhang and Zerva [37] to an iterative algorithm that can be used as an exact method as well as an approximate method and, just like the direct method, does not require additional eigenvalues and eigenvectors. The method assumes that the inverse stiffness matrix exists. Recently, Lin and Lim [38] and Zeng [39] presented an approach to deal with singular stiffness matrices. The convergence rate of the iterative method depends mainly on the ratio of the specified eigenvalue to the lowest unavailable one, and when the ratio approaches 1, the convergence rate of the corresponding eigenvector derivative will reduce quickly and the method becomes more expensive than the direct method. The iterative method used here was derived originally in Ref. [37]. The basic iterative equation after $p(p \geq 1)$ iterations is

$$\{V_{ku}\}_p = \sum_{i=q+1}^{n}\left[1-\left(\frac{\lambda_k}{\lambda_i}\right)^p\right]\frac{\{Q_k\}^T\left(\frac{\partial[K]}{\partial v_j}-\lambda_i\frac{\partial[M]}{\partial v_j}\right)\{Q_i\}}{\lambda_i-\lambda_k}\{Q_k\}+$$

$$+\sum_{i=q+1}^{n}\left(\frac{\lambda_k}{\lambda_i}\right)^p\{Q_k\}^T[M]\{V_{ku}\}_0\{Q_i\},$$

$$k<q; \qquad p=1,2,... \tag{29}$$

where

$\{V_{ku}\}$ = component of $\partial\{Q_k\}/\partial v_j$ in the range of unavailable eigenvectors $\{Q_{q+1}\},...,\{Q_n\}$,

$\{V_{ku}\}_p$ = pth iterative solution for $\{V_{ku}\}$,

$\{V_{ku}\}_0$ = stands for the initial value.

The term $(\lambda_k/\lambda_i)^p$ represents the error because of the ith unknown eigenvector. When p tends to infinity, $(\lambda_k/\lambda_i)^p$ vanishes because $\lambda_k/\lambda_i<1$, and $\{V_{ku}\}_p$ converges to the exact solution with any initial value. Equation (29) also suggests that $\{V_{ku}\}_0$ can be set equal to zero. Note that in each iteration, the roundoff error in the subspace spanned by the lower available eigenvectors $\{Q_1\},...,\{Q_q\}$ will be automatically wiped out, which results in a very stable iterative process.

3. CONCLUDING REMARKS

This paper reviewed several methods for eigensensitivity analysis with respect to design variables. These were the finite-difference method, the modal method, a modified modal method, Nelson's method, an improved first-order approximation of eigenvalues and eigenvectors and an iterative method. Nelson's method was the least computationally intensive, and since it is an exact method, it is the method recommended. When the original mode shapes were used as initial approximations to the subspace eigensolution of the perturbed problem, the finite-difference method was competitive with Nelson's method. The modified modal method always converged faster than the modal method when at least as many modes were used in the approximation as the number of the mode shape being differentiated. The modified modal method can compete with Nelson's method for the first mode shape derivative when the number of modes needed in the summation was known before the eigensolution was performed. Detailed comparasion an improved first-order approximation [33] with other approximation techniques indicate that significant improvements are achieved with a relatively small extra computational effort. An iterative method is simple, systematic, efficient and numerically stable.

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АНАЛИЗА ОСЕТЉИВОСТИ СОПСТВЕНИХ ВРЕДНОСТИ И СОПСТВЕНИХ ВЕКТОРА МЕХАНИЧКИХ СИСТЕМА

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Резиме. *Неколико метода за израчунавање извода сопствених вектора у односу на конструкционе параметре описане су у овом раду. То су метода коначних разлика, модални метод, модификовани модални метод, Нелсонов метод, побољшана апроксимација првог реда и итеративни метод. Комбиновањем других техника реанализе и неког од ових метода, могуће је да се повећа ефикасност и тачност техника оптимизације за одређивање оптималних услова механичког система одређеног од стране аналитичара. Анализа сензитивности се заснива на селекцији конструкционих параметара у почетном коначноелементном моделу чијом модификацијом би дошло до поправљања динамичког понашања посматране конструкције.*

Кључне речи*: Анализа осетљивости сопствених вредности и сопствених вектора*

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