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BIOLOGICALLY INSPIRED CONTROL AND MODELING OF (BIO)ROBOTIC SYSTEMS AND SOME APPLICATIONS OF FRACTIONAL CALCULUS IN MECHANICS

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Abstract. *In this paper, the applications of biologically inspired modeling and control of (bio)mechanical (non)redundant mechanisms are presented, as well as newly obtained results of author in mechanics which are based on using fractional calculus. First, it is proposed to use biological analog–synergy due to existence of invariant features in the execution of functional motion. Second, the model of (bio)mechanical system may be obtained using another biological concept called distributed positioning (DP), which is based on the inertial properties and actuation of joints of considered mechanical system. In addition, it is proposed to use other biological principles such as: principle of minimum interaction, which takes a main role in hierarchical structure of control and self-adjusting principle (introduce local positive/negative feedback on control with great amplifying), which allows efficiently realization of control based on iterative natural learning. Also, new, recently obtained results of the author in the fields of stability, electroviscoelasticity, and control theory are presented which are based on using fractional calculus (FC).*

Key words: *biologically inspired systems, control algorithms, modeling, fractional calculus, stability*

1. INTRODUCTION

The field of biomimetics and biologically inspired principles from the application of methods and systems found in nature to engineering and technology, has spawned a number of innovations far superior to what the human mind alone could have devised, [1-3]. Also, the fast growing interest in flexible, versatile and mobile robotic manipulators demands for robots with inherent high passive safety suited for direct human-robot interaction. Traditional robotic systems and industrial manipulators demonstrate outstanding specifications regarding, for example, precision and speed movement. Some complex industrial – and especially non-industrial tasks – recently induced a new approach to robot design and control in order to achieve very stable, fast, and accurate systems._Biologically inspired approaches have recently succeeded in design and control in robotics [2-4]. Biological systems have been evolved to optimize themselves under selective pressures for a long time. Biological organisms have evolved to perform and survive in a world characterized by rapid changes, high uncertainty, indefinite richness, and limited availability of information. General biomechanical systems including the human body as well as the bodies of mammals and insects are also redundantly actuated. For example, mobility of the human upper-extremity (arm) can be considered as 7 DOF`s, while it has 29 human actuators (i.e, muscles) and accordingly, it has 22 redundant actuators,_[3]. A robotic manipulator is called kinetically redundant if it has more degrees of freedom (DOF) then required for a realization of a prescribed task in a task space. The kinematic redundancy in a manipulator structure yields increased dexterity and versality and also allows avoiding collisions with obstacles by the choice of appropriate configurations,_[5]. Also, redundant actuation can be also found in many robotic applications,_[6].

First, it is proposed using biological analog–synergy due to existence of invariant features in the execution of functional motion, Bernstein_(1967)_(i.e. rule(s) that can be developed by central nervous system (CNS) based on some principles),_[1]. New, synergy approach allows resolving redundancy control problem i.e. actuator redundancy, in the framework of optimal control problem which it is solved by Pontryagin`s maximum principle. It is suggested joint actuator synergy approach which is established by optimization law at coordination level, where it is introduced a central control,_[7]. In that way, one may obtain a specific constraint(s) on the control variables. Also, modeling and resolving kinematic redundancy of (bio)mechanical/robotical system in synergy like fashion, can be achieved using optimization law with suitable kinematic and dynamic criteria which are the function of generalized coordinates, velocities, accelerations and control vectors, respectively,_[8,_9]. Second, model of (bio)mechanical system may be obtained using another biological concept called distributed positioning (DP) which is based on the inertial properties and actuation of joints of considered mechanical system,[3],_[8-10]. At last, using other biological principles is proposed, such as: *principle of minimum interaction* which takes a main role in hierarchical structure of control,_[11] and *self-adjusting principle* (introduce local positive/negative feedback on control with great amplifying),_[8], which allows efficiently realization of control based on *iterative natural learning.* In that way, control problem of coordinating segments of (non)redundant (bio)mechanical system can be stated as an optimization problem which is most likely to biological *principle of minimum interaction.* Also, the common observation that human beings can learn perfect skills trough repeated trials motivations the idea of iterative learning control for systems performing repetitive tasks where for improving the properties of tracking is proposed applying *principle of self-adaptability.*

In the second part of this paper, new, recently obtained results of author in fields of stability, electroviscoelasticity, and control theory which are based on using fractional calculus (FC) are presented, [12-20]. FC has attracted attention of researchers from different fields in the recent years and the fractional integro-differential operators are a generalization of integration and derivation to an arbitrary order operators and they provide an excellent instrument for the description of memory and hereditary properties of various materials and processes and, also obtaining more degrees of freedom in the in the model,[3]. First of them, is an example within a new theory of electroviscoelasticity, which describes the behavior of electrified liquid-liquid interfaces in fine dispersed systems, and is based on a new constitutive model of liquids: fractional order model (generalized the Van der Pol equation) with corresponding non-integer time derivative and integral order, especially linear and nonlinear case,[4]. Also, new algorithms for fractional iterative learning control (ILC), a $D^{\alpha}, PD^{\alpha}, PI^{\alpha}D^{\beta}$ types are proposed for fractional time delay system and fractional process control $PI^{\alpha}D^{\beta}$ type which include ILC feedback control is also presented. At last,stability test procedure (finite time and practical stability) is shown for (non)linear (non)homogeneous time-invariant fractional order time delay systems where sufficient conditions of this kind of stability are derived. Specially, previous results can be applied for robotic system where it appears a time delay in PD^{α} _fractional control system,[5].

2.FUNDAMENTALS OF BIOLOGICALLY INSPIRED MODELING AND CONTROL

As we know, control exists everywhere in complex biological systems. Recent rapid development of biological science and technologies will further improve the active applications of control engineering. Meanwhile, system control theory itself will also be promoted by advanced biomimetic researches, [2], [4], [12]. Several theoretical concepts have been evolved in control theory, typified by feedback control, optimal control, sequence control, and so on,[11]. The main roles of feedback control are regulation and adjustment, whereas optimal control involves planning and supervision with a higher level of control state than feedback control. Meanwhile, it becomes more and more important for the artificial systems to have high flexibility, diversity, reliability, and affinity. System control theory, which forms the core foundation for understanding, designing, and operating of systems, is still limited and insufficient to handle complex large-scale systems and to process spatial temporal information in real time as biological systems. Under this background, biomimetic and biologically inspired control research is becoming a very important subject, [2]. In the first approach, technology approximates the end result or function of an organ or organism. In the second approach, the principles extracted from bio-systems may be applied in ways very much unlike those exhibited in the originating organism. The analysis and clarification of functions of complex biological systems mathematically at the system level, and imitation of them in engineering, will lead to a deeper understanding of ourselves and will be significant for constructing the next generation of advanced artificial systems such as human friendly robots.

2.1. Biologically inspired principle-synergy

The organization and development of brain nervous system's motor control functions largely depend on the physical interaction with the external environment. Self-organization of the environmental adaptive motor function is one of the most interesting characteristics that we should learn in biomimetic control research. From a mechanical point of view, any human or animal represents a redundant mechanism, [2], [3]. The nervous system takes advantage of kinematic and multi-muscle redundancies to control actions in a flexible way so that, for example, the same motor goal can be reached differently depending on our intentions, external environmental (e.g. obstacles) or intrinsic (neural) constraints. Despite this flexibility, the central control of actions is unambiguous: each time the body moves, a unique action is produced despite the possibility of using other actions leading to the same goal. It is amazing how these seemingly opposite aspects – flexibility and uniqueness- are combined in the control of actions. Following Bernstein [1], we refer to these aspects of action production as the "redundancy problem". In other words, it was observed in the execution of functional motions that certain trajectories are preferred from the infinite number of options [1], [11]. Such behavior of organisms can be only explained by the existence of *inherent optimization laws* in self-organized systems governing the acquisition of motor skills. Existence of invariant features in the execution of functional motions points out that central nervous system (CNS) uses *synergy* (i.e rule(s) that can be developed by the CNS based on some principles). In fact, such behavior implies that it obeys the optimization at the coordination level where the goal is to minimize efforts in terms of synergy patterns. Speaking mathematically, the synergy imposes specific constraints on the control variables of joints which are related to the task dependent functions pertaining to classes of motor acts. For example, the control of arm movement in humans also relies very much on distributed usage of different joints, and inherent optimization of muscles which are active. Arm muscles are found grouped in pairs about simple hinge joints where even in the simplest case of two antagonist muscles about a joint there are two distinct control variables. Moreover, muscles should be regarded as functional units with more than one control and activation parameter. Also, the biological muscle is the starting point for many new approaches by the development of new actuators for robotics. Beside the direct simulation of biological systems [2], there are different approaches to mimic biological operational principles in technical systems, [11].

Here, the redundancy control problem has been discussed in the framework of optimal control problem which is solved by Pontryagin's maximum principle. Joint actuator synergy approach is suggested which is established by optimization law at coordination level, where is introduced a central control as suggested Bernstein in [1]. In that way, one may obtain a specific constraint(s) on the control variables. The dynamic model of robot can be described with application set of the $2n$ Hamiltonian equations with respect to Hamiltonian phase variables q_i, p_i [13] where conjugate (canonical) momenta p_i

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \qquad \dot{p}_i = -\frac{\partial H}{\partial q_i} + Q_i(u), \quad i = 1, 2, \ldots, n \tag{1}$$

where is $H(\boldsymbol{q},\boldsymbol{p})$ Hamiltonian and $Q_i,\ i=1,2,..,n$ be non-conservative control forces. For a global optimization, the problem is set up as following

$$J=\int_{t_0}^{t_k} f_o(q,p,u)dt \rightarrow min \tag{2}$$

The goal is to find $u(t),\ t_o \le t \le t_k$ which drives system from given initial state (q_o,p_o) to a final state (q_k,p_k) under the condition that the whole trajectory minimizes the performance criterion. Performance criterion is introduced at coordination level as the energy criterion which is, in our case, functional sum of weighted controls of the robot

$$f_o(u)=\frac{1}{2}u^T Ru \tag{3}$$

Alternatively, the control can be smoothed by minimizing an energy function, quadratic in control, in addition to time. Here, t_o, t_k are the initial and final time of an end-effector movement, which are known and fixed. The control weighting matrix $R=diag\{r_1,r_2,...,r_m\}$ is symmetric positive definite matrix; $u(t)$ must be entry of a given subset U of admissible controls of m-dimensional Euclidean space: $u(t)\in U\subset R^m$. It is also assumed that optimal control problem has a solution. Applying biologically inspired concept of control, and introducing central control u_c as suggested Bernstein [1], one may introduce control vector $u=\left[u_1,u_2,u_3,u_c\right]^T$. Also, generalized forces can be presented as functions of components of control u as

$$Q_i=u_i+u_c,\ i=1,2,3 \quad u=[u_1,u_2,u_3,u_c]^T \tag{4}$$

It means that we have four motors, a "central" motor which produces u_c, and rest of motors (corresponding controls u_1, u_2, u_3) are placed at each joint separately. In that way, one of possible control strategies is established. Taking in a account condition of optimal control based on the Pontryagin's maximum principle and applying the matrix theory it implies that following condition must be fulfilled:

$$\det\begin{bmatrix}1 & 0 & 0 & u_1^* r_1\\ 0 & 1 & 0 & u_2^* r_2\\ 0 & 0 & 1 & u_3^* r_3\\ 1 & 1 & 1 & u_c^* r_0\end{bmatrix}=0 \tag{5}$$

After some algebraic operations it yields

$$u_c^* r_0 = u_1^* r_1 + u_2^* r_2 + u_3^* r_3 \tag{6}$$

Equation (6) presents an *invariant on control variables "control synergy"*- which is established by optimization law at coordination level. In order to obtain finite solutions of the problem mentioned, it is necessary to solve two-point boundary value problem for a system of ordinary differential equations or, even in particular cases, to solved complicate algebraic problems. The proposed biologically inspired optimal control is illustrated by simulation results of a robot with 3 DOF`s (Fig.1) and 4 control variables (Fig.2-5),see [7].

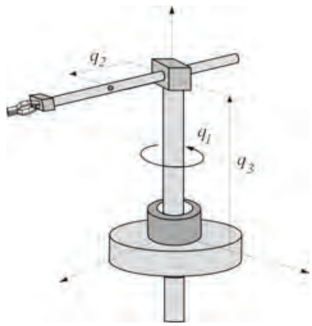


Fig 1. Autolemec ACR with three DOFs

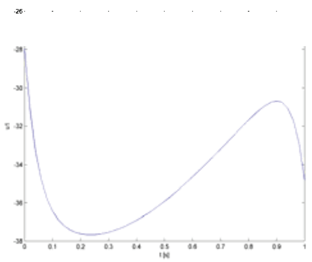


Fig.2. Optimal control *u1*

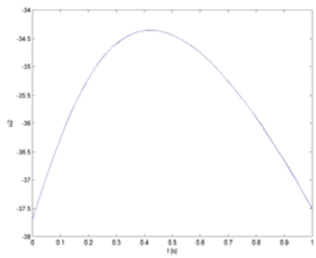


Fig.3. Optimal control *u2*

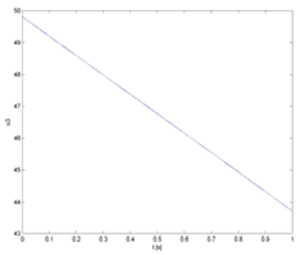


Fig.4. Optimal control *u3*

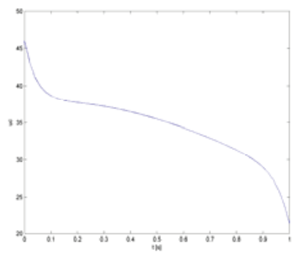


Fig.5. Optimal control *uc*

2.2 Biologically inspired principle - distributed positioning (DP)

The relatively new approach in modeling redundant mechanism is based on biological analog i.e, the modeling is based on the separation of the prescribed movement into two motions: smooth global, and fast local motion, called distributed positioning (DP). Distributed positioning is an inherent property of biological systems. It is based on the inertial properties and actuation capabilities of joints. Humans, when writing, as shown in literature control their proximal joints, while the movement of distal joints follow them. Writing is a good representative of task that is characteristic for humans, but at the same time interesting for robots. It is fast and accordingly very demanding from the viewpoint of dynamics (high accelerations produce high inertial loads). In humans highly inertial arm joints (shoulder and elbow) provide smooth global motion, and low inertial hand joints (fingers) perform fast and precise local motions, [3],[10]. Acceleration of massive segments leaded to drive overload and required redundancy. Let, the position of the arm be defined by the vector of joint (internal) coordinates of dimension $n=8: q=\left[q_1\ q_2\ ...q_8\right]^T$. The position of the terminal device (pencil) is defined by the vector of external coordinates of dimension $n_e=5: X=\left[x\ y\ z\ \theta\ \varphi\right]^T$, where *x,y,z* define the tip position and angles θ,φ define the pencil axis. The kinematic model of the arm-hand complex i.e. the transformation of coordinates (internal to external and vice versa) is highly nonlinear

$$X = f(q) \tag{7}$$

where *f* is the function: $R^8 \rightarrow R^5$. The inverse kinematics (calculation of q for given X) has an infinite number of solutions since (7) represents a set of 5 equations with 8 unknowns. this is due to presence of redundancy. The dimension of redundancy is $n_r=n-n_e=8-5=3$. The kinematic model can be written in the Jacobian form of the first or of the second order

$$\dot{X} = J(q)\dot{q}, \quad \ddot{X} = J(q)\ddot{q} + A(q,\dot{q}) \tag{8}$$

where *J* is $n_e \times n$ (i.e. 5x8) Jacobian matrix and A is $n_e \times 1$ (i. e. 5x1) adjoint vector containing the derivative of the Jacobian. Let X_1 be the subvector containing the accelerated motions (dimension n_a), and X_2 be the subvector containing the smooth motions (n_e-n_a). Now

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \tag{9}$$

The redundant robot ($n=8$ DOFs) is now separated into two subsystems,Fig.6. The subsystem with $n_e=5$ DOFs with greatest inertia is called *the basic configuration.* The other subsystem is *the redundancy* having $n_r=3$ DOFs. It holds that $n=n_e+n_r$. Analysing the plane writing task one finds that there are n_a=2 accelerated external motions : *x(t)* and *y(t)*. The others (z,θ,φ) are constant or smooth. According to DP concept we introduce $X_1=\left[x\ y\right]^T$ and $X_2=\left[z\ \theta\ \varphi\right]^T$. It can be defined the basic configuration as a mechanism $q_b=\left[q_1\ q_2\ \ ....q_5\right]^T$.The resting joints, one wrist joint q_6 and "fingers " (q_7, q_8), form the redundancy and $q_r=\left[q_6\ q_7\ \ q_8\right]^T$ defines the position of the redundancy. The DP concept solves the inverse kinematics of a redundant robot in two steps. At the first step the motion of basic configuration is calculated (q_b) using kinematic model and properties DP concept, and at the second step the motion of redundancy (q_r) is determined, [3].

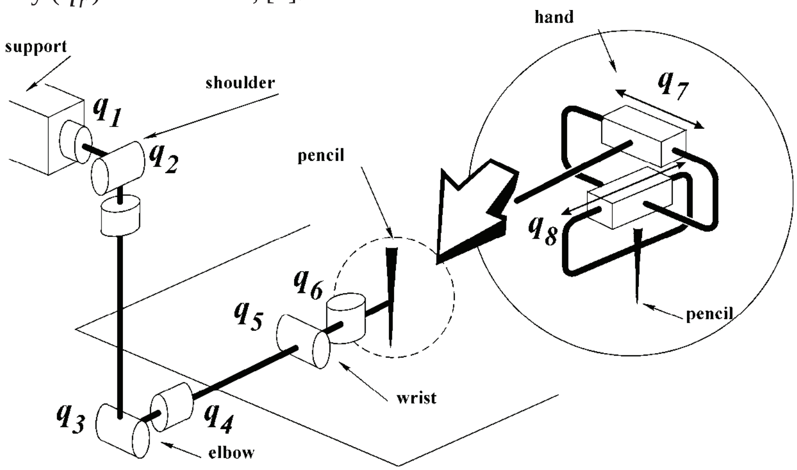


Fig. 6. Eight-DOF arm-hand complex

2.2. Principle of minimum interaction in hierarchical control

Also,motor control is organized as a multilevel structure, is generally accepted. In assistive system involves man as the decision maker, a hierarchical control structure can be proposed with three levels from the left to right: voluntary level, coordination level, actuator level. This imposes the robotic system is decomposed into several subsystems with strong coupling between subsystems. For an instance, the system dynamics of redundant robot are described by:

$$F_o = \{(U,Y,Z): F_1 = 0, F_2 = 0\}, \quad (10)$$

where $U \in R^m$ is the control input vector, $Y \in R^n$ the output vector, and $Z \in R^n$ the vector representing interactions between the two subsystems(segments),Fig.7. The cost function of a multiple-system is the sum of the cost functions of all subsystems:

$$J(U,Z,Y) = \sum_{i=1}^{N} J_i, \quad (11)$$

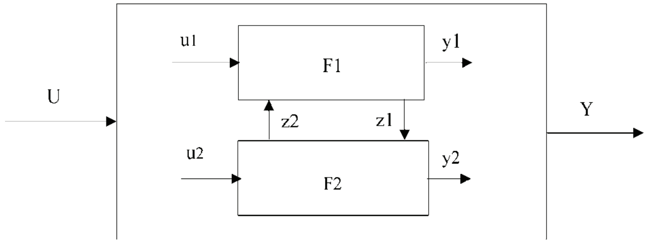


Fig.7. Coordination of two subsystems

The problem of coordinating multiple systems can be stated as an optimization problem: minimize the cost function J subject to the constraint F_o. In monograph [8], it is stated and solved the Bernstein problem which is related to kinematic redundancy of ARA. It is treated control of a anthropomorphic robot arm (ARA) with three degrees of freedom. The optimal control problem of continuous nonlinear dynamic systems - (redundant robotic manipulators), with quadratic performance index can be stated as follows. Determine $U \in L_2(t_o, t_k)$ such that under system constraints is minimized.

$$J = \frac{1}{2}\int_{t_o}^{t_f}\left[\left\|x(t)-x_d(t)\right\|_Q^2 + \left\|u(t)\right\|_R^2 + \left\|z(t)\right\|_S^2\right]dt \to min, \tag{12}$$

$$\dot{x}(t) = \frac{dx(t)}{dt} = F(x(t),u(t)) = g(x(t)) + h(x(t))u(t),\ x(t_o) = x_o, \tag{13}$$

$$z(t) = x(t) \quad \dim x = n, \dim u = m, \dim z = n, \tag{14}$$

where x, u state and control vectors and z is interaction vector; weighting matrices Q, R, S are all block diagonal. So, problem of coordinating multiple systems can be state as an optimization problem which is most likely to biological *"principle of minimum interaction"* which is formulated by Gelfand and Tsetlin, [11]: "*For complex controlling systems, the typical structure permits the separation of individual, relatively automatic subsystems. For each subsystem of that type all the remaining subsystems belong to the outside environment and the expediency of the subsystems appears in the minimization of interaction among them so that in stable conditions these subsystems function as if independently, autonomously*." A major consequence of this principle is that the complexity of each subsystem does not depend on the complexity of the whole system. The application of the minimum interaction principle also leads to a structural form for the "self-organizing" controller. The solution of stated problem of control is generated in a sequence of steps involving a heuristic techniques of genetic algorithm that provides reliable initial guesses. Genetic algorithms are stochastic adaptive algorithms whose search method is based on simulation of natural genetic inheritance and striving for survival. To solve local problems, the minimum principle is used where the multi-level univariate hierarchical strategies is proposed. The problem is divided into two-level optimization problem which is solved iteratively until the desired performance is achieved, [8].

2.3. Iterative learning control, self – adaptability

Recently, there have been extensive research activities in the topic of learning control for controlling dynamics non-linear systems in a iterative manner. The learning control concept differs from conventional control methodologies in that the control input can be appropriately adjusted to improve its future performance by learning from the past experimental information as the operation is repeated. The common observation that human beings can learn perfect skills trough repeated trials motivations the idea of iterative learning control for systems performing repetitive tasks. Therefore, iterative learning control requires less *a priori* knowledge about the controlled system in the controller design phase and also less computational effort than many other kinds of control. Learning control for controlling dynamics systems, a class of tracking systems is applied where it is required to repeat a given task to desired precision.

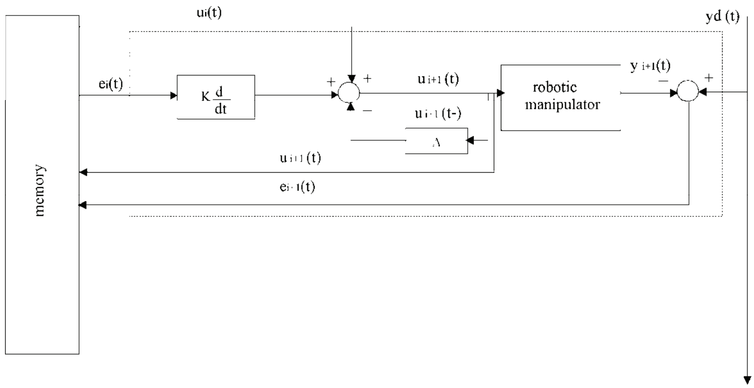


Fig.8. Block diagram of iterative learning control

In these equations t denotes time, $t \in [0,T]$, $t \in \Re$, x_i the state vector, $x_i \in \Re^n$, u_i the control vector, v_i the vector uncertainties, $u_i \in \Re^m$, y_i the output vector of the system, $y_i \in R^r$ and *i* denotes the *i*-the repetitive operation of the system. The learning controller for generating the present control input is based on the previous control history and a learning mechanism. Motivated by human learning, the basic idea of iterative learning control is to use information from previous executions of the task in order to improve performance from trial to trial in the sense that the tracking error $e_i(t)$ is sequentially reduced. It is proposed applying biological analog - *principle of self-adaptability* which introduce, here, local negative feedback on control with great amplifying. In the simplest case learning control law can be shown such as (see Fig.8):

$$u_{i+1}(t) = -\Delta u_{i+1}(t^-) + u_i(t) + K(t)\dot{e}_i(t)\,, \tag{15}$$

where $u_{i+1}(t^{-}) = u_{i+1}(t-\tau)$, $\tau -$ the small time delay, denotes a vector of the delayed control signal. If the feedback delay can be neglected, (for example using very fast processors) then: $u_{i+1}(t^{-}) = u_{i+1}(t)$.

3. Some applications of fractional calculus in mechanics

In the second part of this presentation are presented, new, recently obtained results of author in fields of stability, electroviscoelasticity, and control theory which are based on using fractional calculus (FC). FC has attracted attention of researchers from different fields in the recent years and the fractional integro-differential operators is a generalization of integration and derivation to an arbitrary order operators and they provide an excellent instrument for the description of memory and hereditary properties of various materials and processes and, also obtaining more degrees of freedom in the in the model,[14-18].

3.1. Brief historical introduction

When in the 17th century the integer calculus had been developed, Leibniz and L'Hospital probed into the problems on the fractional calculus (FC) and the simplest fractional differential equations (FOEs) through letters. Leibniz asked in a letter addressed to L'Hospital:

Can the meaning of derivatives of integral order $d^n f(x)/dx^n$ *be extended to have meaning when n is not an integer but any number (irrational, fractional or even complex-valued)? L'Hospital responded: What if n be 1/2?* $d^{1/2} f(x)/dx^{1/2} = ?$ *for* $f(x) = x$. *Leibniz, in a letter dated from Sept. 30, 1695, replied: It will lead to a paradox, from which one day useful consequences will be drawn.In these words fractional calculus was born.*

Following L'Hopital's and Liebniz's first inquisition, fractional calculus was primarily a study reserved for the best minds in mathematics. Futher, the theory of fractional-order derivative was developed mainly in the 19th century. In his 700 pages long book on Calculus, 1819 Lacroix [19] developed the formula for the n-th derivative of $y = x^m$, m – is a positive integer,

$$D^n x^m = \frac{m!}{(m-n)!} x^{m-n}, \tag{16}$$

where $n\ (\le m)$ is an integer. Replacing the factorial symbol by the Gamma function, he further obtained the formula for the fractional derivative

$$D^{\alpha}x^{\beta} = \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)}x^{\beta-\alpha}, \tag{17}$$

where α and β are fractional numbers and Gamma function $\Gamma(z)$ is defined for $z > 0$ as:

$$\Gamma(z) = \int_{0}^{\infty} e^{-x}x^{z-1}dx, \quad \Gamma(z+1) = z\Gamma(z), \tag{18}$$

In particular, he calculated

$$D^{1/2}x = \left[\frac{\Gamma(2)}{\Gamma(3/2)}\right]x^{1/2} = 2\sqrt{x/\pi}, \tag{19}$$

On the other hand, Liouville (1809-1882) formally extended the formula for the derivative of integral order *n*

$$D^{n}e^{ax} = a^{n}e^{ax} \Rightarrow D^{\alpha}e^{ax} = a^{\alpha}e^{ax}, \ \alpha - arbitrary\, order, \tag{20}$$

Using the series expansion of a function, he derived the formula known as *Liouville`s first formula for fractional derivative,* where α may be rational, irrational or complex.

$$D^{\alpha}f(x) = \sum_{n=0}^{\infty} c_{n}a_{n}^{\alpha}e^{a_{n}x}, \tag{21}$$

where

$$f(x) = \sum_{n=0}^{\infty} c_{n}\exp(a_{n}x), \ \mathrm{Re}\, a_{n} > 0, \tag{22}$$

However, it can be only used for functions of the previous form. Also, Liouville formulated another definition of a fractional derivative based on the Gamma function (see below) such as:

$$D^{\alpha}x^{-\beta} = (-1)^{\alpha}\frac{\Gamma(\alpha+\beta)}{\Gamma(\beta)}x^{-\beta-\alpha}, \ \beta > 0, \tag{23}$$

which is known as *Liouville`s second definition of fractional derivative.*Also his second definition is useful only for rational functions.Neither of his definitions was found to be suitable for a wide class of definitions.The derivative of constant function $\beta = 0$ is zero because $\Gamma(0) = \infty$. On the other hand, the Lacroix definition gives:

$$D^{\alpha}1 = \frac{1}{\Gamma(1-\alpha)}x^{-\alpha} \neq 0,, \tag{24}$$

But,Lacroix's method could not be applied to many functions, so was not useful in a broad context. The modern epoch started in 1974 when a consistent formalism of the fractional calculus had grown to such extent, that in 1974 the first conference was held in New Haven. In the same year the first book on fractional calculus by Oldham and Spanier [15] was published after a joint collaboration starting in 1968. Applications of FC are very wide nowadays, in rheology, viscoelasticity, acoustics, optics, chemical physics, robotics, control theory of dynamical systems, electrical engineering, bioengineering and so on, [14-18].The main reason for the success of applications FC is that these new fractional-order models are more accurate than integer-order models, i.e. there are more degrees of freedom in the fractional order model. Furthermore, fractional derivatives provide an excellent instrument for the description of memory and hereditary properties of various materials and processes due to the existence of a ”memory” term in a model. This memory term insure the history and its impact to the present and future.

3.2. Fundamentals of fractional calculus

Fracional calculus is a name for the theory of integrals and derivatives of arbitrary order, which unify and generalize the notions of integer-order differentiation and n-fold integration. At present, based on the different background and purpose there are some other definitions of FC. There exist today many different forms of fractional integral operators, ranging from divided-difference types to infinite-sum types, Riemann-Liouville fractional derivative, Grunwald–Letnikov fractional derivative, Caputo’s, Weyl’s and Erdely-Kober left and right fractional derivatives and so on, Kilbas *et al.*[16]. At first, one can generalize the differential and integral operators into one fundamental D_t^p operator t which is known as fractional calculus:

$$ {}_aD_t^p = \begin{cases} \dfrac{d^p}{dt^p} & \Re(p) > 0, \\ 1 & \Re(p) = 0 \quad , \\ \int_a^t (d\tau)^{-p} & \Re(p) < 0. \end{cases} \tag{25} $$

The two definitions generally used for the fractional differintegral are the Grunwald-Letnikov (GL) definition and the Riemann-Liouville (RL) definition [14-16]. The original Grunwald-Letnikov definition of fractional derivative is given by a limit, i.e

$$ `` \; {}_aD_t^p f(t) = \lim_{h\to 0} \frac{1}{h^p} \sum_{j=0}^{[(t-a)/h]} (-1)^j \binom{p}{j} f(t-jh)\,, \tag{26} $$

where a, t are the limits of operator and [x] means the integer part of x . Integral version of GL is defined by

$$ {}_aD_t^p f(t) = \sum_{k=0}^{n-1} \frac{f^{(k)}(0) t^{-p+k}}{\Gamma(-p+k+1)} + \frac{1}{\Gamma(n-p)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{p-n+1}} d\tau, \tag{27} $$

The Riemann-Liouville definition of fractional derivative is given by

$$ {}_aD_t^p f(t) = \frac{1}{\Gamma(n-p)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{p-n+1}} d\tau, \tag{28} $$

for ($n-1 < p < n$) and for the case of ($0 < p < 1$), the fractional integral is defined as

$$ {}_0D_t^{-p} f(t) = \frac{1}{\Gamma(p)} \int_0^t \frac{f(\tau)}{(t-\tau)^{1-p}} d\tau, \tag{29} $$

where $\Gamma(.)$ is the well known Euler's gamma function. Also, the *chain rule* has the form

$$ \frac{d^\beta f(g(t))}{dt^\beta} = \sum_{k=0}^{\infty} \binom{\beta}{k}_\Gamma \left(\frac{d^{\beta-k}}{dt^{\beta-k}} 1 \right) \frac{d^k}{dt^k} f(g(t)) \tag{30} $$

Where $k \in \mathbb{N}$ and $\binom{\beta}{k}_\Gamma$ are the coefficients of the generalized binomial

$$ \binom{\beta}{k}_\Gamma = \frac{\Gamma(1+\beta)}{\Gamma(1+k)\Gamma(1-k+\beta)} \tag{31} $$

For convenience, Laplace domain is usually used to describe the fractional integro-differential operation for solving engineering problems. The formula for the Laplace transform of the RL fractional derivative has the form:

$$ \int_0^\infty e^{-st} {}_0D_t^p f(t) dt = s^p F(s) - \sum_{k=0}^{n-1} s^k {}_0D_t^{p-k-1} f(t)\big|_{t=0} \tag{32} $$

In practical applications, the initial conditions ${}_0D_t^{p-k-1} f(t)\big|_{t=0}$ are frequently not available. Also, Caputo,[20] has proposed that one should incorporate the integer order (classical) derivative of function x, as they are commonly used in initial value problems with integer-order equations. In that way, one can use the derivatives of the Caputo type such as:

$$ {}_0^C D_t^p \left[f(t) \right] = \frac{d^p f}{dt^p} = \frac{1}{\Gamma(1-p)} \int_0^t \frac{f^{(1)}(\tau)}{(t-\tau)^p} d\tau \quad ,0 < p < 1 \quad , \quad f^{(1)}(\tau) = df / d\tau \tag{33} $$

From definition of Riemann-Liouville and Caputo derivatives one may observe that the relation between the two fractional derivatives is as follows:

$$ {}_0^C D_t^p \left[f(t) \right] = {}_0D_t^p \left[(f - T_{n-1}[f])(t) \right] \quad , \tag{34} $$

where $T_{n-1}[f]$ is the Taylor polynomial of order $(n-1)$ for f, centered at 0. So, one can specify the initial conditions in the classical form

$$f^{(k)}(0) = f_0^{(k)}, \quad k = 0,1,\dots,n-1\,, \tag{35}$$

The two Riemann-Liouville and Caputo formulation coincide when the initial conditions are zero. For numerical calculation of FC one can use relation which has the following form:

$$_{((t-L)}D_t^{\pm p} f(t) \approx h^{\mp p} \sum_{j=0}^{N(t)} b_j^{(\pm p)} f(t-jh)\,, \quad b_0^{(\pm p)} = 1, \quad b_j^{(\pm p)} = \left(1 - \frac{1 \pm p}{j}\right) b_{j-1}^{(\pm p)} \tag{36}$$

where L is the *"memory length"*, h is the step size of the calculation, $N(t) = \min\left\{\left[\frac{t}{h}\right], \left[\frac{L}{h}\right]\right\}$ $[x]$ is the integer part of x and $b_j^{(\pm p)}$ is the binomial coefficient.

3.3 Electroviscoelasticity of Liquid/Liquid Interfaces: Fractional-Order Model

Also, number of theories that describe the behavior of liquid-liquid interfaces have been developed and applied to various dispersed systems e.g., Stokes, Reiner-Rivelin, Ericksen, Einstein, Smoluchowski, Kinch, etc. According to this model liquid-liquid droplet or droplet-film structure (collective of particles) is considered as a macroscopic system with internal structure determined by the way the molecules (ions) are tuned (structured) into the primary componentns of a cluster configuration. How the tuning/structuring occurs depends on the physical fields involved, both potential (elastic forces) and nonpotential (resistance forces). All these microelements of the primary structure can be considered as electromechanical oscillators assembled into groups, so that excitation by an external physical field may cause oscillations at the resonant/characteristic frequency of the system itself (coupling at the characteristic frequency),[21-24]. Up to now, there are three possible mathematical formalisms discussed related to the theory of electroviscoelasticity,where the first is tension tensor model, the second is Van der Pol derivative model,and the third model presents an effort to generalize the previous Van der Pol equation, i.e. the ordinary time derivative and integral are now replaced with corresponding fractional-order time derivative and integral of order $p < 1$. Hence, the study of the electro-mechanical oscillators is based on electromechanical and electrodynamic principles. At first, during the droplet formation it is possible that the serial analog circuits are more probable, but later, as a consequence of tuning and coupling processes the parallel circuitry become dominant. Also, since the transfer of entities by tunneling (although with low energy dissipation) is much less probable it is sensible to consider the transfer of entities by induction (medium or high energy dissipation). A nonlinear differential equation of the Van der Pol type represents the initial electromagnetic oscillation

$$C\frac{dU}{dt} + \left(\frac{U}{R} - \alpha U\right) + \gamma U^3 + \frac{1}{L}\int U dt = 0\,, \tag{37}$$

where U is the overall potential difference at the junction point of the spherical capacitor C and the plate, L is the inductance caused by potential difference, and R is the ohmic resistance. The α and γ are constants determining the linear and nonlinear parts of the characteristic current and potential curves. The noise in this system, due to linear amplification of the source noise, causes the oscillations of the "continuum" particle (molecule surrounding the droplet or droplet-film structure), which can be represented by the particular integral

$$C\frac{dU}{dt}+\left(\frac{1}{R}-\alpha\right)U+\gamma U^{3}+\frac{1}{L}\int Udt+=-2A_{n}\cos\omega t\ , \tag{38}$$

where ω is the frequency of the incident oscillations, [21]. The noise output appears as an induced anisotropic effect. In an effort to generalize the previous equation the ordinary time derivative and integral are now replaced with corresponding fractional-order time derivative and integral of order p ,[22]. Here, the capacitive and inductive elements, using Riemann-Liouville definition of differintegral forms, fractional–order $p\in[0,1)$ enable formation of the fractional differintegral equation, i.e. more flexible or general model of liquid-liquid interfaces behaviour, as follows (linear case):

$$C_0D_t^p\left[U(t)\right]+\left(\frac{1}{R}-\alpha\right)U+\frac{1}{L}\,{}_0D_t^{-p}\left[U(t)\right]=i(t)\ , \tag{39}$$

Using Laplace transform of (39) leads to

$$G(s)=\frac{U(s)}{i(s)}=\frac{s^{p}}{Cs^{2p}+\left(1/R-\alpha\right)s^{p}+1/L}=s^{p}G_{3}(s),\ G_{3}(s)=\frac{1}{as^{2p}+bs^{p}+c}, \tag{40}$$
$$a=C,\ b=(1/R-\alpha),\ c=1/L$$

The term-by-term inversion, based on the general expansion theorem for the Laplace transform, [2] produces

$$G_{3}(t)=\frac{1}{a}\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}\left(\frac{c}{a}\right)^{k}t^{2p(k+1)-1}E_{p,2p+pk}^{(k)}\left(-\frac{b}{a}t^{p}\right)\ , \tag{41}$$

where $E_{\lambda,\mu}(z)$ is the Mittag-Leffler function in two parameters.Laplace transform of the Mittag-Leffler function in two parameters is:

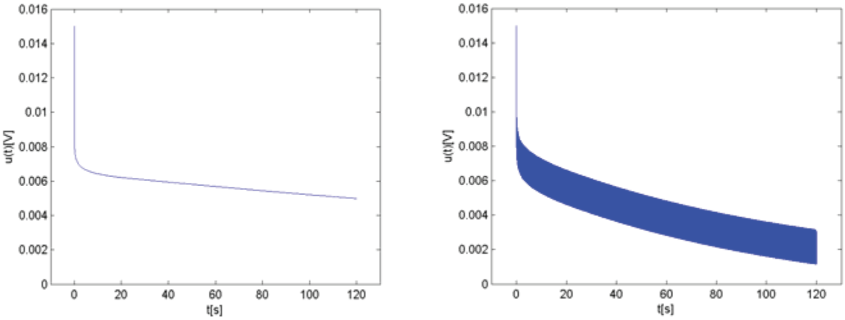
$$\int_{0}^{\infty}e^{-t}t^{\beta-1}E_{\alpha,\beta}(zt^{\alpha})dt=\frac{1}{1-z},\quad (|z|<1)\ , \tag{42}$$

Using inverse Laplace transform of $G(s)$ one can obtain an explicit representation of the solution (39) such as:

$$U(t)=\int_{0}^{t}G(t-\tau)i(\tau)d\tau\ , \tag{43}$$

So,the initial electromagnetic oscillation is represented by the equation (43) i.e, a (non)homogeneous solution (Fig.9) may be obtained in following manner using numerical procedure (Grunwald definition). Also, one can obtain equivalent nonlinear problem applying differentiation of (37) such as:

$$C\frac{d^2U}{dt^2}+\left(\frac{1}{R}-\alpha+3\gamma U^2\right)\frac{dU}{dt}+\frac{1}{L}U=0\,, \qquad (44)$$



homogenous case Fig.9 *nonhomogenous case*

$\alpha=0.9995$ $Uo=15mV$, $p=0.95$, $T=0.001s$

$\alpha=0.95$ $Uo=15mV$, $p=0.95$, $r=0.95$ $T=0.01s$ $An=0.05nm$

Taking into account of Caputo definition [4] and introducing vector

$$x_1(t)=U(t),\ x_2(t)={}_0D_t^C U(t),\quad p\in\mathbb{R}\quad \mathbf{x}(t)=\left(x_1,x_2\right)^T, \qquad (45)$$

one can get:

$${}_0D_t^p\mathbf{x}(t)=\begin{bmatrix}0 & 1\\ -1/LC & -\left(1/R-\alpha\right)/C\end{bmatrix}\begin{Bmatrix}x_1(t)\\ x_2(t)\end{Bmatrix}+\begin{bmatrix}0 & 0\\ 0 & -3\gamma x_1^2(t)/C\end{bmatrix}\begin{Bmatrix}x_1(t)\\ x_2(t)\end{Bmatrix}, \qquad (46)$$

It is easily observed that previous case is a one of the general case for this nonlinear problem which can be obtained in the form:

$${}_0^CD_t^p\mathbf{x}(t)=f\left(t,\mathbf{x}(t)\right)\ \mathbf{x^{(k)}}(0)=\mathbf{x_0^{(k)}},\quad k=0,1,...,\left[p\right], \qquad (47)$$

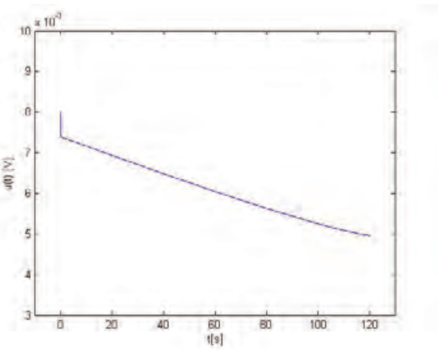


Fig. 10. Homogeneous solution of (Eq. 37)

The initial electromagnetic oscillation is represented by the non linear fractional differential equation (39), homogeneous solution may be obtained using numerical calculation of Caputo derivative and predictor-corrector algorithm as it is shown in Fig.10. The calculation has been done for the following parameters: $\alpha = 8 \cdot 10^{-7}$, $Uo = 8mV$, $p = 1.2$, $T = 0.004\,s$, $\gamma = 3 \cdot 10^{-3}$

3.4 Iterative learning feedback control algorithms of $PI^{\alpha}D^{\beta}$ type in process control systems

In classical control theory, state feedback and output feedback are two important techniques in system control. Also, in recent years, there has been a great deal of study to overcome limitations of conventional controllers against uncertainty due to inaccurate modelling and/or parameter variations. As one of alternatives, the iterative learning control (ILC) method has been developed [25], where the concept of ILC was originally proposed by Arimoto [26] for accurate tracking of robot trajectories. Motivated by human learning, the basic idea of iterative learning control is to use information from previous executions of the task in order to improve performance from trial to trial in the sense that the tracking error is sequentially reduced. Therefore, iterative learning control requires less *a priori* knowledge about the controlled system in the controller design phase and also less computational effort than many other kinds of control. The basic strategy is to use an iteration of the form:

$$u_{i+1}(t) = f(u_i(t), e_i(t)), \quad _i\,(t) = y_d(t) - y_i(t)\,, \tag{48}$$

where $f(.,.)$ defines the learning algorithm and remains to be specified, $y_i(t)$ is the output at the ith operation resulting from the input $u_i(t)$ and $y_d(t)$ represents the desired output. The new control input $u_{i+1}(t)$ should make the system closer to the desired result in the next execution cycle. Here, it is suggested the learning control scheme comprises two types of control laws: a $PI^{\alpha}D^{\beta}$ feedback law and a feed-forward control law,[27]. In the feedback loop, the $PI^{\alpha}D^{\beta}$ controller provides stability of the system and keeps its state errors within uniform bounds. In the feed-forward path, a learning control rule/strategy is exploited to track the entire span of a reference input over a sequence of iterations i.e:

$$\boldsymbol{u}_{i+1}(t) = f(\boldsymbol{u}_i(t), \boldsymbol{e}_i(t), \boldsymbol{e}_i^{(\alpha)}(t))\,, 0 < \alpha \le 1 \tag{49}$$

where $u_i(t)$ is the control vector at the i-the iteration, while $e_i(t) = y_d(t) - y_i(t),$ is the tracking error signal between the desired signal $y_d(t)$ and the actual output trajectory one $y_i(t)$ at the i-the iteration. Here, $t \in [0, T]$, where T presents terminal time which is known and finite. Here, it is considered the non-integer (fractional) linear system described in the form of state space and output equations.

$$\begin{aligned} & \boldsymbol{x}_i^{(\alpha)}(t) = A\boldsymbol{x}_i(t) + B\boldsymbol{u}_i(t), \ \ x_i(0) = x_d(0) = 0, \quad 0 < \alpha < 1 \\ & \boldsymbol{y}_i(t) = C\boldsymbol{x}_i(t), \end{aligned} \tag{50}$$

where is $f(.,.)$ fractional order derivative, A,B, and C are matrices with appropriate dimensions. Here, it is suggested the learning control scheme which comprises two types of control laws: a PD^{α} feedback law and a feed-forward control law (Fig.11). In the feedback loop, the $PI^{\alpha}D^{\beta}$ controller provides stability of the system and keeps its state errors within uniform bounds.

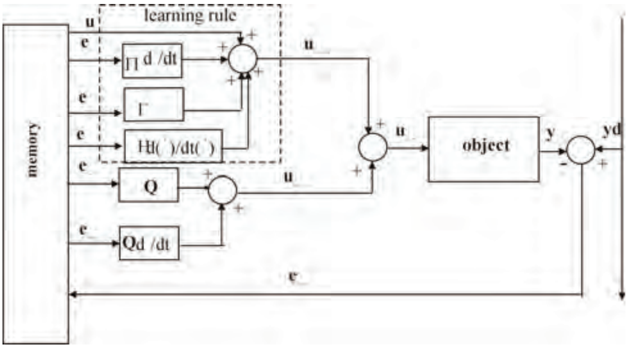


Figure11. Block diagram of $PI^{\alpha}D^{\beta}$ iterative learning feedback control for a LTI system

Here, it is introduced feedback control as follows:

$$u_{fbi+1}(t) = Q(D_t^{\alpha} e_{i+1}(t) + \gamma\, e_{i+1}(t)), \tag{51}$$

and in feed-forward it is proposed a new $PI^{\beta}D^{\alpha}$-type ILC updating law for given system such as:

$$u_{fi+1}(t) = u_i(t) + \Gamma e_i(t) + \Pi\, {}_0D_t^{\alpha} e_i(t) + \mathrm{H}\, {}_0D_t^{-\beta} e_i(t), \tag{52}$$

and

$$u_{i+1}(t) = u_{fi+1}(t) + u_{fbi+1}(t), \tag{53}$$

where Q, Γ, Π, H are gain matrices appropriate dimensions, where $\gamma > 0$ is real constant; $u_{fb}(t)$ the feedback control input, $u_f(t)$ the feed-forward input; $u(t)$ the value of the function at time t . A sufficient condition for convergence of a proposed feedback ILC is given by the main theorem and proved as follows.

Main theorem: Suppose that the update law Eqs.(51-53), is applied to the system (50) and the initial state at each iteration satisfies (50). If matrices Π, Q , exist such that

$$\left\| [I - \Pi CB][I - D] \right\| \le \rho < 1, \tag{54}$$

then, when $i \to \infty$ the bounds of the tracking errors $\|x_d(t) - x_i(t)\|, \|y_d(t) - y_i(t)\|,\ \|u_d(t) - u_i(t)\|,$ converge asymptotically to zero.

3.5. Finite time stability analysis of linear autonomous fractional order systems with delayed state –example PD^{α} fractional control of robotic time delay systems

The problem of time delay system has been discussed over many years and time delay is very often encountered in different technical systems. The existence of pure time delay, regardless of its presence in a control and/or state, may cause undesirable system transient response, or generally, even an instability. Here, another approach is presented, i.e system stability from the non-Lyapunov point of view is considered. In practice one is not only interested in system stability (e.g. in the sense of Lypunov), but also in bounds of system trajectories. A system could be stable but still completely useless because it possesses undesirable transient performances. Thus, it may be useful to consider the stability of such systems with respect to certain subsets of state-space which are defined a priori in a given problem. Besides that, it is of particular significance to concern the behavior of dynamical systems only over a finite time interval. Recently, there have been some advances in control theory of fractional differential systems for stability questions. However, for fractional order dynamic systems, it is difficult to evaluate the stability by simply examining its characteristic equation either by finding its dominant roots or by using other algebraic methods. The problem of sufficient conditions is examined that enable system trajectories to stay within the a priori given sets for the particular class of linear fractional order time-delay systems in state space form. A linear, ordinary, multivariable time-delay system can be represented by differential equation:

$$\frac{d\mathbf{x}(t)}{dt} = A_0\,\mathbf{x}(t) + A_1\,\mathbf{x}(t-\tau), \qquad (55)$$

and with associated function of initial state:

$$\mathbf{x}(t) = \psi_x(t), \quad -\tau \le t \le 0, \qquad (56)$$

or

$$\|\psi\|_C = \max_{-\tau \le \theta \le 0} \|\psi(\theta)\| \qquad (57)$$

where $\tau > 0$ is a pure time delay. Dynamical behavior of an autonomous system (55) is defined over time interval $J = \{t_o, t_o + T\}$, . Time invariant sets, used as bounds of system trajectories, are assumed to be open, connected and bounded. Let index "ε" stands for the set of all allowable states of system and index "δ" for the set of all initial states of the system, such that the set $S_\delta \subseteq S_\varepsilon$,and $S_\rho = \left\{x : \|x(t)\|_Q^2 < \rho\right\}$, where Q is assumed to be symmetric, positive definite, real matrix. It is assumed that the usual smoothness conditions are present so that there are no difficulties with questions of existence, uniqueness, and continuity of solutions with respect to initial data. Here, it presented a result of sufficient conditions that enable system trajectories to stay within the a priori given sets for the particular class of linear autonomous fractional order time-delay systems.System given by (55) satisfying initial condition (56) is finite stable w.r.t $\{t_o, J, \delta, \varepsilon, \tau\}$, $\delta < \varepsilon$ if and only if:

$$\|\psi_x\|_C < \delta \tag{58}$$

implies:

$$\|\mathbf{x}(t)\| < \varepsilon \quad , \qquad \forall t \in J \tag{59}$$

where δ is a real positive number $\varepsilon \in R$, $\delta < \varepsilon$.

Here, it is considered a class of fractional linear autonomous system with time delay described by the state space equation:

$$\frac{d^{\alpha}\mathbf{x}(t)}{dt^{\alpha}} = A_0\mathbf{x}(t) + A_1\mathbf{x}(t-\tau) \tag{60}$$

with associated function of initial state (57), where it is discussed the case $0 < \alpha < 1$.

Main theorem [28]:

A) Autonomous system given by (60) satisfying initial condition (57) is finite time stable *w.r.t.* $\{\delta, \varepsilon, \tau, t_o, J,\}$, $\delta < \varepsilon$, if the following condition is satisfied:

$$\left[1 + \frac{\sigma_{\max}^{A}\left(t-t_0\right)^{\alpha}}{\Gamma\left(\alpha+1\right)}\right] \cdot e^{\frac{\sigma_{\max}^{A}(t-t_0)^{\alpha}}{\Gamma(\alpha+1)}} \le \varepsilon / \delta, \qquad \forall t \in J. \tag{61}$$

where $\sigma_{\max}(.)$ being the largest singular value of matrix (.), namely:

$$\sigma_{\max}^{A} = \sigma_{\max}\left(A_0\right) + \sigma_{\max}\left(A_1\right), \tag{62}$$

Here, particular attention is paid to the finite time stability of robotic system *Newcastle robot* where a time delay appears in PD^{α} fractional control system,[29].The equation of motion of Newcastle robot with one degree of freedom in case of PD^{α} controller is:

$$\begin{aligned} & m\ddot{q}(t) + c\dot{q}(t) + kq(t) = \\ & = Q_d(t) + K_p\left[q_d(t-\tau) - q(t-\tau)\right] + K_d\left[q_d^{(\alpha)}(t-\tau) - q^{(\alpha)}(t-\tau)\right] \end{aligned} \tag{63}$$

For the small $y = q_d - q$ perturbation and after linearization leads to the linear time delay-differential equation as follows:

$$\ddot{y}(t) + 2\beta\dot{y}(t) + \omega^2 y(t) = k_p y(t-\tau) + k_d y^{(\alpha)}(t-\tau) \tag{64}$$

So,one may convert some linear equations with commensurate multiple fractional derivatives into linear system of fractional differential equations of low order, one can obtain:

$$D_t^{1/2}\boldsymbol{x}(t)=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -\omega^2 & 0 & -2\beta & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\\ x_4(t)\end{bmatrix}+\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ k_P & k_D & 0 & 0\end{bmatrix}\begin{bmatrix}x_1(t-\tau)\\ x_2(t-\tau)\\ x_3(t-\tau)\\ x_4(t-\tau)\end{bmatrix} \quad (65)$$

In that way using results of previous theorem, one can easily check stability condition for this system. Besides, it is also established new stability result for the particular class of nonlinear perturbed autonomous fractional order time-delay systems described by the state space equation,[30],[31]:

$$\frac{d^{\alpha}\mathbf{x}(t)}{dt^{\alpha}}=\left(\mathbf{A}_0+\Delta\mathbf{A}_0\right)\mathbf{x}(t)+\left(\mathbf{A}_1+\Delta\mathbf{A}_1\right)\mathbf{x}(t-\tau)+\mathbf{f}_0\left(\mathbf{x}(t)\right), \quad (66)$$

$$\left\|\mathbf{f}_0(\mathbf{x}(t))\right\|\le c_0\left\|\mathbf{x}(t)\right\|,\quad t\in\left[0,\infty\right) \quad (67)$$

with the initial functions (57)of the system and vector functions f_0 satisfied (67).

Main theorem: Nonlinear autonomous system given by (66) satisfying initial condition (57) and (67) is finite time stable w.r.t. $\{\delta,\varepsilon,t_o,J,\},\quad \delta<\varepsilon$, if the following condition is satisfied:

$$\left(1+\frac{\mu_p(t-t_0)^{\alpha}}{\Gamma(\alpha+1)}\right)e^{\frac{\mu_p(t-t_0)^{\alpha}}{\Gamma(\alpha+1)}}\le\varepsilon/\delta,\ \forall t\in J \quad (68)$$

where

$$\mu_{Aoco}=\sigma_{Ao}+\gamma_{\Delta A_0}+c_0,$$
$$\sigma_{A1\Delta}=\sigma_{A1}+\gamma_{\Delta A_1},\ \mu_p=\mu_{Aoco}+\sigma_{A1\Delta} \quad (69)$$

4. CONCLUSION

Proposed synergy approach allows resolving redundancy control problem i.e. actuator redundancy, in the framework of optimal control problem which it is solved by Pontryagin`s maximum principle._Also, modeling and resolving kinematic redundancy of (bio)mechanical/robotically system in synergy like fashion, can be achieved using optimization law with suitable kinematic and dynamic criteria which are the function of generalized coordinates, velocities, accelerations and control vectors, respectively. Besides that, model of (bio)mechanical system may be obtained using another biological concept called distributed positioning (DP) which is based on the inertial properties and actuation of joints of considered mechanical system. Also, they are presented other biological principles such as: *principle of minimum interaction* which takes a main role in hierarchical structure of control and *self-adjusting principle* (introduce local positive/negative feedback on control with great amplifying), which allows efficiently realization of control based on *iterative natural learning*. In the second part of this paper, newly, recently obtained results of author in fields of stability, electroviscoelasticity, and control theory which are based on using fractional calculus are presented. First of them, it is an example within a new theory of electroviscoelasticity, which describes the behavior of electrified liquid-liquid interfaces in fine dispersed systems, and is based on a new constitutive model of liquids: fractional order model -generalized the Van der Pol equation. Also, a new algorithms for fractional iterative learning control (ILC), a $PI^{\alpha}D^{\beta}$ types are proposed for fractional time (delay) systems are also presented. At last, new stability test procedure (finite time and practical stability) is shown for (non)linear (non)homogeneous time-invariant fractional order time delay systems where sufficient conditions of this kind of stability are derived.

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БИОЛОШКИ ИНСПИРИСАНО УПРАВЉАЊЕ И МОДЕЛИРАЊЕ (БИ)РОБОТСКИХ СИСТЕМА И НЕКЕ ПРИМЕНЕ ФРАКЦИОНОГ РАЧУНА У МЕХАНИЦИ

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Апстракт: *У овом раду, презентоване су примене биолошки инспирисаног моделирања и управљања (био)механичким (не)редундантним механизмима, као и новодобијени резултати аутора у области примењене механике који су засновани на примени рачуна нецелобројног реда. Прво, предложено је коришћење биолошког аналогона-синергије захваљујући постојању непроменљивих одлика у извршавању функционалних покрета. Друго, модел (био)механичког система може се добити применом другог биолошког концепта познатим под називом дистрибуирано позиционирање (ДП), који је заснован на инерцијалним својства и покретању зглобова разматраног механичког система. Такође,предлаже се коришћење других биолошких принципа као што су: принцип минималне интеракције, који има главну улогу у хијерархијској структури управљања и принцип самоподешавања (уводи локалне позитивну/негативну повратну спрегу у управљачкој петљи и то са великим појачањем), који омогућава ефикасно остваривање управљања на бази итеративног природног учења. Такође, нови, недавно публиковани резултати аутора су такође представљени у области стабилности, електро-вискоеластичности и теорији управљања а који су засновани на коришћењу рачуна нецелобројног реда.*

Кључне рчи: *биолошки инспирисани системи, алгоритми управљања, фракциони рачун, стабилност.*

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