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Control System Modeling of Hydraulic Actuator With Compressible Fluid Flow

The paper treats Riemann's partial differential equations in the form (1), where variable μ is defined by relation (2). Corresponding boundary conditions are defined in various forms, such as boundary conditions of pressure and flow on the fixed and movable boundaries. Problem formulation is constructed in order to describe hydraulic actuator dynamics in a complete form, including its real flow and geometric characteristics. Special algorithm is generated and a corresponding computer package for simulation of the complete hydraulic actuator dynamics, including the existing wave effects, using the method of characteristics to obtain the desired problem solution. Results of computer simulation of hydraulic actuator dynamics are presented in 3-D diagrams.

Keywords: *hydraulic actuator, servo-valve, boundary conditions, method of characteristics.*

1. INTRODUCTION

Wave propagation effects can be evaluated by partial differential equations of continuity and momentum with additional fluid compressibility law for one dimensional flow[1,3,6]. The effect of fluid viscosity can be entered as a friction between fluid streamline and pipeline wall. Local viscous effects at control servo valve and flow inlet and outlet of actuator cylinder represented by corresponding pressure loss coefficients are involved as boundary conditions. Fast hydraulic actuator can be assumed as serial connected compressible fluid flows controlled by supply and return variable restrictors enclosed in the control servo-valve and separated by the actuator piston [5]. The solution of presented mathematical model is evaluated using the method of characteristics.

Corresponding computer package for actuator simulation enables it for the arbitrary states of function by using the same computational procedure. It also enables relatively high computational accuracy of pressure and velocity distribution. For fast actuator its state is characterized by strongly expressed wave traveling effects and high gradients of flow velocity and pressure changes along the fluid streamline. These effects are placed mostly in the source part of the actuator. The paper also presents the results of visualization of computer simulation of generalized fast hydraulic actuator.

2. MATHEMATICAL MODELLING OF ACTUATOR

Equations of continuity and momentum for one dimensional fluid flow including the effects of wall friction are presented in the form of Riemann's partial differential equations of small wave's propagation through compressible medium in addition with the corresponding boundary conditions.

By using the method of characteristics the corresponding system model is presented as a system of algebraic linear equations. Riemann's partial differential equations including the wall friction are given in the form:

$$u_t + uu_x = -c\mu_x + u^2\xi_x,$$
$$\mu_t + u\mu_x = -cu_x, \tag{1}$$

$$\mu = \int_{\rho_0}^{\rho} \sqrt{\frac{dp}{d\rho}}\frac{d\rho}{\rho}, \tag{2}$$

where u is the fluid velocity, p is fluid static pressure, ρ is fluid density, c is velocity of sound, ξ is coefficient of pressure loses, x is co-ordinate along streamline and time. The presented partial differential equations (1) can be written in the form of characteristics, where λ is the coefficient of pressure losses per unit length of streamline. The corresponding equations of characteristics are given in the following form:

$$\frac{\delta_+ P}{\delta t} = \frac{\delta_+}{\delta t}(\mu+u) = \left[\frac{\partial}{\partial t} + (u+c)\frac{\partial}{\partial x}\right](\mu+u) = \frac{1}{2}\lambda u^2 sign(u),$$
$$\frac{\delta_- Q}{\delta t} = \frac{\delta_-}{\delta t}(\mu-u) = \left[\frac{\partial}{\partial t} + (u-c)\frac{\partial}{\partial x}\right](\mu-u) = -\frac{1}{2}\lambda u^2 sign(u). \tag{3}$$

In the case of small fluid compressibility, where ρ_0 is fluid density for zero fluid static pressure and χ is fluid bulk module, fluid density ρ and variable μ can be presented in the linear mathematical form as follows:

$$\rho = \rho_0\left(1+\frac{p}{\chi}\right) \cong \rho_0, \mu = c\ln\left(1+\frac{p}{\chi}\right) \cong \frac{p}{c\rho_0}. \tag{4}$$

2.1 Boundary Conditions

Any direction change of actuator motion produces pressure discontinuity which is caused by the inversion of fluid flow between supply pipeline and both actuator

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chambers. Possible pressure drop or surge is also caused by geometric asymmetry of the servo valve. Complete actuator (points B through E'), connected into hydraulic system into points A (by supply pipeline A-B) and F' (by return pipeline E'-F'), is presented in Figure 1. Static pressure in supply and return pipelines is presented as p_s and p_0, respectively.

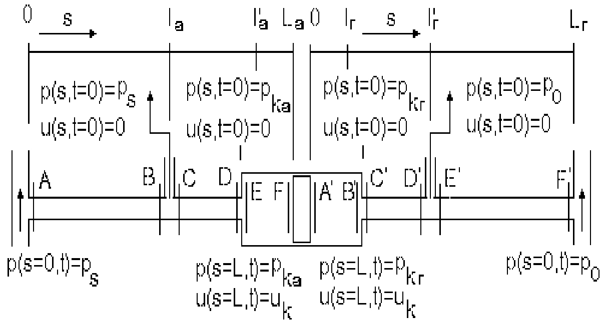

Figure 1. Complete actuator boundary conditions

A complete problem formulation includes the corresponding boundary and initial conditions for each of the assumed system sub domains. These domains correspond to the inlet and outlet pipelines, supply and return fluid flow sections between the control servo valve and the actuator piston. Wave effects of return flow can be assumed to be completely separated by the actuator piston. The corresponding boundary conditions are defined in the form of continuity and Bernoulli equations or piston momentum equation in addition to the pressure and velocity conditions. Left boundaries are determined with Q-characteristics, and right ones by P-characteristics. At the boundaries B-C, D-E, B'-C' and D'-E' the corresponding values of μ and u must be determined by interpolation. At points F and A' some of numeric integration methods must be applied for determination of corresponding pressure value on the movable piston surfaces. Boundary condition in the point A is caused by the performances and behavior of the power pump of hydraulic system and its connection with a relief valve. The corresponding boundary conditions can be defined in an alternate form as follows:

$$p_0 = p_{p\max};\, 0 \le u(t) \le V_{\max},$$
$$u_0 = V_{\max};\, p(t) \le p_{p\max}. \tag{5}$$

At points B-C the corresponding boundary conditions are defined as flow continuity and pressure loss caused by control servo valve throttle leakage. At points D-E the boundary conditions are of the same type as for the points B-C the difference being only in pressure loss caused by fluid viscous effects at the cylinder flow inlet. At point F boundary conditions of static pressure are defined as well as piston velocity, whose relation is defined by equivalent actuator stiffness. At point A' boundary conditions of static pressure are defined as well as velocity from the other side of the piston. At points B'-C' boundary conditions are of the same type as for the points D'-E', the difference being only in pressure loss caused by fluid viscous effects at cylinder flow outlet. In points D'-E' corresponding boundary conditions are defined as flow continuity, and pressure lose caused by control servo valve throttle leakage. As a boundary condition at point F' the nominal return pressure of hydraulic system can be assumed. Initial conditions are defined at each of the domains (A-B through E'-F'). Initial conditions of fluid static pressure are assumed to be supply static pressure (A-B) and cylinder static supply pressure (C-D and E-F).

2.2 Boundary conditions of inlet and outlet control servo- valve

Boundary conditions are given by the following Bernoulli's equation with additional equation of continuity in the form:

$$\frac{p_M}{\rho_M} - \left(\frac{A_M}{\eta_M d_M x_r}\right)^2 \frac{u_M^2}{2}\operatorname{sgn}(u_M) =$$
$$\frac{p_N}{\rho_N} + \left(\frac{A_N}{\eta_N d_N x_r}\right)^2 \frac{u_N^2}{2}\operatorname{sgn}(u_N),$$
$$\rho_M A_M u_M = \rho_N A_N u_N. \tag{6}$$

Given equations can be approximated by assuming the equivalence of pipeline area on the inlet and outlet section of the servo valve, and equivalence of fluid velocity caused by the change of fluid density on the inlet and outlet servo-valve section:

$$\rho_M = \rho_0\left(1 + \frac{p_M}{\chi}\right) \cong \rho_0,$$
$$\rho_N = \rho_0\left(1 + \frac{p_N}{\chi}\right) \cong \rho_0,$$
$$A_M = A_N, u_M = u_N. \tag{7}$$

2.3 Boundary conditions on the fluid inlet and outlet of actuator chamber

Previous relations for the actuator servo-valve can be used to determine the mentioned boundary conditions in the following form:

$$\frac{p_M}{\rho_M} - \frac{p_N}{\rho_N} \cong \frac{p_M}{\rho_0} - \frac{p_N}{\rho_0} =$$
$$\eta_0\left(\frac{u_M^2}{2}\operatorname{sgn}(u_M) + \frac{u_N^2}{2}\operatorname{sgn}(u_N)\right) =$$
$$\eta_0(\operatorname{sgn}(u_N))\left[1 + \left(\frac{A_N}{A_M}\right)^2\right]\frac{u_N^2}{2}\operatorname{sgn}(u_N),$$
$$A_M u_M \cong A_N u_N, \tag{8}$$

where η_0 is the corresponding coefficient of local pressure losses at these points of streamline.

2.4 Boundary conditions of the actuator piston

Equations of piston equilibrium are given in the form:

$$p_M = p_N + \frac{k_a}{A_k}x_k + \frac{\delta_k}{A_k}u_k + \frac{m_k}{A_k}\dot{u}_k,$$
$$u_k = u_M = u_N. \tag{9}$$

3. NON-DIMENSIONAL PROBLEM FORMULATION

It is more convenient to present the given equations in non dimensional form by introducing the following non dimensional system co-ordinates and variables:

$$\xi_x=\frac{x}{L},\xi_u=\frac{u}{V_{\max}},\xi_p=\frac{p}{p_{\max}},$$
$$T=\frac{L}{c},\tau=\frac{t}{T},k_u=\frac{V_{\max}}{c},$$
$$k_p=\frac{p_{\max}}{c\rho_0 V_{\max}},k_\lambda=\frac{1}{2}\lambda k_u L, \qquad (10)$$

where $V_{\max}$ is maximal fluid velocity along the streamline caused by the system pump, $p_{\max}$ is maximal fluid nominal static pressure and L is total length of the assumed streamline. Non-dimensional form of equations of characteristics can be presented as:

$$\frac{\delta_+ P_\xi}{\delta t}=\left[\frac{\partial}{\partial\tau}+\left(k_u\xi_u+1\right)\frac{\partial}{\partial\xi_x}\right]\left(k_p\xi_p+\xi_u\right)=k_\lambda\xi_u{}^2\operatorname{sgn}\xi_u,$$
$$\frac{\delta_- Q_\xi}{\delta t}=\left[\frac{\partial}{\partial\tau}+\left(k_u\xi_u-1\right)\frac{\partial}{\partial\xi_x}\right]\left(k_p\xi_p-\xi_u\right)=-k_\lambda\xi_u{}^2\operatorname{sgn}\xi_u \qquad (11)$$

with the corresponding non dimensional characteristics:

$$\frac{d\xi_x{}^+}{d\tau}=k_u\xi_u+1,$$
$$\frac{d\xi_x{}^-}{d\tau}=k_u\xi_u-1. \qquad (12)$$

Co-ordinates of cross point R and corresponding values of non dimensional velocity x_u and parameter x_m are given by the following relations:

$$\tau_R=\frac{\xi_x^B-\xi_x^A+(k_u\xi_u^A+1)\tau_A-(k_u\xi_u^B-1)\tau_B}{2+k_u\xi_u^A-k_u\xi_u^B},$$
$$\xi_x^R=\xi_x^A+(k_u\xi_u^A+1)(\tau_R-\tau_A),$$
$$\xi_\mu^R=\frac{\xi_\mu^A+\xi_\mu^B+\xi_u^A-\xi_u^B+k_\lambda(\xi_u^A)^2(\tau_R-\tau_A)}{2}+$$
$$+\frac{-k_\lambda(\xi_u^B)^2(\tau_R-\tau_B)}{2}$$
$$\xi_u^R=\frac{\xi_\mu^A-\xi_\mu^B+\xi_u^A+\xi_u^B+k_\lambda(\xi_u^A)^2(\tau_R-\tau_A)}{2}+$$
$$+\frac{k_\lambda(\xi_u^B)^2(\tau_R-\tau_B)}{2}. \qquad (13)$$

For the left boundary non dimensional variables are defined along Q characteristics only. For the left boundary static pressure condition, the corresponding parameters of non dimensional time τ and velocity ξ_u are defined in the form:

$$\tau_R=\tau_B+\frac{\xi_x^R-\xi_x^B}{k_u\xi_u^B-1},$$
$$\xi_u^R=\xi_\mu^R-\xi_\mu^B+\xi_u^B+k_\lambda(\xi_u^B)^2(\tau_R-\tau_B). \qquad (14)$$

For the left boundary velocity condition, the corresponding parameters of non dimensional time τ and parameter ξ_μ are defined in the form:

$$\tau_R=\tau_B+\frac{\xi_x^R-\xi_x^B}{k_u\xi_u^B-1},$$
$$\xi_\mu^R=\xi_\mu^B+\xi_u^R-\xi_u^B-k_\lambda(\xi_u^B)^2(\tau_R-\tau_B). \qquad (15)$$

For the right boundary non dimensional variables are defined along P characteristics only. For the right boundary static pressure condition, the corresponding parameters of non dimensional time τ and velocity ξ_u are defined in the form:

$$\tau_R=\tau_A+\frac{\xi_x^R-\xi_x^A}{k_u\xi_u^A+1},$$
$$\xi_u^R=-\xi_\mu^R+\xi_\mu^A+\xi_u^A+k_\lambda(\xi_u^A)^2(\tau_R-\tau_A). \qquad (16)$$

For the right boundary velocity condition, the corresponding parameters of non dimensional time τ and parameter ξ_μ are defined in the form:

$$\tau_R=\tau_A+\frac{\xi_x^R-\xi_x^A}{k_u\xi_u^A+1},$$
$$\xi_\mu^R=-\xi_u^R+\xi_\mu^A+\xi_u^A+k_\lambda(\xi_u^A)^2(\tau_R-\tau_A). \qquad (17)$$

Combined boundary conditions for the left and right side are presented at the points of inlet and outlet hydraulic servo-valve, inlet and outlet of the actuator chamber and piston position. This case can be formulated for the fixed and movable boundaries.

For the fixed boundaries the corresponding conditions of characteristics can be solved as non-dimensional time co-ordinates of generalized points B and E:

$$\tau_B=\tau_A+\frac{\xi_x^B-\xi_x^A}{k_u\xi_u^A+1},\tau_E=\tau_D+\frac{\xi_x^E-\xi_x^D}{k_u\xi_u^D-1}. \qquad (18)$$

With additional following system of two linear algebraic equations:

$$\xi_\mu^B+\xi_u^B=\xi_\mu^A+\xi_u^A+k_\lambda(\xi_u^A)^2(\tau_B-\tau_A),$$
$$\xi_\mu^D-\xi_u^D=\xi_\mu^E-\xi_u^E-k_\lambda(\xi_u^E)^2(\tau_D-\tau_E). \qquad (19)$$

The corresponding two boundary conditions have to be added to previous equations (19), because the generalized points B and E have different non-dimensional time co-ordinates. They must be interpolated by introducing the following relations:

$$\xi_\mu^B=\xi_\mu^G+(\xi_\mu^M-\xi_\mu^G)\frac{\tau_B-\tau_G}{\tau_M-\tau_G},$$
$$\xi_\mu^D=\xi_\mu^H+(\xi_\mu^N-\xi_\mu^H)\frac{\tau_D-\tau_H}{\tau_N-\tau_H},$$
$$\xi_u^B=\xi_u^G+(\xi_u^M-\xi_u^G)\frac{\tau_B-\tau_G}{\tau_M-\tau_G},$$
$$\xi_u^D=\xi_u^H+(\xi_u^N-\xi_u^H)\frac{\tau_D-\tau_H}{\tau_N-\tau_H}. \qquad (20)$$

Interpolation point can be assumed by the following relation:

$$\tau_M = \tau_N = \max(\tau_B, \tau_D). \tag{21}$$

By changing the presented relations we can give the following algebraic equations:

$$c_M = \xi_\mu^M + \xi_u^M = \xi_\mu^G + \xi_u^G +$$
$$+\frac{\tau_M - \tau_G}{\tau_B - \tau_G}\left[-\xi_\mu^G - \xi_u^G + \xi_\mu^A + \xi_u^A + k_\lambda (\xi_u^A)^2 (\tau_B - \tau_A)\right],$$
$$c_N = \xi_\mu^N - \xi_u^N = \xi_\mu^H - \xi_u^H +$$
$$+\frac{\tau_N - \tau_H}{\tau_D - \tau_H}\left[-\xi_\mu^H + \xi_u^H + \xi_\mu^E - \xi_u^E - k_\lambda (\xi_u^E)^2 (\tau_D - \tau_E)\right]. \tag{22}$$

If the boundaries are in motion along the streamline with the corresponding non-dimensional velocity $u_{uk}(t)$, then we can write the following algebraic relations:

$$\xi_{xk}(\tau_{M=N}) = \xi_{xk}(\tau_{M_0=N_0}) +$$
$$+\xi_{uk}(\tau_{M_0=N_0})(\tau_{M=N} - \tau_{M_0=N_0}),$$
$$\xi_x^B = \xi_{xk}(\tau_{M_0=N_0}) + \xi_{uk}(\tau_{M_0=N_0})(\tau_B - \tau_{M_0=N_0}),$$
$$\xi_x^D = \xi_{xk}(\tau_{M_0=N_0}) + \xi_{uk}(\tau_{M_0=N_0})(\tau_D - \tau_{M_0=N_0}). \tag{23}$$

Where index 0 denotes previous position of the movable boundary. By changing the relations (23) into the relations (18) we can give:

$$\tau_B = \frac{-\xi_x^A + \xi_{xk}(\tau_{M_0=N_0}) + (k_u \xi_u^A + 1)\tau_A}{k_u \xi_u^A + 1 - \xi_{uk}(\tau_{M_0=N_0})} +$$
$$+\frac{-\xi_{uk}(\tau_{M_0=N_0})\tau_{M_0=N_0}}{k_u \xi_u^A + 1 - \xi_{uk}(\tau_{M_0=N_0})},$$
$$\tau_E = \frac{-\xi_x^E + \xi_{xk}(\tau_{M_0=N_0}) + (k_u \xi_u^D - 1)\tau_D}{k_u \xi_u^D - 1 - \xi_{uk}(\tau_{M_0=N_0})} +$$
$$+\frac{-\xi_{uk}(\tau_{M_0=N_0})\tau_{M_0=N_0}}{k_u \xi_u^D - 1 - \xi_{uk}(\tau_{M_0=N_0})}. \tag{24}$$

In other piston position procedure is the same as for the fixed boundaries. To these equations have to be added boundary conditions which exist at points M=N. Previous relations (6) in non dimensional form can be written as:

$$\left(\xi_p^M - \xi_p^N\right)\theta^2 = \frac{1}{2\left(\xi_{xr}\right)^2}\left[(\xi_u^M)^2 \operatorname{sgn}(\xi_u^M) +\right.$$
$$\left.+(\xi_u^N)^2 \operatorname{sgn}(\xi_u^N)\right], \xi_u^M = \xi_u^N. \tag{25}$$

By changing c_M and c_N from relations (22), we can give:

$$\xi_u^M = \frac{\theta^2 (\xi_{xr})^2}{\operatorname{sgn}(\xi_u^M)}\left[-\frac{1}{k_p} + \sqrt{\frac{1}{k_p^2} + \frac{(c_M - c_N)}{\theta^2 k_p (\xi_{xr})^2}\operatorname{sgn}(\xi_u^M)}\right]. \tag{26}$$

In non-dimensional form follows from relations (8):

$$\xi_p^M - \xi_p^N = k_{rg}\theta_A (\xi_u^N)^2 \operatorname{sgn}(\xi_u^N), \xi_u^M = \frac{A_N}{A_M}\xi_u^N, \tag{27}$$

where:

$$k_{rg} = \frac{\rho_0 \eta_0 V_{\max}^2}{2 p_{p\max}}, \phi_A = 1 + \frac{A_N}{A_M}, \theta_A = 1 + \left(\frac{A_N}{A_M}\right)^2. \tag{28}$$

From the previous relations we can give the following solution:

$$\xi_u^N = \frac{1}{2k_{rg}\theta_A}\left[-\frac{\phi_A}{k_p} \pm \sqrt{\left(\frac{\phi_A}{k_p}\right)^2 + 4k_{rg}\theta_A \frac{c_M - c_N}{k_p}}\right]. \tag{29}$$

By introducing the equivalent piston position co-ordinate previous relations (9) becomes in the following form:

$$\xi_p^M - \xi_p^N = k_{xx}\xi_x^M + k_{xu}\xi_u^M + k_{xup}\dot{\xi}_u^M, \xi_u^M = \xi_u^N. \tag{30}$$

Where the corresponding coefficients are:

$$k_{xx} = \frac{k_a H_{kl}}{A_k p_{p\max}} = \frac{k_a L \theta_{kl}}{A_k p_{p\max}},$$
$$k_{xu} = \frac{\delta_k V_{\max}}{A_k p_{\max}} = \frac{\delta_k V_{k\max} a_\theta}{A_k p_{p\max}}, k_{xup} = \frac{m_k c V_{\max}}{A_k p_{\max} L}. \tag{31}$$

The piston position can be calculated from the following relation in non dimensional form:

$$\xi_x^M = \xi_x^G + k_u \xi_u^G (\tau_M - \tau_G). \tag{32}$$

If non dimensional piston acceleration is assumed as:

$$\dot{\xi}_u^M = \left.\frac{d\xi_u}{d\tau}\right|_M = \frac{\xi_u^M - \xi_u^G}{\tau_M - \tau_G}. \tag{33}$$

Then there follows the final form of the previous equations (9) written as:

$$\frac{c_M - c_N}{k_p} - k_{xx}\left[\xi_x^G + k_u \xi_u^G (\tau_M - \tau_G)\right] + k_{xup}\frac{\xi_u^G}{\tau_M - \tau_G} =$$
$$\left(\frac{2}{k_p} + k_{uu} + \frac{k_{xup}}{\tau_M - \tau_G}\right)\xi_u^M. \tag{34}$$

4. SIMULATION MODEL PERFORMANCES

Actual hydraulic actuators have the frequency range upon 100 Hz in correlation with the conventional one, whose frequency is limited to 10 - 20 Hz. The presented results of computer simulation prove that wave effects are of influence for the faster types of actuators [4].

For compact actuators, whose frequency range is significantly greater than 100 Hz, we must include wave effects with its full influence. The main difference is the fact that wave reflection and corresponding velocity and pressure gradients are of the same order as the frequency range of the actuator input servo-valve control throttle. In that case, wave propagation effect takes the main role in system behavior. Inclusion of wave effects makes actuator mathematical model more complicated. Boundary conditions change its relatively simplified formulation for the conventional models into a very difficult procedure, commonly presented in iterative form including the interaction of two-connected boundaries with coupled parameters on its both sides, which can be also movable with arbitrary velocity and position.

In the paper [2] is presented simplified one-sided actuator approximation whose outlet servo-valve part and reverse actuator chamber were neglected. This approximation enables simplified problem formulation because the effects of characteristics delay between inlet and outlet actuator servo-valve parts are not present. This approximation can be of interest, but the given results are not too qualitative. This fact is compensated with a relatively simplified procedure of feedback analysis and corresponding active control synthesis. In a complete system model formulation, we must solve efficiently the problem of reverse fluid flow in the return actuator part. The inversion of fluid flow between direct and reverse actuator modes is assumed as inversion of the points of inlet and outlet servo-valve parts. Compensation of characteristics time-delay needs an iterative procedure for calculation the corresponding parameters in the domain of characteristics time-delay between the points of inlet and outlet servo-valve zero throttles.

The density of nodal point distribution along the stream-line and simulation time is dependant of maximal velocity and pressure gradients. For usual actuator geometry, 200 points along the stream-line produce about 130.000 iterations for one second, or the total of 26 millions nodal points. Each step of computations has the same number of medium nodal points. The method of accuracy is very high because the maximal errors of velocity and pressure distribution are small. The corresponding differences between the results of one or two-step iterative procedure for the case of 10.000 Hz actuator is less than $1x10^{-5}$ of its maximal values. In accordance the assumed units ratio of total streamline length and the corresponding time step ratio of simulation loop, the problem of numeric stability and convergence is of interest. If time and coordinate discretization step equals 0,005, for simulation of one second of the actuator activity approximately $1.3x10^5$ time steps and loop iterations are needed. For that number of iterations numerical accuracy of calculation, spatially on the fixed and movable boundaries, is critical. For solving this problem, the corresponding procedure is established, expressed by relation (21), in order to reduce computational oscillations around the 'exact' solution.

In Figure 2 are presented diagrams of velocity and static pressure ratio distribution along streamline and during 52 units of time (2a and 2b), for 2.3 units of time (2c and 2d) and for 7.2 units of time (2e and 2f) by using the same steps of numerical integration. In Figures 2c and 2d the propagation of waves is visible in the starting period of the actuator function. In Figures 2e and 2f the effects of waves propagation are less visible, and in Figures 2a and 2b the effects are practically invisible. This is the proof that same computational procedure with equal step parameters can be used for actuator simulation during short or long time period without arbitrarily increasing the cumulative numerical integration errors.

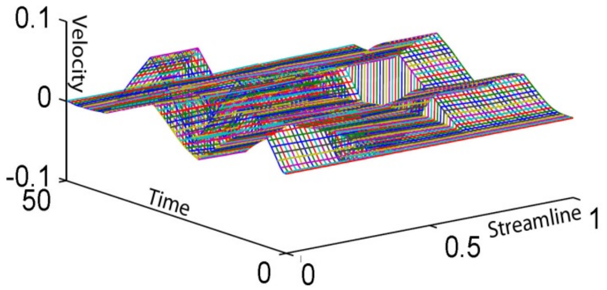

Figure 2a. Velocity distribution ratio

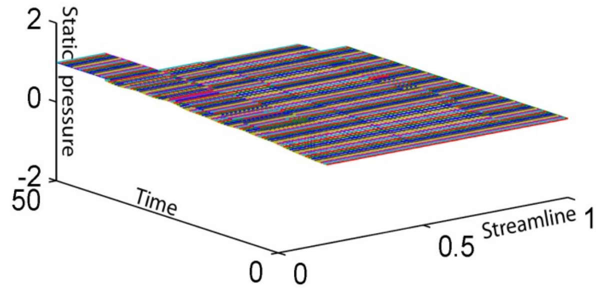

Figure 2b. Static pressure distribution ratio

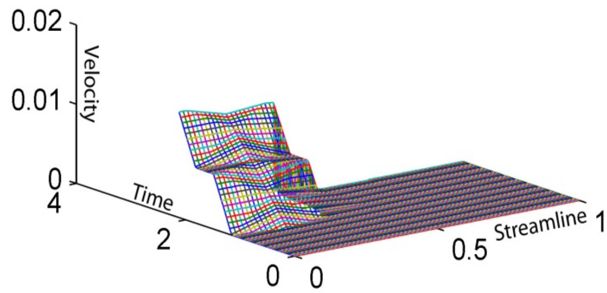

Figure 2c. Velocity distribution ratio

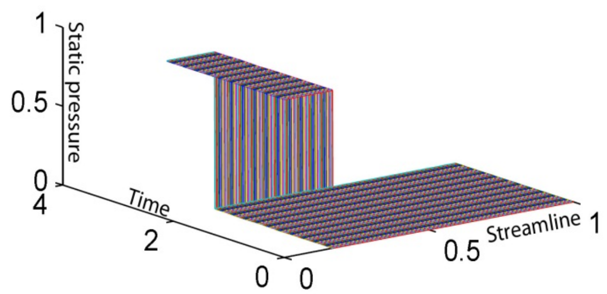

Figure 2d. Static pressure distribution ratio

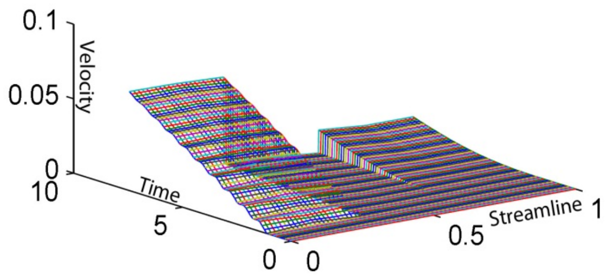

Figure 2e. Velocity distribution ratio

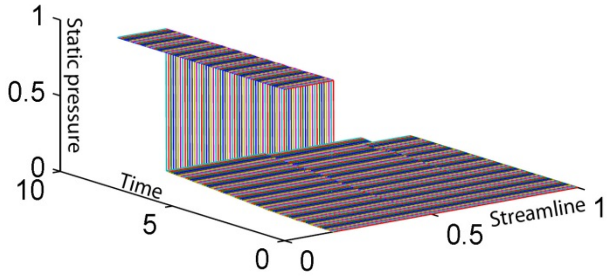

Figure 2f. Static pressure distribution ratio

5. CONCLUSION

Computer simulation of generalized fast hydraulic servo-actuator including effects of fluid compressibility introduces the additional wave propagation problem, which can be solved by some numerical methods. For the simulation purposes, is the method of characteristics is recommended for system modeling. The presented results of actuator simulation prove that the exposed system model and the corresponding computer package enables its simulation for arbitrary actuator configuration and states of function.

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МОДЕЛ УПРАВЉАЊА ХИДРАУЛИЧКИМ АКТУАТОРОМ СА ЕФЕКТИМА СТИШЉИВОСТИ

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У раду су третиране Риманове парцијалне диференцијалне једначине у облику израза (1) где је промењлива μ дефинисана релацијом (2). Одговарајући гранични услови дефинисани су у различитим облицима, као што су гранични услови притиска и протока и то на фиксним и на покретним границама. Проблем је формулисан са циљем да опише комплетну динамику хидрауличког актуатора, укључујући његов стварни проток и геометријске карактеристике. У ту сврху направљен је посебан алгоритам и одговарајући рачунарски пакет за симулацију комплетне динамике хидрауличког актуатора, укључујући и присутне ефекте таласа. За решење проблема коришћена је метода карактеристика. Резултати рачунарске симулације динамике хидрауличког актуатора представљени су 3-Д дијаграмима.