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Fractional Order Control of a Robot System Driven by DC Motors

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This paper presents the new algorithms of PID control based on fractional calculus (FC) and an optimal procedure in the position control of a 3 DOF robotic system driven by DC motors. The objective of this work is to find out the optimal settings for a fractional $PI^{\alpha}D^{\beta}$ controller in order to fulfill the proposed design specifications for the closed-loop system, taking advantage of the fractional orders, α and β. The effectiveness of the suggested optimal fractional PID control is demonstrated with a suitable robot with three degrees of freedom as an illustrative example. In addition, this paper proposes a robust fractional-order sliding mode control of a 3-DOF robot system driven by DC motors. Primarily, a conventional sliding mode controller based on a PD^{α} sliding surface is designed. Numerical simulations have been carried out to show the proposed control system's robustness properties as well as the significance of the proposed control which resulted in reducing output oscillations (*chattering-free*) of the given robot. The simulations also include a comparison of the fractional-order PD^{α} sliding mode controller with the standard PD sliding-mode controller.

Key words: **robots, DC motor, robust control, control algorithm, PID algorithm, fractional order control, vibration setting.**

Introduction

FRACTIONAL calculus (FC) is a mathematical topic with more than 300 years old history, but its application to physics and engineering has been reported only in the recent years. The fractional integro-differential operators are a generalization of integration and derivation to non-integer order (fractional) operators. There is an increasing number of studies related to the application of fractional controllers in many areas of science and engineering, where especially fractional-order systems are of interest for both modeling and controller design purposes.

In the classical control theory, state feedback and output feedback are two important techniques in the system control. Due to its functional simplicity and performance robustness, the PID controller has been widely used in the process industries. The design and the tuning of PID controllers have been a large research area ever since Ziegler and Nichols presented their methods in 1942 [1]. Specifications, stability, design, applications and performance of the PID controller have been widely treated since then [2,3]. On the other hand, fractional calculus has the potential to accomplish what integer-order calculus cannot. Moreover, there is an increasing number of studies related to the application of fractional controllers in many areas of science and engineering, where especially fractional-order systems are of interest for both modeling and controller design purposes. It has been found that in interdisciplinary fields, many systems can be described by the fractional differential equations i.e. in the fields of continuous-time modeling, fractional derivatives have proved useful in linear viscoelasticity, acoustics, rheology, polymeric chemistry, biophysics robotics, control theory of dynamical systems, electrical engineering, bioengineering, etc.[4-6].

However, in recent years, the emergence of effective methods of solving differentiation and integration of noninteger order equations makes fractional-order systems more and more attractive for the systems control community. The fractional PD^{α} controller [7], the fractional PI^{α} controller [8], the fractional controller $PI^{\beta}D^{\alpha}$ [6], the CRONE controllers [9,10], and the fractional lead-lag compensator [11] are some of the well-known fractional order controllers. In this paper, we suggest and obtain new algorithms of PID control based on fractional calculus (FC) in the control of a robotic system driven by DC motors. The objective of this work is to find out suitable settings for a fractional $PI^{\alpha}D^{\beta}$ controller in order to achieve a better transient response as well as to fulfill proposed design specifications for the closed-loop system, taking advantage of the fractional orders, α and β.

In addition, a sliding-mode controller (SMC) is a powerful tool to robustly control incompletely modeled or uncertain systems [12] which has many attractive features such as fast response, good transient response and asymptotic stability. The conventional SMC law guarantees robustness of the sliding manifold if the model uncertainties are bounded with known bounds and comply with the matching condition. Once the system states hit the sliding surface, they stay there, and the equivalent system dynamics is predefined with a reduced order. However, an SMC has some disadvantages related to well-known chattering in the system. The main causes of chattering due to the discontinuous control action are neglected high order control plant dynamics, actuator dynamics, sensor noise, and discrete-time implementation in computer controlled applications. Chattering is undesirable in the control of mechanical systems, since it causes excessive control action leading to increased wear on the actuators and to the

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excitation of the high order nonmodeled dynamics. Consequently, the demanded performance cannot be achieved, or even worse – the mechanical parts of the servo system can be destroyed. Therefore, chattering must be eliminated from the SMC system. Since chattering is caused by the discontinuous control, there exist several techniques to reduce a high switching amplitude [13]. Recently, a fractional-order sliding mode control technique by Monje et al. [14] has been successfully applied for a robot manipulator as well as in [15], or in [16] the results of combining the sliding mode control and the fractional order derivative are considered in two different approaches. In this paper, we suggest and obtain a chattering-free fractional PD^{α} sliding-mode controller in the control of a robotic system driven by DC motors. In that way, one has used a fractional-order sliding surface to design a fractional-order sliding mode controller for a chattering-free tracking robot system.

Preliminaries on the fractional calculus

The fractional integro-differential operators-(fractional calculus) are a generalization of integration and derivation to non-integer order (fractional) operators. The idea of FC has been known since the development of the regular calculus, with the first reference probably being associated with Leibniz and Marquis de l'Hopital in 1695. Both Leibniz and L`Hospital, aware of ordinary calculus, raised the question of a noninteger differentiation (order $n = 1/2$) for simple functions. It had always attracted the interest of many famous ancient mathematicians, including L'Hospital, Leibniz, Liouville, Riemann, Grünward, and Letnikov [4-6]. In that way, the theory of fractional-order derivative was developed mainly in the 19th century. As a foundation of fractional geometry and fractional dynamics, the theory of FO, in particular, the theory of FC and FDEs and research of application have been developed rapidly in the world since 19th century. The modern epoch started in 1974 when a consistent formalism of the fractional calculus was developed by Oldham and Spanier[4], and later Podlubny [6]. The applications of FC are very wide nowadays, in rheology, viscoelasticity, acoustics, optics, chemical physics, robotics, control theory of dynamical systems, electrical engineering, bioengineering, etc. [4-12]. In fact, real world processes generally or most likely are fractional order systems. The main reason for the success of FC applications is that these new fractional-order models are more accurate than integer-order models, i.e. there are more degrees of freedom in the fractional order model. Furthermore, fractional derivatives provide an excellent instrument for the description of memory and hereditary properties of various materials and processes due to the existence of a "memory" term in a model. This memory term ensures the history and its impact on the present and future. A typical example of a non-integer (fractional) order system is the voltage-current relation of a semi-infinite lossy transmission line or diffusion of the heat through a semi-infinite solid, where heat flow is equal to the half-derivative of the temperature [4,6].

The modern epoch started in 1974 when a consistent formalism of the fractional calculus was developed by Oldham and Spanier [4]. The theory of FC is a well-adapted tool to the modeling of many physical phenomena, allowing the description to take into account the same peculiarities that classical integer-order models simply neglect. The main reason for the success of FC applications is that these new fractional-order models are more accurate than integer-order models and fractional derivatives provide an excellent instrument for the description of the memory and hereditary properties of various materials and processes due to the existence of a "memory" term in a model. There are today many different forms of fractional integral operators, ranging from divided-difference types to infinite-sum types, Riemann-Liouville fractional derivative, Grunwald - Letnikov fractional derivative, Caputo's, Weyl's and Erdely-Kober left and right fractional derivatives, etc, Kilbas *et al.*[48]. The three most frequently used definitions for the general fractional differintegral are: the Grunwald-Letnikov (GL) definition, the Riemann-Liouville (RL) and the Caputo definitions, [4-6]. The first one is the GL definition i.e Grunwald-[17] and Letnikov [18] developed an approach to fractional differentiation based on the definition

$$
\begin{aligned}
{}_{GL}D_x^{\alpha} f(x) &= \lim_{h\to 0}\frac{\left(\Delta_h^{\alpha} f(x)\right)}{h^{\alpha}},\\
\Delta_h^{\alpha} f(x) &= \sum_{0\le |j|<\infty} (-1)^{|j|}\binom{\alpha}{j} f(x-jh),\ h>0,
\end{aligned}
\tag{1}
$$

which is the left Grunwald-Letnikov (GL) derivative as a limit of a fractional order backward difference. Similarly, we have the right one as

$$
\begin{aligned}
{}_{GL}D_x^{\alpha} f(x) &= \lim_{h\to 0}\frac{\left(\Delta_{-h}^{\alpha} f(x)\right)}{h^{\alpha}},\\
\Delta_{-h}^{\alpha} f(x) &= \sum_{0\le |j|<\infty} (-1)^{|j|}\binom{\alpha}{j} f(x+jh),\ h<0,
\end{aligned}
\tag{2}
$$

As indicated above, the previous definition of GL is valid for $\alpha > 0$ (fractional derivative) and for $\alpha < 0$ (fractional integral) and, commonly, these two notions are grouped into one single operator called *differintegral*. The GL derivative and RL derivative are equivalent if the functions they act on are sufficiently smooth. For the generalized binomial coefficients calculation for $\alpha \in R$ and $k \in \mathbb{N}_0$ we can use the relation between Euler's Gamma function and factorial, defined as

$$
\begin{aligned}
\binom{\alpha}{j} &= \frac{\alpha!}{j!(\alpha-j)!} = \frac{\alpha(\alpha-1)\ldots(\alpha-j+1)}{j!} =\\
&= \frac{\Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)},\quad \binom{\alpha}{0}=1
\end{aligned}
\tag{3}
$$

If we consider $n = t - a/h$, where a is a real constant which expresses a limit value, one may write

$$
{}_{GL}D_{a,t}^{\alpha} f(t) = \lim_{h\to 0}\frac{1}{h^{\alpha}}\sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^{j}\binom{\alpha}{j} f(t-jh),
\tag{4}
$$

where $[x]$ means the integer part of x, a and t are the bounds of the operation for ${}_{GL}D_{a,t}^{\alpha} f(t)$. For the numerical calculation of fractional-order derivatives we can use the following relation (5) derived from the GL definition (4). This approach is based on the fact that for a wide class of functions, three definitions - GL, RL, and Caputo's - are equivalent. The relation to the explicit numerical approximation of the α-th derivative at the points *kh*, (k=1,2,...) has the following form, [6]

$$ {}_{((x-L)}D_x^{\pm\alpha} f(x) \approx h^{\mp\alpha} \sum_{j=0}^{N(x)} b_j^{(\pm\alpha)} f(x-jh) \quad (5) $$

where L is the "memory length", h is the step size of the calculation,

$$ N(t) = min\left\{\left[\frac{x}{h}\right],\left[\frac{L}{h}\right]\right\}, \quad (6) $$

$[x]$ is the integer part of x and $b_j^{(\pm\alpha)}$ is the binomial coefficient given by

$$ b_0^{(\pm\alpha)} = 1, \quad b_j^{(\pm\alpha)} = \left(1 - \frac{1\pm\alpha}{j}\right) b_{j-1}^{(\pm\alpha)} \quad (7) $$

For the expression of the Riemann-Liouville definition, we will consider the Riemann-Liouville n-fold integral for $n \in N, n > 0$ defined as

$$ \underbrace{\int_a^t \int_a^{t_n} \int_a^{t_{n-1}} \cdots \int_a^{t_3} \int_a^{t_2}}_{n-fold} f(t_1)\, dt_1 dt_2 \ldots dt_{n-1} dt_n = $$
$$ = \frac{1}{\Gamma(n)} \int_a^t (t-\tau)^{n-1} f(\tau)\, d\tau, \quad (8) $$

The fractional Riemann-Liouville integral of the order α for the function $f(t)$ for $\alpha, a \in R$ can be expressed as follows

$$ {}_{RL}I_a^{\alpha} f(t) \equiv {}_{RL}D_a^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} f(\tau)\, d\tau, \quad (9) $$

Here, $\Gamma(.)$ is the well known Euler's gamma function which is defined by the so-called *Euler integral of the second kind*:

$$ \Gamma(z) = \int_0^{\infty} e^{-t} t^{z-1} dt, \quad z \in \mathbb{C} \quad (10) $$

For this function the *reduction formula* holds, for $z \in \mathbb{C} \setminus \{0,-1,-2,-3,\ldots\}$:

$$ \Gamma(z+1) = z\Gamma(z), \ \Rightarrow \Gamma(n+1) = n(n-1)! = n!, \quad n \in \mathbb{N}_0 \quad (11) $$

The second important property of the gamma function is that it has simple poles at the points $z = -n, \ (n = 0,1,2,\ldots)$. Another important relationship for the gamma function is the Legendre formula:

$$ \Gamma(z)\Gamma(z+1/2) = \sqrt{\pi}\, 2^{2z-1} \Gamma(2z), \quad 2z \neq 0,-1,-2,\ldots \quad (12) $$

Taking $z = n + 1/2$ in the previous relation, one can obtain a set of particular values of the gamma function:

$$ \Gamma(n+1/2) = \frac{\sqrt{\pi}\,\Gamma(2n+1)}{2^{2n}\Gamma(n+1)} = \frac{\sqrt{\pi}(2n)!}{2^{2n} n!}, \quad (13) $$

For the case of $0 < \alpha < 1, t > 0$, and $f(t)$ being a a causal function of t, the fractional integral is presented as

$$ {}_{RL}D_a^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau, \quad 0 < \alpha < 1, \quad t > 0 \quad (14) $$

Moreover, the left Riemann-Liouville fractional integral and the right Riemann-Liouville fractional integral are defined respectively as

$$ {}_{RL}I_a^{\alpha} f(t) \equiv {}_{RL}D_a^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} f(\tau)\, d\tau, \quad (15) $$

$$ {}_{RL}I_b^{\alpha} f(t) \equiv {}_{RL}D_b^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_t^b (\tau-t)^{\alpha-1} f(\tau)\, d\tau, \quad (16) $$

where $\alpha > 0$, $n-1 < \alpha < n$. Furthermore, the *left Riemann-Liouville fractional derivative* is defined as

$$ {}_{RL}D_{a,t}^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t (t-\tau)^{n-\alpha-1} f(\tau)\, d\tau, \quad (17) $$

and the right Riemann-Liouville fractional derivative is defined as

$$ {}_{RL}D_{t,b}^{\alpha} f(t) = \frac{(-1)^n}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_t^b (\tau-t)^{n-\alpha-1} f(\tau)\, d\tau, \quad (18) $$

where $n-1 \leq \alpha < n$, a, b are the terminal points of the interval $[a,b]$, which can also be $-\infty, \infty$. Also, for the RL derivative, we have

$$ \lim_{\alpha \to (n-1)^+} {}_{a}^{RL}D_t^{\alpha} f(t) = \frac{d^{n-1} f(t)}{dt^{n-1}} \quad (19a) $$

and

$$ \lim_{\alpha \to n^-} {}_{a}^{RL}D_t^{\alpha} f(t) = \frac{d^n f(t)}{dt^n} \quad (19b) $$

The RL fractional derivative of a constant C takes the form

$$ {}_{RL}D_{a,t}^{\alpha} C = C \frac{(t-a)^{-\alpha}}{\Gamma(1-\alpha)} \neq 0 \quad (20) $$

However, the definitions of the fractional differentiation of Riemann-Liouville type create a conflict between the well-established and polished mathematical theory and proper needs, such as the initial problem of the fractional differential equation, and the nonzero problem related to the Riemann-Liouville derivative of a constant.

A certain solution to this conflict was proposed by Caputo first in his paper [19]. The Caputo fractional derivatives are defined as follows. The *left Caputo fractional derivative* is

$$ {}_{C}D_{a,t}^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau)\, d\tau, \quad (21) $$

and the right Caputo fractional derivative is

$$ {}_C D_{t,b}^{\alpha} f(t) = \frac{(-1)^n}{\Gamma(n-\alpha)} \int_t^b (\tau - t)^{n-\alpha-1} f^{(n)}(\tau) d\tau, \quad (22)$$

where $f^{(n)}(\tau) = d^n f(\tau)/d\tau^n$ and $n-1 \le \alpha < n \in \mathbb{Z}^+$. By definition, the Caputo fractional derivative of a constant is zero. The previous expressions show that the fractional-order operators are *global* operators having a memory of all past events, making them adequate for modeling hereditary and memory effects in most materials and systems. Moreover, for the Caputo derivative, we have

$$ \lim_{\alpha \to (n-1)^+} {}_a^C D_t^{\alpha} x(t) = \frac{d^{n-1}x(t)}{dt^{n-1}} - D^{(n-1)} x(a) \quad (23)$$

and

$$ \lim_{\alpha \to n^-} {}_a^C D_t^{\alpha} x(t) = \frac{d^n x(t)}{dt^n} \quad (24)$$

where obviously, ${}_{RL}D_a^{\alpha}$, $n \in (-\infty, +\infty)$ varies continuously with n, but the Caputo derivative cannot do this. Obviously, the Caputo derivative is stricter than the Riemann-Liouville derivative; one reason is that the n-th order derivative is required to exist. On the other hand, the initial conditions of fractional differential equations with the Caputo derivative have a clear physical meaning and the Caputo derivative is extensively used in real applications. The Riemann-Liouville fractional derivatives and the Caputo fractional derivatives are connected with each other by the following relations:

$$ {}_{RL}D_{t,a}^{\alpha} f(t) = \\ = {}_C D_{t,a}^{\alpha} f(t) + \sum_{k=0}^{n-1} \frac{(-1)^k f^{(k)}(a)}{\Gamma(k-\alpha+1)} (t-a)^{k-\alpha} \quad (25)$$

$$ {}_{RL}D_{t,b}^{\alpha} f(t) = \\ = {}_C D_{t,b}^{\alpha} f(t) + \sum_{k=0}^{n-1} \frac{(-1)^k f^{(k)}(b)}{\Gamma(k-\alpha+1)} (b-t)^{k-\alpha} \quad (26)$$

The Caputo and Riemann-Liouville formulation coincide when the initial conditions are zero[4-6], [20]. Besides, the RL derivative is meaningful under weaker smoothness requirements. In addition, the RL derivative can be presented as:

$$ {}_{RL}D_t^{\alpha} f(t) = D^n D_{a,t}^{\alpha-n} f(t), \quad \alpha \in [n-1, n), \quad (27)$$

and the Caputo derivative

$$ {}_C D_{a,t}^{\alpha} f(t) = D_{a,t}^{\alpha-n} D^n f(t), \quad \alpha \in n(n-1, n), \quad (28)$$

where $n \in Z^+, D^n$ is the classical n-order derivative. For convenience, the Laplace domain is usually used to describe the fractional integro-differential operation for solving engineering problems. The formula for the Laplace transform of the RL fractional derivative has the form:

$$ \int_0^{\infty} e^{-st} {}_{RL}D_{0,t}^{\alpha} f(t) dt = s^{\alpha} F(s) - \sum_{k=0}^{n-1} s^k {}_{RL}D_{0,t}^{\alpha-k-1} f(t)_{|t=0} \quad (29)$$

where for $\alpha < 0$ (*i.e.*, for the case of a fractional integral) the sum in the right-hand side must be omitted). Also, the Laplace transform of the Caputo fractional derivative is:

$$ \int_0^{\infty} e^{-st} {}_C D_{0,t}^{\alpha} f(t) dt = \\ = s^{\alpha} F(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1} f^{(k)}(0), \quad n-1 < \alpha < n \quad (30)$$

which implies that all the initial values of the considered equation are presented by a set of only classical integer-order derivatives. Besides that, a geometric and physical interpretation of fractional integration and fractional differentiation can be found in Podlubny's work [21].

Mathematical model of a robotic system with DC motors

A robotic system is considered as an open linkage consisting of $n+1$ rigid bodies $[V_i]$ interconnected by n one-degree-of–freedom joints forming kinematical pairs of the fifth class, Fig.1, where the robotic system possesses n degrees of freedom. Here, the Rodriguez\` method [22], is proposed for modeling the kinematics and dynamics of the robotic system. The configuration of the robot mechanical model can be defined by the vector of the joint (internal) generalized coordinates q of the dimension n, $(q) = (q^1, q^2, ..., q^n)^T$, with the relative angles of rotation (in case of revolute joints) and relative displacements (in case of prismatic joints). The geometry of the system has been defined by the unit vectors $\vec{e}_i$, $i = 1,2,...,j,..,n$ where the unit vectors $\vec{e}_i$ describe the axis of rotation (translation) of the i-th segment with respect to the previous segment as well as the position vectors $\vec{\rho}_i$ and $\vec{\rho}_{ii}$ usually expressed in local coordinate systems connected with the bodies $(\vec{\rho}_i^{(i)})$, $(\vec{\rho}_{ii}^{(i)})$. The parameters $\xi_i, \overline{\xi}_i = 1 - \xi_i$ denote the parameters for recognizing the joints $\xi_i, \overline{\xi}_i = 1 - \xi_i$, ξ_i =1-prismatic, 0-revolute. For the entire determination of this mechanical system, it is necessary to specify the masses m_i and the tensors of inertia J_{Ci} expressed in local coordinate systems. In order that the kinematics of the robotic system may be described, the points O_i, O_i' are noticed somewhere at the axis of the corresponding joint (i) such that they coincide in the reference configuration. The point O_i is immobile with respect to the $(i-1)$-th segment and O_i' does so with respect to the i-th one; obviously, for a revolute joint (i), the points O_i and O_i' will coincide all the time during robotic motion. For example, the position vector of the end-effector $\vec{r}_H$ can be written as a multiplication of the matrices of transformation $[A_{j-1,j}]$, the vectors $\vec{\rho}_{ii}$ and $\xi_i q^i \vec{e}_i$,and it is expressed by

$$ \vec{r}_H(q) = \sum_{i=1}^{n} \left(\vec{\rho}_{ii} + \xi_i q^i \vec{e}_i \right) = \\ = \sum_{i=1}^{n} \left(\prod_{j=1}^{i} [A_{j-1,j}] \right) \left((\vec{\rho}_{ii}^{(i)}) + \xi_i q^i (\vec{e}_i^{(i)}) \right) \quad (31)$$

where the appropriate Rodriguez' matrices of transformation are

$$\left[A_{j-1,j}\right]=\left[I\right]+\left[e_j^{d(j)}\right]^2\left(1-\cos q^j\right)+\left[e_j^{d(j)}\right]\sin\left(q^j\right) \quad (32)$$

and

$$\left(e_j^{(j)}\right)=\left(e_{\xi j},e_{\eta j},e_{\zeta j}\right)^T,\ \left[e_j^{d(j)}\right]=\begin{bmatrix}0 & -e_{\zeta j} & e_{\eta j}\\ e_{\zeta j} & 0 & -e_{\xi j}\\ -e_{\eta j} & e_{\xi j} & 0\end{bmatrix} \quad (33)$$

It is also shown [23], regardless of the chosen theoretical approach, that we could start from different theoretical aspects (e.g. general theorems of dynamic, d`Alembert`s principle, Langrange`s equation of second kind, Appell`s equations, etc.) and get the equations of motion of the robotic system, which can be expressed in the identical covariant form as follows

$$\sum_{\alpha=1}^{n}a_{\alpha i}(q)\ddot{q}^{\alpha}+\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\Gamma_{\alpha\beta,i}(q)\dot{q}^{\alpha}\dot{q}^{\beta}=Q_i \quad . \quad (34)$$
$$i=1,2,...,n.$$

where the coefficients $a_{\alpha\beta}$ are the covariant coordinates of the basic metric tensor $\left[a_{\alpha\beta}\right]\in R^{n\times n}$ and $\Gamma_{\alpha\beta,\gamma}$ $\alpha,\beta,\gamma=1,2,...,n$ presents Christoffel symbols of the first kind. The generalized forces Q_i can be presented in the following expression (35) where $Q_i^c,Q_i^g,Q_i^\beta,Q_i^w,Q_i^a$ denote the generalized spring forces, gravitational forces, viscous forces, semi-dry friction and generalized control forces, respectively.

$$Q_i=Q_i^c+Q_i^g+Q_i^\beta+Q_i^w+Q_i^a,\ \ i=1,2,...,n \quad (35)$$

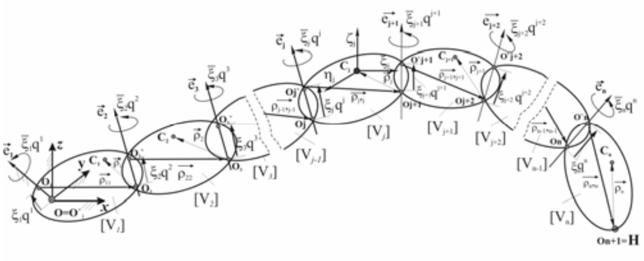

Figure 1. Open-chain structure of a robotic multi-body system

Fig.2 shows the equivalent circuit of a DC motor represented.

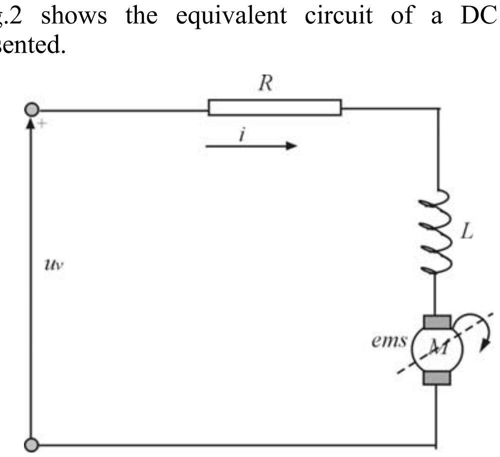

Figure 2. The equivalent circuit of a DC motor

The next equation describes the given circuit

$$R_i i_i(t)+L_i\frac{di_i(t)}{dt}+ems_i(t)=u_{vi}(t),\ i=1,2,3 \quad (36)$$

where R_i, L_i, i_i and u_{vi} are resistance, inductivity, electrical current and voltage, respectively. The electromotive force is $ems_i(t)=k_e dq_m/dt$ where $k_e=const$ and $q_m(t)$ is the generalized coordinate of a DC motor. If there is a reductor with a degree of reduction N_i then $q_{mi}(t)=N_i q_i(t),\ i=1,2,3$. It can be assumed that

$$Q_i^u(t)=N_i k_m i_i(t) \quad (37)$$

where $k_m=const$ is the torque constant. If the equation of a robotic system is combined with (37), the next equation can be written

$$A(q)\ddot{q}+C(q,\dot{q})=NK_m i \ \Rightarrow$$
$$i=\left[NK_m\right]^{-1}A(q)\ddot{q}+\left[NK_m\right]^{-1}C(q,\dot{q}) \quad (38)$$

this in a combination with (10) becomes (39)

$$+R\left[NK_m\right]^{-1}\left(A(q)\ddot{q}+C(q,\dot{q})\right)+K_e N\dot{q}=u_v(t) \quad (39)$$

In the state space, equation (39) is given with

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\end{bmatrix}=\begin{bmatrix}x_2(t)\\ x_3(t)\\ -A^{\bullet -1}(x_1(t))n(x(t))\end{bmatrix}+\begin{bmatrix}0\\ 0\\ -A^{\bullet -1}(x_1(t))\end{bmatrix}u_v(t) \quad (40)$$

where

$$A^{\bullet}(q)=L\left[NK_m\right]^{-1}A(q)$$
$$n(q,\dot{q},\ddot{q})=L\left[NK_m\right]^{-1}\dot{A}(q)\ddot{q}+L\left[NK_m\right]^{-1}\dot{C}(q,\dot{q})+ \quad (41)$$
$$+R\left[NK_m\right]^{-1}\left(A(q)\ddot{q}+C(q,\dot{q})\right)+K_e N\dot{q}$$

and

$$x(t)=[x_1(t),x_2(t),x_3(t)]=[q(t),\dot{q}(t),\ddot{q}(t)]^T\in R^{3n} \quad (42)$$

Main results

Optimal conventional and non-integer order PID control algorithm

Here, a 3 DOF robotic system driven by 3 DC motors is used, Fig.3.

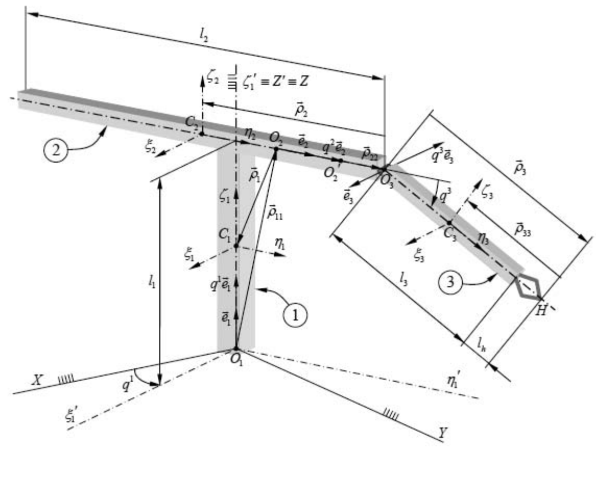

Figure 3. A 3 DOF robotic system

New algorithms of PID control are proposed, based on fractional calculus (FC) in the control of a robotic system driven by DC motors. Here, we introduce the next optimality criterion

$$J = \int |e(t)| dt \tag{43}$$

where is $e(t) = q_z(t) - q(t)$. The conventional PID control algorithm is

$$u(t) = k_p e(t) + k_d \frac{d}{dt} e(t) + k_i \int e(t) dt \tag{44}$$

while the fractional PID control algorithm is given by

$$u(t) = k_p e(t) + k_d D^{\alpha_D} [e(t)] + k_i D^{-\alpha_I} [e(t)]; \alpha_D, \alpha_I \in [0,1] \tag{45}$$

The integrator term is s^{α}, i.e. on a semi-logarithmic plane, there is a line having a slope −20αdB. /dec. Clearly, by selecting $\alpha = \beta = 1$, a classical PID controller can be recovered. The selections of $\alpha = 1,\ \beta = 0,\ \alpha = 0,\ \beta = 1,$ respectively, correspond to conventional PI & PD controllers. All these classical types of PID controllers are special cases of the fractional $PI^{\alpha}D^{\beta}$ controller. It can be expected that the controller $PI^{\alpha}D^{\beta}$ may enhance the systems control performance.

In order to determine the optimal parameters, a simulation of a given robotic system with three degrees of freedom driven by DC motors was made in *Simulink-Matlab* environment [24]. For the system control, voltage is used where parameters are set for each DC motor. The idea was to determine the optimal parameters for the conventional PID control algorithm (its gains) first and then to use these optimal parameters (gains) as known parameters for the fractional PID control algorithm in order to determine the optimal exponents of differentiation and integration. For the calculation of fractional derivatives and integrals the Crone approximation of the second order was used

$$s^{0.3} \approx \frac{3.981s^2 + 20.15s + 1}{s^2 + 20.15s + 3.981},\ s^{0.6} \approx \frac{15.85s^2 + 40.21s + 1}{s^2 + 40.21s + 15.85} \tag{46}$$

$$s^{0.9} \approx \frac{63.1s^2 + 80.23s + 1}{s^2 + 80.23s + 63.1},\ s^{-0.3} \approx \frac{25.12s^2 + 50.62s + 1}{s^3 + 50.62s^2 + 25.12s}$$

Simulation results for the position control

A desired value of the vector of generalized coordinate was $q_d = (1\ \ 1\ \ 1)$. The optimal parameters for the conventional PID and optimality criterion in this case had the following values (the results are given for each DC motor):

$$k_{p1} = 50, k_{d1} = 8, k_{i1} = 4, J_1 = 0.4672, \tag{47}$$

$$k_{p2} = 50, k_{d2} = 12, k_{i2} = 4, J_2 = 0.8591$$

$$k_{p3} = 50, k_{d3} = 4, k_{i3} = 8, J_3 = 0.3602$$

The optimal parameters for the fractional PID and optimality criterion had the following values:

$$k_{p1} = 50, k_{d1} = 8, k_{i1} = 4, \alpha_{d1} = 1, \alpha_{i1} = 0.2, J_1 = 0.3836$$

$$k_{p2} = 50, k_{d2} = 12, k_{i2} = 4, \alpha_{d2} = 1, \alpha_{i2} = 0.2, J_2 = 0.7401 \tag{48}$$

$$k_{p3} = 50, k_{d3} = 4, k_{i3} = 8, \alpha_{d3} = 1, \alpha_{i3} = 0.8, J_3 = 0.3555$$

Figs.4-6 give the coordinate-time diagrams for the previous optimal parameters –position control.

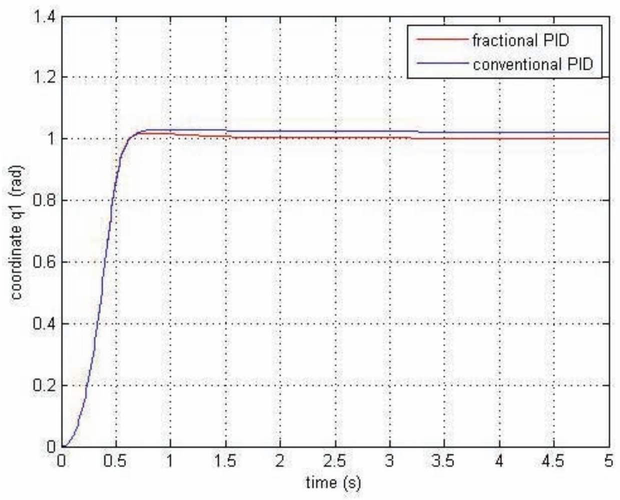

Figure 4. Optimal trajectory q_1

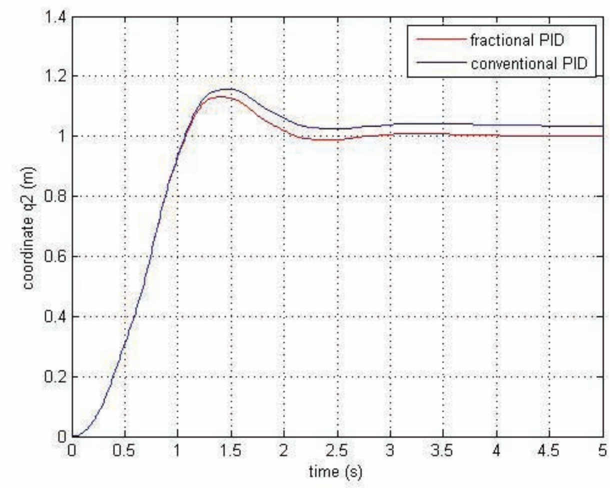

Figure 5. Optimal trajectory q_2

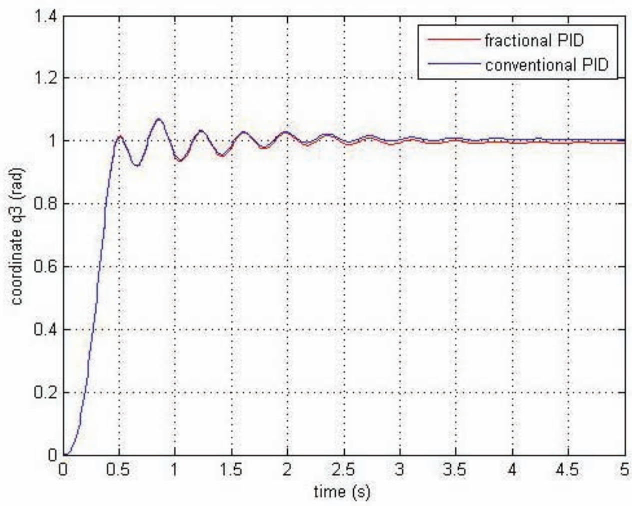

Figure 6. Optimal trajectory q_3

As it is expected, the controller $PI^{\alpha}D^{\beta}$ may enhance the systems control performance. It has been shown that using the fractional PID gives a better transient response as well as a steady state error, and better tracking performances in the position control of a 3 DOF robotic system driven by DC motors.

A chattering-free sliding mode controller design based on the fractional order PD^{α} sliding surface

Moreover, we suggested and obtained here a chattering-free fractional PD$^{\alpha}$ sliding-mode controller in the control of a robotic system driven by DC motors. It is well-known that the sliding-mode control is used to obtain high-performance robust control nonsensitive to disturbances and parameter variations. For a nonlinear MIMO system represented in a so-called normal form

$$\dot{x} = f(x) + G(x)u \tag{49}$$

one general sliding mode control law is, [25]

$$u = -[\Lambda G(x)]^{-1}\Lambda[f(x) - \dot{x}_d] - [\Lambda G(x)]^{-1}Q\operatorname{sgn}(s) \tag{50}$$

consisting of a continuous and discontinuous control part where switching surfaces $s = [s_1\, s_2 \ldots s_n]^T$ are defined as $s = \Lambda(x - x_d)$, x_d being the vector of the desired states and the Q positive definite diagonal matrix. The elements of the matrix Λ are chosen so that the *i*-th component of the sliding hypersurface has the structure

$$s_i = \left(\frac{d}{dt} + \lambda_i\right)^{(r_i - 1)}(x_i - x_{di}), \quad i = 1, 2, \ldots, n \tag{51}$$

where r_i is the order of the *i*-th subsystem and $\lambda_i > 0$. More generally, considering (14) as a nominal (known) plant dynamics, we can write

$$\dot{x} = f(x) + \tilde{f}(x) + \left[G(x) + \tilde{G}(x)\right]u \tag{52}$$

where $\tilde{f}(x)$ and $\tilde{G}(x)$ represent uncertainties or unknown plant dynamics. Choosing, as it is common, the Lyapunov function candidate to be

$$V = \frac{1}{2}s^T s \tag{53}$$

we have

$$\dot{s} = -PQ\operatorname{sgn}(s) + (P - I)\Lambda[\dot{x}_d - f(x)] + \Lambda\tilde{f}(x) \tag{54}$$

where $P := \Lambda\left(G + \tilde{G}\right)(\Lambda G)^{-1}$. Regardless whether $\tilde{G} \neq 0$ and/or $\tilde{f} \neq 0$, with an appropriate choice of Q, we can obtain $s^T\dot{s} < 0$ for $\|s\| > 0$, and this result indicates that the error vector defined by the difference $x - x_d$ is attracted by the subspace characterized by $s = 0$ and moves toward the origin according to what is prescribed by $s = 0$, [25]. In most cases, this leads to good results but there are some disadvantages such as a *chattering* phenomenon. This problem could be overcome by approximating the *sgn* (.) function in the control law (50) with *saturation*(.) or *tanh*(.) function, but here we suggest one other solution. Instead of replacing the *sgn*(.) function, we suggested the application of the fractional sliding surface in order to decrease output signal oscillations. In this paper, it can be shown that, without a special tuning of Q for the perturbed plant case, model uncertainties can be successfully compensated using just the fractional order sliding surface and the values of Q suitable for the nominal plant. For a 3-DOF robotic system, a conventional sliding manifold is of the first order *PD* structure $s_i = d\tilde{x}_i/dt + \lambda_i\tilde{x}_i$, $i = 1, 2, 3$ where $\tilde{x}_i = x_i - x_{id}$. There were some examples of using the fractional *PI* and *PID* structures, [26] and now we propose a fractional PD^{α} structure as follows:

$$s_i = d^{\alpha}\tilde{x}_i/dt^{\alpha} + \lambda_i\tilde{x}_i, \; i = 1, 2, 3 \tag{55}$$

Simulation results for the position control based on fractional PD^{α} sliding-mode control

Simulation studies have been carried out to verify the effectiveness of the proposed fractional PD$^{\alpha}$ sliding-mode control. Some experimental simulations were undertaken for $\alpha = 0.7, 0.8, 0.9, 0.95, 0.99$, and we have found that the best results are obtained with $\alpha = 0.95$, (Fig.7). The transfer function s^{ν} was realized by Crone's approximation, [27] and the matrix $Q_{nom} = diag[5,5,5]$ as well as $\lambda = (5, 2.5, 2.5)^T$. The parameters of a robot system and DC motors are set as:

$$m_1 = 6.2712 \text{ kg}, \; m_2 = 5.5575 \text{ kg}, \; m_3 = 1.8970 \text{ kg} \tag{56}$$

$$J_{x1} = 0.5273,\; J_{y1} = 0.5273,\; J_{z1} = 0.0164 \;\left[kgm^2\right]$$
$$J_{x2} = 1.0441,\; J_{y2} = 1.0441,\; J_{z2} = 0.0073$$
$$J_{x3} = 0.1016,\; J_{y3} = 0.1016,\; J_{z3} = 0.0016$$

$$K_{ei} = 2,\; K_{mi} = 1,\; N_i = 1,\; R_i = 1$$

To verify the robustness of the proposed fractional sliding/mode control we have applied the corresponding parameters variation as follows:

$$\frac{\Delta m_1}{m_1} = 9.92\%,\; \frac{\Delta m_2}{m_2} = 9.47\%,\; \frac{\Delta m_3}{m_3} = 9.75\% \tag{57}$$

$$\frac{\Delta K_{ei}}{K_{ei}} = 5\%,\; \frac{\Delta K_{mi}}{K_{mi}} = 10\%,\; \frac{\Delta K_{ei}}{K_{ei}} = 10\%,\; \frac{\Delta K_{ei}}{K_{ei}} = 20\%$$

$$\frac{\Delta J_{x1}}{J_{x1}} = \frac{\Delta J_{y1}}{J_{y1}} = 14.39\%,\; \frac{\Delta J_{z1}}{J_{z1}} = 17.88\%,$$
$$\frac{\Delta J_{x2}}{J_{x2}} = \frac{\Delta J_{y2}}{J_{y2}} = 9.39\%,\; \frac{\Delta J_{z2}}{J_{z2}} = 12.83\%$$
$$\frac{\Delta J_{x3}}{J_{x3}} = \frac{\Delta J_{y3}}{J_{y3}} = 14.20\%,\; \frac{\Delta J_{z3}}{J_{z3}} = 17.32\%$$

The simulation results are depicted in Figs.7 to 11, where the black lines ($h(t)$) are the desired trajectories. Here, the simulation data are presented for the case $i = 2$, q_2, s_2, (Fig-s.7-11). In particular, we present the comparison results for the second coordinate q_2 responses with the PD and fractional PD^{α} cases with all other conditions being the same, for the nominal object, Fig.7 and the perturbed object, Fig.10.

Nominal case:

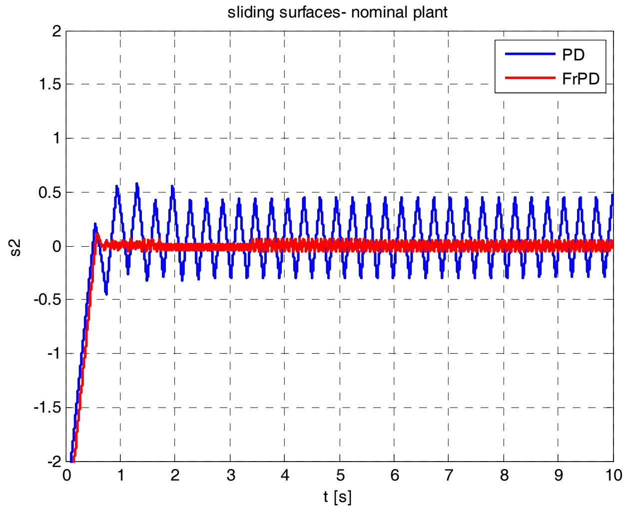

Figure 7. Sliding surface *s2*- nominal case

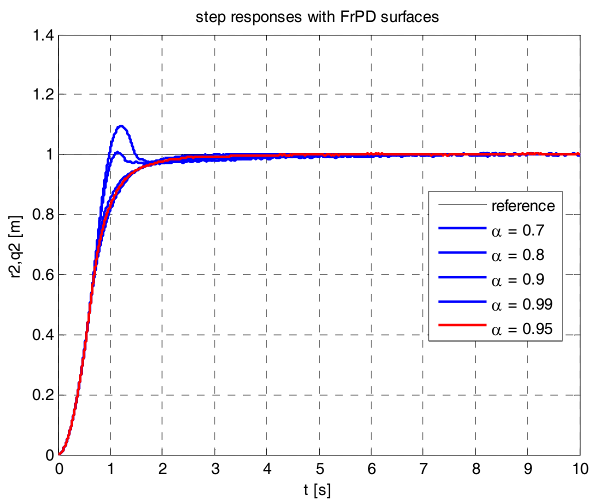

Figure 8. Step response $q_2(t)$ with PD^{α} surface

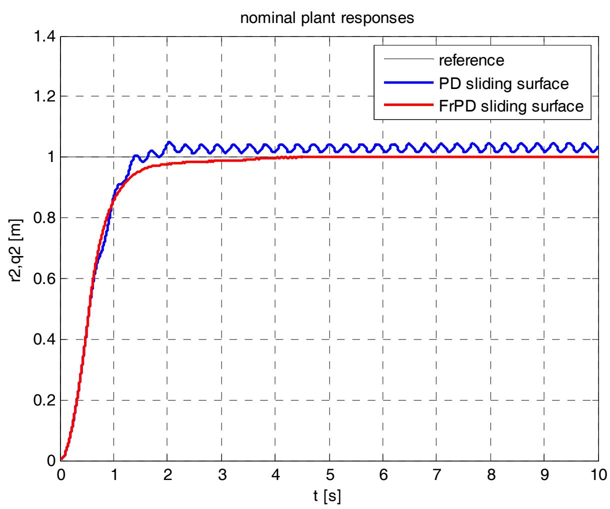

Figure 9. Stabilizing using the sliding mode control *PD* and the fractional PD^{α} - nominal case

Perturbated case:

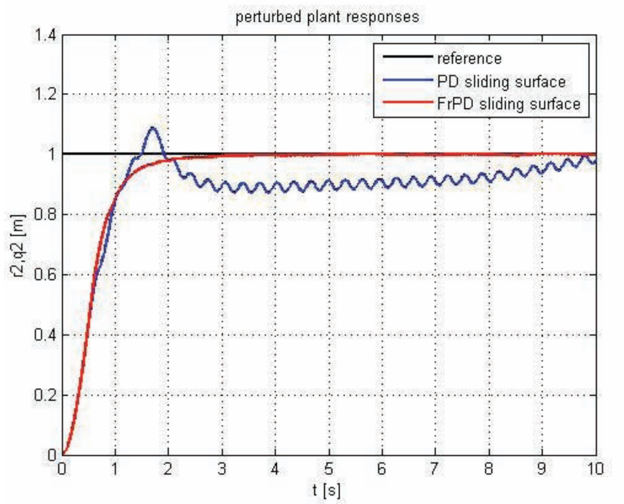

Figure 10. Stabilizing using the sliding mode control *PD* and the fractional PD^{α} -perturbed case

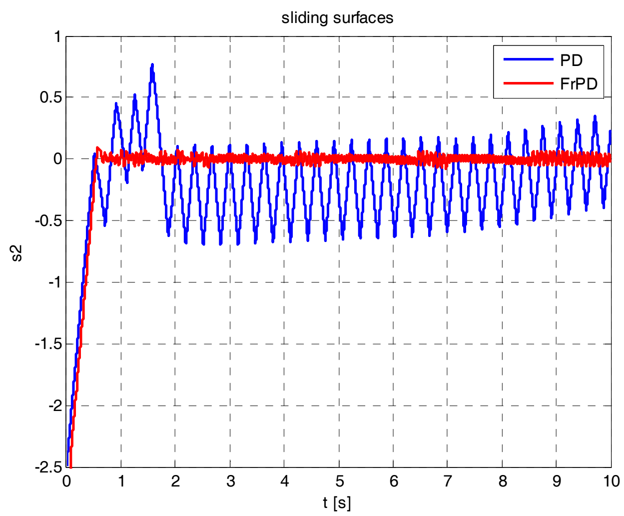

Figure 11 Sliding surface s_2-perturbated case

As it can be seen from the previous figures, the sliding mode control with the fractional sliding surface is more robust to parameter perturbations and, what is most important to emphasize, the output oscillations are almost completely attenuated and the overall quality of the transient response is much better. In that way, we obtain the chattering-free tracking of the given robot system.

Conclusion

In this paper, new algorithms of PID control based on fractional calculus (FC) are studied and presented. We introduced an optimal procedure in the position control of a 3 DOF robotic system driven by DC motors as well as a robust fractional-order sliding mode control. As expected, the controller $PI^{\alpha}D^{\beta}$ may enhance the systems control performance. It has been shown that using fractional PID gives a better transient response and a steady state error as well as better tracking performances in the position control of a 3 DOF robotic system driven by DC motors. The optimal parameters for the conventional PID control algorithm (its gains) are determined first and they are used as initial, known parameters for the fractional PID control algorithm in order to determine the optimal fractional exponents of differentiation and integration. After that, the effectiveness of the suggested optimal fractional PID control is demonstrated with a suitable robot with three degrees of freedom as an illustrative example. In addition, we proposed a robust fractional-order sliding mode control of a given robot system driven by DC motors where a fractional order sliding surface PD^{α} is introduced. It is shown that a sliding mode control with the fractional sliding surface is more robust to parameter perturbations and, what is most important to emphasize, the output oscillations are almost completely attenuated and the overall quality of the transient response is much better. Finally, numerical simulations have been carried out to illustrate the validity of the proposed procedure.

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References

[1] ASTROM,K., HAGGLUND,T.: *PID controllers: Theory, design, and tuning. Instrument Society of America*, North Carolina, 1995.

[2] CHEN,Y., HU,C., MOORE,K.L.: *Relay Feedback Tuning of Robust PID Controllers with Iso-Damping Property*, IEEE 2003 Conference on Decision and Control, Maui, HI, USA, December 9-12, 2003.

[3] ASTROM,K.J., HAGGLUND,T.: *The Future of PID Control*, in IFAC Workshop on Digital Control. Past, Present and Future of PID Control, (Terrassa, Spain), April 2000, pp.19–30.

[4] OLDHAM,K.B., SPANIER,J.: *The Fractional Calculus: Theory and Applications of Differentiation and Integration to Arbitrary Order*, Academic Press, New York, NY, USA, 1974.

[5] ED HILFER,R.: *Applications of Fractional Calculus in Physics*, World Scientific, River Edge, NJ, USA, 2000.

[6] PODLUBNY,I.: *Fractional Differential Equations. Academic Press*, San Diego,1999.

[7] LAZAREVIĆ,M.P.: *Finite time stability analysis of PD^{α} fractional control of robotic time-delay systems,* Mech. Resch.Commun. 2006, 33, pp.269–279.

[8] MAIONE,G., LINO,P.: *New tuning rules for fractional PI^{α} controllers*, Nonlinear Dyn. 2007, 49, pp.251–257.

[9] OUSTALOUP,A., SABATIER,J., LANUSSE,P.: F*rom fractal robustness to CRONE control*, Fractional Calc. Appl. Anal. 1999, 2(1), pp.1-30.

[10] OUSTALOUP,A.: *La Commande CRONE*, Hermes, Paris,1991.

[11] MONJE,C.A., FELIU,V.: *The fractional-order lead compensator*, In: IEEE International Conference on Computational Cybernetics. Vienna, Austria, August 30-September 1, 2004.

[12] EDWARDS,C., SPURGEON,S.K.: *Sliding mode control theory and applications*, Taylor & Fran., NewYork, 1998.

[13] Hace,A., Jezernik,K.,: *Robust position tracking control for direct drive motor*, Industrial Electronics Society, 2000. IECON 2000. 26th Annual Confjerence of the IEEE, 2000.

[14] MONJE,C.A., et al.: *Fractional-order Systems and Controls*, Springer - Verlag, London, 2010.

[15] DELAVARI,H., GHADERI,R., RANJBAR,A., MOMANI,S.: *Fractional Order Controller for Two-degree of Freedom Polar Robot*, 2nd Conference on Nonlinear Science and Complexity, Ankara, Turkey, 2008.

[16] EFE,M.O.: *Fractional Order Sliding Mode Controller Design for Fractional Order Dynamic Systems*, New Trends in Nanotech. and Frac. Calculus Applications 2009.

[17] GRUNWALD,A.K.: *Uber "begrenzte" Derivationen und deren Anwendung*, Zeit fur Mathematik und Physik, 1867, 12, pp.441-480.

[18] LETNIKOV,A.V.: *Theory of differentiation with an arbitrary index*, (Russian), Moscow, Matem. Sbornik, 1868, 3:1-66.

[19] CAPUTO,M.: Elasticit´a e Dissipazione, Zanichelli, Bologna, 1969.

[20] KILBAS,A.A., SRIVASTAVA,H.M., TRUJILLO,J.J.: *Theory and Applications of Fractional Differential Equations*, Elsevier, Amsterdam, 2006.

[21] PODLUBNY,I.: *Geometric and physical interpretation of fractional integration and fractional differentiation*, Fract Calc Appl Anal 2002, 5:pp.367–386.

[22] LAZAREVIĆ,M.: *Optimal control of redundant robots in human-like fashion: general considerations*, FME Transcation, Faculty of Mechanical Engineering, University of Belgrade, 2005, 33(2), pp.53-64.

[23] LURIE,I.: *Analytical Mechanics*, (in Russian), G. Publishing, Moscow, 1961.

[24] SAMARDŽIĆ,J., LAZAREVIĆ,M.P., CVETKOVIĆ,B.: *Optimal conventional and fractional PID control algorithm for a robotic system with three degrees of freedom driven by DC motors*, SISY 2011 • 2011 IEEE 9th International Symposium on Intelligent Systems and Informatics •, Subotica, Serbia, September 8-10, 2011.

[25] MEHMET,Ö.E., KASNAKOGLU,C.: *A fractional adaptation law for sliding mode control*, Int. J. Adapt. Control Signal Process., John Wiley & Sons. 2008.

[26] CALDERON,A.J., VINAGRE,B.M., FELIU,V.: *Fractional Sliding Mode Control of a DC-DC Buck Converter with Application to DC Motor Drives*, Proceedings of ICAR 2003, Coimbra, Portugal, 2003.

[27] VALERIO,D.: *Fractional robust system control*, PhD Thesis, Technical University of Lisboa, Lisbon, 2005.

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Upravljanje necelobrojnog reda jednim robotskim sistemom pogonjenog jednosmernim motorima

U ovom radu, predstavljeni su novi PID algoritmi upravljanja zasnovani na računu necelobrojnog reda i optimalnoj proceduri u zadatku pozicioniranja robotskog sistema sa tri stepena slobode pogonjen jednosmernim motorima. Cilj je bio odrediti optimalno podešavanje $PI^{\alpha}D^{\beta}$ kontrolera necelobrojnog reda da bi se ispunili željeni zahtevi zatvorenog sistema upravljanja, uzimajući u obzir prednosti korišćenja necelobrojnog reda α i β. Efikasnost predloženog optimalnog *PID* upravljanja necelobrojnog reda je demonstriran na pogodno usvojenom robotskom sistemu sa tri stepena slobode kao jednom ilustrativnom primeru. Takođe, ovaj rad predlaže jedno robustno upravljanje u režimu klizanja necelobrojnog reda datim robotom pogonjen jednosmernim motorima. Prvo je projektovan klasični kontroler u kliznom režimu zasnovan na PD^{α} kliznoj površini. Numeričke simulacije su sprovedene da predstave robusne osobine predloženog upravljačkog sistema kao i da istakne značaj datog upravljanja koji se ogleda i u smanjenju oscilacija datog robota u radnom prostoru (*chattering-free*). Simulacije uključuju i poređenje kontrolera PD^{α} u režimu klizanja necelobrojnog reda sa standardnim PD kontrolerom u režimu kilizanja.

Ključne reči: **roboti, jednosmerni motor, robustno upravljanje, algoritam upravljanja, *PID* algoritam, račun necelobrojnog reda, podešavanje vibracije.**

Управление дробного порядка робот оснащённых системой двигателей постоянного тока

В данной работе представлены новые алгоритмы ПИД-регулирования, основанные на учёте дробного порядка и оптимальной процедуры в деле размещения роботизированной системы с тремя степенями свободы, оснащённой двигателями постоянного тока. Цель состоялась в том, чтобы определить оптимальные настройки $PI^{\alpha}D^{\beta}$ для дробных контроллеров для достижения желаемых требований замкнутой системы управления, принимая во внимание преимущества использования дробного порядка α и β. Эффективность предлагаемого оптимального управления PID дробного порядка демонстрируется в принятой подходящей роботизированной системе с тремя степенями свободы, как наглядном примере. Кроме того, этот документ предлагает надёжное управление в режиме скольжения дробного порядка данным роботом, оснащённым двигателями постоянного тока. Первым был разработан классический контроллер в режиме скольжения, основан на скользящей поверхности PD^{α}. Численное моделирование проводится чтобы представить надёжные характеристики предлагаемой системы управления и подчеркнуть значение данного управления, которое находит своё отражение в уменьшении колебаний данного робота в рабочем пространстве (chattering-free). Моделирование включает и сравнение контроллера PD^{α} в скользящем режиме дробного порядка со стандартным PD контроллером в скользящем режиме.

Ключевые слова: **роботы, двигатель постоянного тока, надёжное управление, алгоритм управления, ПИД алгоритм, управления дробного порядка, настройки, вибрации.**

Le contrôle de l’ordre fractionnel à l’aide d’un système robotique conduit par les moteurs à courant continu

Les nouveaux algorithmes PID de contrôle basés sur les calculs de l’ordre fractionnel et sur la procédure optimale pour positionner le système robotique à trois degrés de liberté et conduits par les moteurs à courant continu sont présentés dans cet article. Le but en était de déterminer le réglage optimale $PI^{\alpha}D^{\beta}$ des contrôleurs de l’ordre fractionnel pour réaliser les exigences désirées du système fermé de contrôle considérant les avantages de l’emploi de l’ordre fractionnel α et β. L’efficacité du contrôle optimale de PID proposé a été démontrée sur le système robotique adopté à trois degrés de liberté qui a servi comme un cas de figure. Ce papier propose aussi un contrôle robuste dans le mode de glissement de l’ordre fractionnel par un robot conduit à l’aide des moteurs à courant continu. On a conçu d’abord le contrôleur classique dans le mode glissant basé sur la surface glissante PD^{α}. Les simulations numériques ont été effectuées afin de présenter les caractéristiques robustes du système de contrôle proposé et pour souligner l’importance du contrôle donné qui se reflète aussi dans le diminution des oscillations du robot donné dans l’espace de travail. Les simulations comprennent aussi les contrôleurs PD^{α} dans le mode glissant de l’ordre fractionnel avec le contrôleur PD standard dans le mode glissant.

Mots clés: **robots, moteur à courant continu, contrôle robuste, algorithme de contrôle, algorithme PID, calcul de l’ordre fractionnel, réglage des vibrations.**