

## STABILIZATION CONTROL OF INVERTED PENDULUM SYSTEMS BY FRACTIONAL ORDER PD CONTROLLER BASED ON D-DECOMPOSITION TECHNIQUE

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### ABSTRACT

Many systems in nature are inherently under-actuated, with fewer actuators than degrees of freedom. However, even with reduced number of actuators, these systems are able to produce complex movements. To be capable of performing such motions, complex control algorithms must be implemented. Classical benchmark examples for studying problems of this kind include inverted pendulum systems. This paper deals with stability problem of two types of inverted pendulum controlled by a fractional order PD controller. Rotational and cart inverted pendulum are highly nonlinear mechanical systems with one control input and two degrees of freedom. Detailed mathematical model of both pendulums are derived using the Rodriguez method. Stabilization of pendulum around its unstable equilibrium point is achieved by using the fractional order PD <sup>$\alpha$</sup>  controller, in combination with partial feedback linearization technique. There are several methods for determining stability region of a closed loop system, and D-decomposition is one of them. Herein, D-decomposition method is applied to the inverted pendulum case, and determining its stability regions in parameters space of a fractional order PD controller is presented. D-decomposition for linear fractional systems is investigated, and for the case of linear parameters dependence. Fractional order control laws are represented by a transfer functions which are not rational, which gives rise to a problem of practical implementation of the corresponding control algorithms. A method for rational approximation of linear fractional order systems used in this paper is computationally efficient, accurate, and relies on the interpolation of the frequency characteristics of the system on a predefined set of target frequencies. The performance of the proposed method is demonstrated with experimental verification of the stabilization control of the cart pendulum system.

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