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Natural Roll Period of River-sea Ships

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ABSTRACT

The natural roll period is an important component of stability assessment. The need for development of a proper estimation method is particularly important for river-sea ships, which represent inland vessels intended for operations in maritime environment. The overall goal of this study is to contribute to the development of a suitable method for the natural roll period estimation for river-sea ships. For this purpose, a method based on the linear potential hydrodynamics is applied on a database consisted of 31 river-sea ships. The obtained results are analysed and compared with those calculated using semi-empirical methods, and a practical natural roll period estimation method for river-sea ships is proposed.

Keywords: natural roll period, added inertia, river-sea ships, inland vessels, roll motion

1 INTRODUCTION

The natural roll period T_φ (or alternatively the natural roll frequency ω_φ) is one of the basic parameters in stability assessment in rough weather due to the phenomenon of resonance. Resonant rolling in beam waves was thoroughly examined in the past and was considered as the most important (if not the only important) phenomenon for stability assessment (Blume, 1979; IMO, 2008; Yamagata, 1959). Even IMO regulations currently in force explicitly consider only this dynamic condition (IMO, 2021a). Furthermore, the roll period is a fundamental parameter also in the framework of Second Generation Intact Stability Criteria, and the same formula given in the 2008 IS Code is still used as the reference estimation method (IMO, 2020), with some alternatives available in the explanatory notes (IMO, 2021b).

Although the stability assessment procedures have considerably improved recently, the methods for the natural roll period assessment have not been revised, and the formula given in the 2008 IS Code is commonly used nowadays, even though it was developed in 1980s. The natural roll period of inland vessels has received even less attention, considering that very few formulae have been developed for this purpose. However, the need for a proper natural roll period estimation method becomes particularly important when inland vessels are employed in maritime navigation as is the case in the river-sea shipping (Bačkalov, 2019; Rudaković & Bačkalov, 2019).

In this paper, different methods for natural roll frequency estimation are presented, compared and further discussed with respect to their advantages and disadvantages, with specific attention to river-sea ships. Following the outcomes from the study by Rudaković et al. (2019), attention is given herein to the exploitation of direct estimations based on linear potential seakeeping calculations, with a goal of developing simpler approaches. For this purpose, a suitable database of vessel in different loading conditions is used. The possibility of improving the IMO method for the natural roll frequency estimation is considered and an alternative formula is proposed. Indications are also provided regarding typical values of added inertia as determined for the consider type of ships.

2 NATURAL ROLL FREQUENCY ESTIMATION

In this study, the following natural roll period estimation methods are considered:

- Predictions using 3-degrees-of-freedom direct calculations based on linear seakeeping hydrodynamics (reduced 3DOF approach);
- Predictions using two semi-empirical methods.

The mentioned approaches are described in details in the following sections.

2.1 Direct calculation based on linear seakeeping hydrodynamics

The direct calculation of natural roll frequency in this paper is carried out by means of linear hydrodynamics. The natural roll frequency is influenced by sway and yaw motions, therefore, using a three-degree-of-freedom model could improve estimation results. The linear 3DOF model for sway-roll-yaw is:

$$\left(\mathbf{M}+\mathbf{A}(\omega)\right)\ddot{\mathbf{x}}+\mathbf{B}(\omega)\dot{\mathbf{x}}+\mathbf{C}\mathbf{x}=\hat{\mathbf{F}}(\omega) \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{A}$ is the added mass matrix, $\mathbf{B}$ is the damping matrix, $\mathbf{C}$ is the restoring matrix, $\mathbf{F}$ is the vector of complex generalised forces, ω is the wave frequency, $\mathbf{x}$ is the complex state vector and dots indicate time derivatives. By neglecting the damping matrix $\mathbf{B}$, and following the principle explained in Bulian et al. (2008), Bulian & Francescutto (2009, 2011) and Rudaković et al. (2019), equation (1) becomes:

$$\begin{cases}\mathbf{Q}(\omega)\ddot{\mathbf{x}}+\mathbf{C}\mathbf{x}=\hat{\mathbf{F}}(\omega)\\ \text{with }\ \mathbf{Q}(\omega)=\mathbf{M}+\mathbf{A}(\omega)\end{cases} \tag{2}$$

which corresponds to the following set of equations:

$$\begin{cases}Q_{22}\ddot{y}+Q_{24}\ddot{\varphi}+Q_{26}\ddot{\psi}=\hat{F}_2\\ Q_{42}\ddot{y}+Q_{44}\ddot{\varphi}+Q_{46}\ddot{\psi}+C_{44}\varphi=\hat{F}_4\\ Q_{62}\ddot{y}+Q_{64}\ddot{\varphi}+Q_{66}\ddot{\psi}=\hat{F}_6\end{cases} \tag{3}$$

where subscript indices correspond to the standard 6DOF linear seakeeping nomenclature (2: sway, 4: roll, 6: yaw).

The system of equations (3) may be further reduced to a single equation for roll, which, however, still comprises the coupling effect with other motions:

$$I_{44,c}(\omega)\ddot{\varphi}+C_{44}\varphi=\hat{F}_{44,c}(\omega) \tag{4}$$

where, as usual, the restoring coefficient is

$$C_{44}=g\cdot m\cdot\overline{GM} \tag{5}$$

with g the gravitational acceleration, m the ship mass, and $\overline{GM}$ the metacentric height. The frequency dependent coupled total roll moment of inertia, $I_{44,c}(\omega)$, in equation (4), can be determined as

$$I_{44,C}\left(\omega_\varphi\right) = Q_{44} - \frac{Q_{42}\left(Q_{24}Q_{66} - Q_{64}Q_{26}\right)}{Q_{22}Q_{66} - Q_{26}Q_{62}} + \frac{Q_{46}\left(Q_{24}Q_{62} - Q_{64}Q_{22}\right)}{Q_{22}Q_{66} - Q_{26}Q_{62}} \tag{6}$$

where $Q_{ij} = Q_{ij}\left(\omega\right),\ i, j = 2,4,6$. Equation (4) can be rewritten in the following form:

$$\ddot{\varphi} + \frac{C_{44}}{I_{44,c}\left(\omega\right)}\varphi = \frac{\hat{F}_{44,c}\left(\omega\right)}{I_{44,c}\left(\omega\right)} \tag{7}$$

from which the undamped natural roll frequency can be determined as the solution of

$$\omega_\varphi = \sqrt{\frac{C_{44}}{I_{44,C}\left(\omega_\varphi\right)}} \tag{8}$$

The reported procedure is conceptually equivalent to those reported by, e.g., Tasai (1971) and Fossen (2011).

It is noted, however, that it is not uncommon to consider an approach for deriving a one-degree-of-freedom roll model from seakeeping calculations, by considering a pure roll motion around the centre of gravity G. In this case, the resulting 1DOF equation is

$$\left(I_{xx} + A_{44}\left(\omega\right)\right)\ddot{\varphi} + B_{44}\left(\omega\right)\dot{\varphi} + C_{44}\varphi = F_4 \tag{9}$$

where I_{xx} is the dry roll moment of inertia with respect to G, and G is also the reduction point for the hydrodynamic coefficients. The corresponding undamped natural roll frequency may be determined as the solution of the following equation:

$$\omega_\varphi = \sqrt{\frac{C_{44}}{I_{xx} + A_{44}\left(\omega_\varphi\right)}} \tag{10}$$

However, seakeeping calculations are necessary both for the pure 1DOF (equation (10)) and for the reduced 3DOF (equation (8)) estimation methods (in order to determine added masses $A_{ij}\left(\omega\right),\ i, j = 2,4,6$). Therefore, the complexity of the methods is similar. Furthermore, while the reduced 3DOF method accounts for sway-roll-yaw coupling, the pure 1DOF method considering rotations around G does not properly account for such couplings. Thus, only the reduced 3DOF method will be further analysed in this paper.

2.2 Semi-empirical formulae

The usual way for natural roll period estimation is derived from the assumption of one-degree-of-freedom model. If mass moment of inertia and added mass moment of inertia are, for the sake of simplicity, considered together, the natural roll frequency may be written as follows:

$$\omega_\varphi = \frac{\sqrt{g\overline{GM}}}{k'_{xx}} \tag{11}$$

where k'_{xx} is the "wet", or total, roll radius of inertia (in contrast to the "dry" roll radius of inertia $k_{xx} = \sqrt{I_{xx} / m}$). However, in semi-empirical formulae, it is more common to refer to the corresponding natural roll period, as follows:

$$T_\varphi = \frac{2\pi k'_{xx}}{\sqrt{g\overline{GM}}} \tag{12}$$

Furthermore, the wet roll radius of inertia k'_{xx}, is often referred to in dimensionless form, as fraction of ship breadth, i.e. as k'_{xx} / B.

In this paper two semi-empirical approaches are considered. The first approach is the semi-empirical formula from Weather Criterion (IMO, 2008, 2021a) that is also considered as the reference estimation method in the framework of Second Generation Intact Stability Criteria (IMO, 2020). In this case, the roll period is estimated as follows:

$$\begin{cases} T_\varphi = \dfrac{2 \cdot C \cdot B}{\sqrt{GM}} \\ C = 0.373 + 0.023\dfrac{B}{d} - 0.043\dfrac{L_{wl}}{100} \end{cases} \tag{13}$$

It is noted that, since $\pi / \sqrt{g} \approx 1$, it follows that $C = \pi \cdot k'_{xx} / \left(\sqrt{g} \cdot B\right) \approx k'_{xx} / B$.

The second semi-empirical formula considered herein is the formula given by Bureau Veritas (BV, 2019). The Bureau Veritas rules for inland vessels provide a formulation of the total roll radius of inertia, by introducing an added mass coefficient C_a. According to BV (2019), the roll period is estimated as:

$$\begin{cases} T_\varphi = \dfrac{2 \cdot C_a \cdot k_{xx}}{\sqrt{GM}} \\ C_a = 1.066 + 0.066\dfrac{B}{d} - 0.123\dfrac{L}{100} \end{cases} \tag{14}$$

where L is the “rule length” (BV, 2019). According to BV (2019), when k_{xx} is not known, it can be assumed as 0.4B for the lightship condition and 0.35B for the fully laden vessel. Interestingly, it seems that the coefficient C_a is directly derived from the coefficient C from the IMO formula (13). In fact, when $k_{xx} = 0.35B$ is used in (14), the equation (13) is obtained, except for the small difference regarding the use of the length at waterline in (13) and the “rule length” in (14).

The IMO formula was selected for this study because of its common wide application, while the specific semi-empirical formula from Bureau Veritas rules for inland navigation was selected as it is a method specifically intended for application to inland vessels. It is noted that the IMO formula (13) tries to capture a dependence of the dimensionless wet roll radius of inertia on the B/d ratio and on the ship length, but it does not allow to specify the actual dry radius of inertia, if known.

Instead, the approach in the BV formula (14), using basically the same vessel particulars, allows to explicitly take into account the dry inertia, if known. The wet roll radius of inertia is then predicted as a proportion of the dry roll radius inertia through the added mass coefficient C_a.

It is also noted that a number of other semi-empirical indications for estimation of natural roll period have been developed in the past for seagoing ships (e.g., Kato 1956; Munro-Smith, 1973; Papanikolaou et al., 1997; Benford, 1991; George, 1983; Peach & Brook 1987; just to mention a few).

3 VESSEL DATABASE

The database comprises 31 river-sea ships: 23 tankers, four general cargo vessels, three container vessels and an LPG tanker. Due to their diverse characteristics, the vessels are representative specimens of a wide range of inland vessel types and dimensions. The length of the vessels in the database ranges from 66 m (for small inland navigation vessels) up to 135 m (corresponding to the largest contemporary inland self-propelled vessels in Western Europe). The inland vessels have full hull forms, with block coefficients up to $C_B \approx 0.92$, due to long parallel middle bodies in combination with full midship coefficients (C_M typically exceeding 0.99). Due to restrictions in waterway dimensions, inland vessels have relatively low draught, which is compensated with greater beam, resulting in somewhat high B/d ratios, in the range of $2.5 \div 4.5$ for the scantling draught, and much higher ratios for lower draughts. They often make use of maximum practical length for a certain waterway, resulting in a wide range of L/B ratios, in range of $5.5 \div 12$, for scantling draught. On the other hand, the beam of inland navigation vessels is restricted by the width of locks, resulting in standardised beam dimensions among them (for instance, from the presented database, 19 vessels in total have the beam close to 11.4 m).

For the purpose of the study, three draughts were used for each vessel: a maximum draught corresponding to the scantling draught, a draught corresponding to the minimum draught enabling full immersion of the stern tunnels, and an intermediate draught conventionally set as the average between the previous two. Furthermore, for the purpose of the direct calculation (explained in Section 2.1) different dry roll radii of inertia k_{xx} were used, corresponding to 0.25B, 0.30B, 0.35B, 0.40B and 0.45B, because a method specifically developed for estimation of dry roll radius of inertia of inland vessels is not readily available. The dry yaw radius of inertia k_{zz} was kept constant and equal to 0.25L_{WL}. A wide range of loading conditions was used – the minimum $\overline{KG}$ was taken about half of the scantling draught, whereas the maximum $\overline{KG}$ corresponds to metacentric heights around 0.3 m$\div$0.5 m, depending on the vessel and draught. Therefore, in total 2835 vessel-draught-$\overline{KG}$-k_{xx} combinations were considered. These combinations are selected so as to cover all possible realistic loading conditions, which will be used for the direct calculation of the natural roll period.

A detailed description of the database has been provided by Rudaković et al. (2019), who used the same database and methodology for an analysis addressing the effective wave slope coefficient.

4 APPLICATION

The natural roll frequency was estimated using the 3DOF linear hydrodynamic approach (Section 2.1), the IMO method (equation (13)) and the BV 2019 rules formula for inland navigation vessels (equation (14)), for all variations of vessels and loading conditions in the database. For the purpose of this study, the rule length L in (14) has been taken as the length between perpendiculars, as the two quantities are generally close.

Resulting data have been used to assess differences in predictions and to provide new insight, as described in the following sections. Nevertheless, the problem of an adequate comparison arises, because experimental results of natural roll frequency for river-sea ships are not readily available.

Therefore, herein, results obtained by the 3DOF linear hydrodynamic approach will be deemed as the reference data, since they correspond to the estimation embedding the most accurate modelling of rigid body dynamics and hydrodynamics among the considered approaches.

4.1 Comparison of estimated roll natural frequencies

In Figure 1, the natural roll frequencies estimated by the reduced 3DOF method ($\omega_{\varphi,3DOF}$) and by the IMO method ($\omega_{\varphi,IMO}$) are compared. Because the IMO method implicitly embeds the dry roll radius of inertia, a significant scattering can be seen. Stripes that can be noticed in the plot correspond to the five different dry roll radii of inertia used in the application of the reduced 3DOF method.

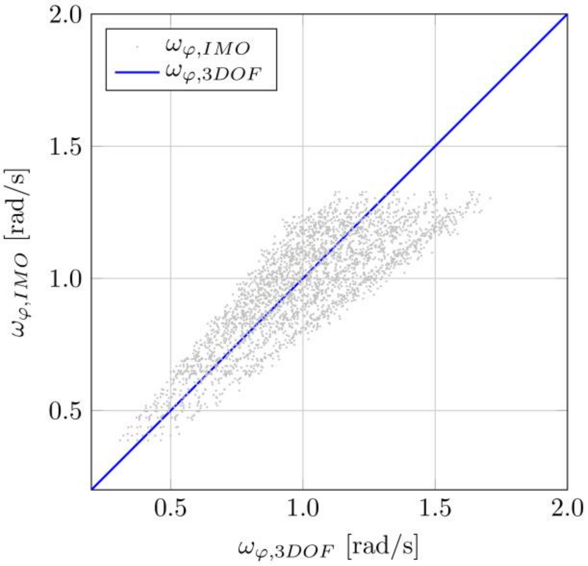

Figure 1 Comparative plot of estimated natural roll frequency: reduced 3DOF method and IMO method.

As already highlighted, in the predictions by the IMO method it is not possible to separate the implicit estimation of dry inertia from the added inertia. Nevertheless, it seems that there may be a certain value of dry roll radius of inertia, for which a best matching could be achieved between predictions by the reduced 3DOF method and the IMO method. This aspect will be further investigated in Section 4.2.

In Figure 2, a similar plot is presented, but this time comparing the natural roll frequencies estimated by the reduced 3DOF method and by the Bureau Veritas 2019 rules formula for inland navigation vessels ($\omega_{\varphi,BV19}$). Because the BV 2019 formula (14) separates dry and additional roll radius of inertia, it was possible to use the same value of k_{xx} for both the reduced 3DOF and the BV 2019 methods. This is clearly reflected in the results in Figure 2, as less scattering is noticeable compared to Figure 1. However, the formula (14) assumes an added inertia proportional to the dry inertia, but from a hydrodynamic perspective, it appears more justifiable to consider the added inertia as a term additive, not proportional, to the dry inertia.

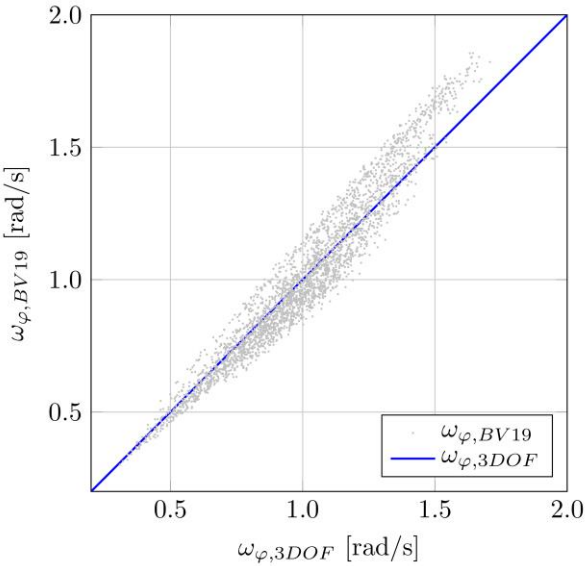

Figure 2 Comparative plot of estimated natural roll frequency: reduced 3DOF method and BV 2019 inland method.

4.2 Modification of IMO formula

As the greatest advantage of the considered semi-empirical methods is their simplicity, it would be interesting to try to improve them in an equally simple way.

For that purpose, the IMO method was selected as a good candidate, having similar trends to the results obtained by the reduced 3DOF method (see Figure 1), but also because it may be possible to make use of its formula as a basis for the total roll radius of inertia. Indeed, it seems that separating the dry from the additional inertia in additive form, rather than in proportional form as in (1), could lead to better estimation results.

Therefore, it is considered that the natural roll frequency can be expressed in the following form:

$$\begin{cases} \omega_\varphi = \sqrt{\dfrac{m \cdot g \cdot \overline{GM}}{I_{xx} + \delta I_{xx}}} = \sqrt{\dfrac{g \cdot \overline{GM}}{k_{xx}^2 + \delta k_{xx}^2}} \\ I_{xx} = m \cdot k_{xx}^2 \\ \delta I_{xx} = \rho \cdot \nabla \cdot \delta k_{xx}^2 = m \cdot \delta k_{xx}^2 \end{cases} \tag{15}$$

where I_{xx} and k_{xx} are the dry inertia and corresponding roll radius of inertia, while δI_{xx} and δk_{xx} are the added inertia and corresponding roll radius of added inertia. Accordingly, the total inertia I'_{xx} and the corresponding wet roll radius of inertia k'_{xx} can be expressed as:

$$\begin{cases} I'_{xx} = I_{xx} + \delta I_{xx} = m \cdot k'^2_{xx} = \rho \cdot \nabla \cdot k'^2_{xx} \\ k'_{xx} = \sqrt{k_{xx}^2 + \delta k_{xx}^2} \end{cases} \tag{16}$$

The modification of the IMO method was therefore carried out as follows. The first step has been to find the value of the dry roll radius of inertia, k^*_{xx}, that, if used in the reduced 3DOF calculations, provides the same natural roll period as that obtained from the IMO method, for each vessel and each loading condition in the database. Then, the average of all identified nondimensional dry roll radii, i.e. k_{xx}^{*}/B, has been determined, corresponding to $k_{xx}^{*}/B = 0.362$ for the presented database of river-sea ships. This average value was then used as reference for the separation of the added inertia effects.

Subsequently, the following formula was developed, which is more versatile for the natural roll frequency estimation, provided that the value of dry roll radius of inertia, k_{xx}, is known:

$$\begin{cases} \omega_{\varphi} = \sqrt{\dfrac{g \cdot \overline{GM}}{k_{xx}^2 + \delta k_{xx}^2}} \\ \delta k_{xx} = \sqrt{\left(\dfrac{C \cdot B \cdot \sqrt{g}}{\pi}\right)^2 - (0.362B)^2} \end{cases} \tag{17}$$

where C is the coefficient defined in the IMO formula, as shown in equation (13). Considering that $\sqrt{g}/\pi \approx 1$, the roll radius of added inertia can be simply written as:

$$\delta k_{xx} = \sqrt{(CB)^2 - (0.362B)^2} \tag{18}$$

The benefit of the modified IMO formula is shown in Figure 3, where the natural roll frequency estimated by reduced 3DOF method is compared to the natural roll frequency estimated by the newly modified IMO formula ($\omega_{\varphi,IMO-mod}$). It can be seen that the semi-empirical estimation is now much more in line with the values estimated by the 3DOF method, with a tendency of larger absolute scattering as the natural frequency increases. However, a check of the data indicates that the relative scattering is practically constant, with a bias tendency towards slight overestimation at low frequencies and slight underestimation at high frequencies. Therefore, with the rather simple modification introduced in (17), the standard IMO formula can be made, in principle, more flexible and more reliable for inland vessels of the type represented by the considered database. Moreover, the modified formula (17) shows less scattering and more reliable estimations than the BV 2019 formula (14) (compare Figure 2 and Figure 3).

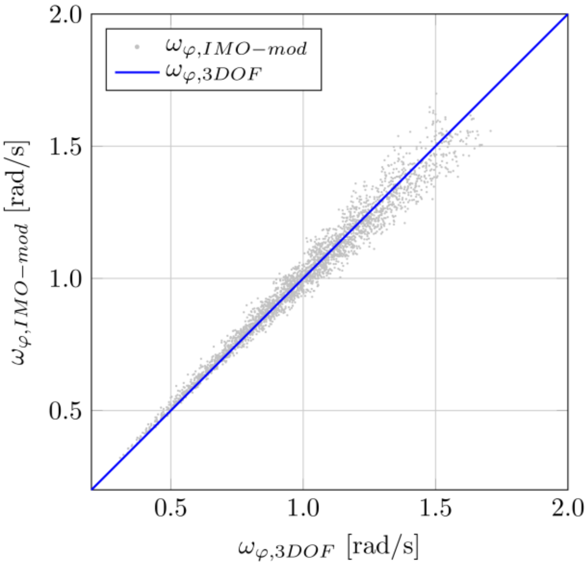

Figure 3 Comparative plot of estimated natural roll frequency: reduced 3DOF method and modified IMO method.

However, a crucial problem persists (or more clearly arises), i.e. the lack of a simplified procedure for the estimation of the dry roll radius of inertia. Such a procedure could significantly improve the accuracy of the natural roll period estimation. Therefore, the proposed modified IMO procedure only improves the estimation of the added moment of inertia.

It should be noted that the value $k_{xx}^{*}/B = 0.362$ is obtained using the described database of river-sea ships. Therefore, the modified IMO formula (17) is to be considered valid only for the considered ship type.

4.3 Average added inertia

Results from reduced 3DOF calculations have been further analysed, in order to identify a typical value for the added inertia for the considered type of ships. To this end, equation (15) was used to derive the ratio $\delta k_{xx} / B$ for each considered ship and loading condition, starting from the predicted roll natural frequency, as follows:

$$\frac{\delta k_{xx}}{B} = \sqrt{\frac{g \cdot \overline{GM}}{B^2 \cdot \omega_\varphi^2} - \left(\frac{k_{xx}}{B}\right)^2} \tag{19}$$

The average value of $\delta k_{xx} / B$ was then computed, and it was found that

$$\left(\frac{\delta k_{xx}}{B}\right)_{\text{average}} = 0.241 \tag{20}$$

The obtained average value of $\delta k_{xx} / B$ shows that δk_{xx} of river-sea ships is not small compared to the dry roll radius of inertia k_{xx}. Moreover, the average value given in (20) may be used as a first approximation of added inertia effects, and as a simple check of validity of a semi-empirical formula for ships of the type considered in this study.

5 CONCLUSIONS

The natural roll period is an important parameter in stability assessment. However, the development of methods for estimation of the natural roll period has not evolved at the same pace as the development of procedures for stability assessment. This observation applies to the natural roll period estimation of inland vessels too, as very few semi-empirical formulae for this type of ships exist.

In the paper, three methods to natural roll period estimation were considered, discussed and applied to a database of inland vessels: a method based on linear hydrodynamics (reduced 3DOF method), which takes into account coupling of sway and yaw with roll motion, and two semi-empirical methods, namely the Weather Criterion formula and a formula from Bureau Veritas rules for inland navigation vessels. The aforementioned methods were applied to a database comprising 31 river-sea ships.

It was shown that the considered semi-empirical methods can provide the same trend as the reduced 3DOF method, but the scattering with respect to the linear potential hydrodynamic predictions was found to be large. It was deemed that the reduced 3DOF method may estimate additional roll radius of inertia of river-sea ships more reliably than the considered semi-empirical formulae, as it is based on a more accurate hydrodynamic model of roll motion and coupling with sway and yaw. Therefore, in general, the reduced 3DOF can be considered to be more appropriate for the natural roll frequency estimation of river-sea ships.

However, the reduced 3DOF method could be somewhat cumbersome for routine design applications, due to the need of performing linear potential hydrodynamic calculations. Therefore, a simple modification of the IMO formula for the natural roll frequency estimation was proposed. The modified formula was derived from the processing of the natural roll frequencies, as obtained from the application of the reduced 3DOF method, for the considered database of river-sea ships.

In addition, indications were also provided regarding the typical added inertia for the considered type of ships. The reported average value of $\delta k_{xx} / B$, corresponding to the presented vessel database, is a suitable first-approximation value for the radius of additional inertia of river-sea ships. This value may be used, along with an assumed dry roll radius of inertia, for a quick initial estimation of the natural roll frequency.

A possible solution for the natural roll period estimation of river-sea ships could be sought in the development of a new semi-empirical formula derived from the analysis of systematic directly calculated results based on the linear potential hydrodynamic approach, such as those obtained for the purpose of this paper. Unfortunately, exploiting accurate hydrodynamic calculations, or tailored semi-empirical methods based on such calculations, improves only part of the estimation of natural roll period of river-sea ships, i.e. the added inertia term. Still, the dry roll radius of inertia k_{xx}, which is a fundamental part of the estimation process, remains associated to a large uncertainty. To the best of the authors' knowledge, there is no quick and simple readily available estimation procedure for the dry inertia of inland vessels in different loading conditions. In this respect, efforts would be necessary to develop simple methodologies, to avoid the necessity of resorting to complex direct calculations.

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7 REFERENCES

Bačkalov, I., 2019, "A probabilistic analysis of stability regulations for river-sea ships", In Contemporary Ideas on Ship Stability, Springer, pp. 707-723.

Benford, H., 1991, "Naval architecture for non-naval architects", Society of Naval Architects.

Blume, P., 1979, "Experimentell bestimmung von koeffizienten der wirksamen rolldämpfung und ihre anwedung zur abschätzung extremer rollwinkel", Schiffstechnik, Vol. 26, pp. 3-23.

Bulian, G. and Francescutto, A., 2009, "Experimental results and numerical simulations on strongly non-linear rolling of multihulls in moderate beam seas", Proceedings of the Institution of Mechanical Engineers, Part M: Journal of Engineering for the Maritime Environment, Vol. 223, pp. 189-210.

Bulian, G. and Francescutto, A., 2011, "Effect of roll modelling in beam waves under multifrequency excitation", Ocean engineering, Vol. 38, pp. 1448-1463.

Bulian, G., Francescutto, A., and Maccari, A., 2008, "Possible simpliffied mathematical models for roll motion in the development of performance-based intact stability criteria - extended and revised version", Quaderno di dipartimento no. 46, Department of Naval Architecture, Ocean and Environmental Engineering, University of Trieste, Trieste.

BV, 2019, "Rules for the classification of inland navigation vessel", NR 217.B1 DT R05 E, Bureau Veritas, Paris.

Fossen, T., 2011, "Handbook of marine craft hydrodynamics and motion control", John Wiley & Sons.

George, W. E. (Ed.)., 1983, "Stability and Trim for the Ship's Officer", Cornell Maritime Press.

IMO, 2008, "MSC.1/Circ.1281: Explanatory Notes to the International Code on Intact Stability, 2008", 9 December, International Maritime Organisation, London.

IMO, 2020, "MSC.1/Circ.1627: Interim guidelines on the second generation intact stability criteria", 10 December, Submitted by Japan, International Maritime Organisation, London.

IMO, 2021a, "International Code on Intact Stability, 2008 (2008 IS Code)", Consolidated Edition, International Maritime Organisation, London.

IMO, 2021b, "Draft explanatory notes to the Interim guidelines on the second generation intact stability criteria", under discussion within the SDC/IMO Intersessional Correspondence Group on Intact Stability.

Kato, H., 1956, "Approximate Methods of Calculating the Period of Roll of Ships", Journal of Zosen Kiokai, Vol. 89, pp. 59-64, in Japanese.

Munro-Smith, R., 1973, "Ships and naval architecture", Institute of Marine Engineers.

Papanikolaou, A., Boulougouris, E., Spanos, D., 1997, "On the Roll Radius of Gyration of RoRo Passenger Ships", Proc. 7th International Offshore and Polar Engineering Conferences (ISOPE 1997), 25-30 May, Honolulu, USA, pp. 499-506.

Peach, R. and Brook, A., 1987, "The radii of gyration of merchant ships", North East Coast Institution of Engineers & Shipbuilders Transactions, 103 (3), pp. 115-117.

Rudaković, S., and Bačkalov, I., 2019, "Operational limitations of a river-sea container vessel in the framework of the second generation intact stability criteria", Ocean Engineering, Vol. 183, pp. 409-418.

Rudaković, S., Bulian, G., and Bačkalov, I., 2019, "Effective wave slope coefficient of river-sea ships", Ocean Engineering, Vol. 192, Article 106427, pp. 1-16.

Tasai, F., 1971, "On the Sway, Yaw and Roll Motions of a Ship in Short Crested Waves", Reports of Research Institute for Applied Mechanics, Kyushu University, Vol. XIX, No. 62, 36p.

Yamagata, M., 1959, "Standard of stability adopted in Japan", Transaction of the Institution of Naval Architects, Vol. 101, pp. 417-443.