ONE APPROACH TO THE NONLINEAR BEHAVIOUR OF THIN WALLED BEAMS WITH SYMMETRICAL OPEN SECTION

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The principle of virtual work is applied to thin walled beams with a cross section having their middle line of an arbitrary curvilinear shape and with continuously varying thickness. Six equilibrium equations and a seventh one relating the constrained torsion are derived taking into account the general sectorial coordinates. The obtained relations are applied to the turbine blade shaped elements with one plane of symmetry. All necessary geometrical characteristiques are calculated for one element of the modified cross section shape and it is shown that it is not recommendable to neglect the influence of those secondary effects.

1. INTRODUCTION

The linear "classical" theory of thin-walled open section beams was extended in [1] by including the secondary sectorial coordinates. Later on in [2] the second order theory was extended to thin-walled members with an open cross section having an arbitrary polygonal middle line. The similar approach is applied in this paper to the sections with a middle line of an arbitrary curvilinear shape and with constantly varying thickness.

The authors tried to investigate the influence of second order terms on the behaviour of the examined beam members.

2. BASIC ASSUMPTIONS

Normal stresses in cross sections in "classical" theory are supposed to be constant across the wall thickness and proportional to the sectorial coordinates. In [1] they obtain more complex distribution if the secondary sectorial coordinate is introduced and if its distribution is supposed to be linear across the wall thickness.

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4.3 *Stresses*

Stress components which appear in the considered case are normal and shear stresses caused by bending and torsion. Particular attention will be paid here to the constrained torsion effects as well as to the stress components which are their consequences because the secondary effects become evident in that case.

Normal stresses are proportional to the sectorial coordinate and it is necessary to remind that the secondary sectorial coordinate will be taken into the consideration together with the total sectorial moment of inertia calculated using (4.22).

5. NUMERICAL EXAMPLE

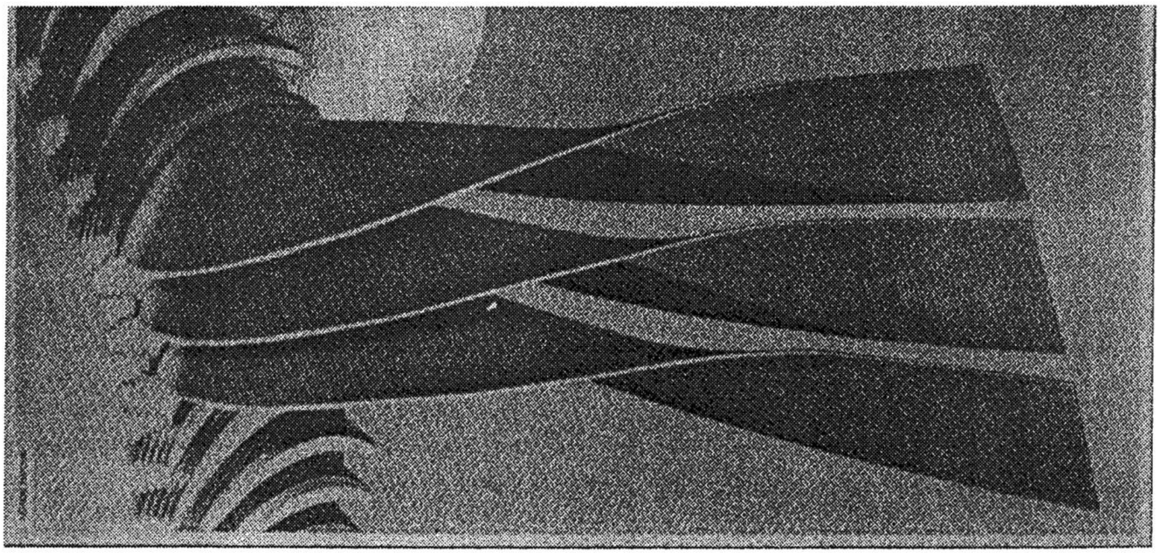

Fig.1. Steam turbine blades

The real steam turbine blades (Fig.1) usually have cross sections of the shapes which are shown [3] in Fig.2 and in most cases they can be treated as thin walled sections of nonconstant thickness.

The main problem consists in the choice of a function by which the variation of the thickness along the section can be described.

In order to apply the derived expressions to thin walled beams with cross sections which are the approximations of those given in Fig.2 one element of the modified shape with one axis of symmetry is chosen and shown in Fig.3.

The chosen shape of the cross section is supposed to have nonconstant thickness and to be defined by the expression

$$t(s)=t(\varphi)=t_0-(t_0-t_1)\varphi/\alpha, \tag{5.1}$$

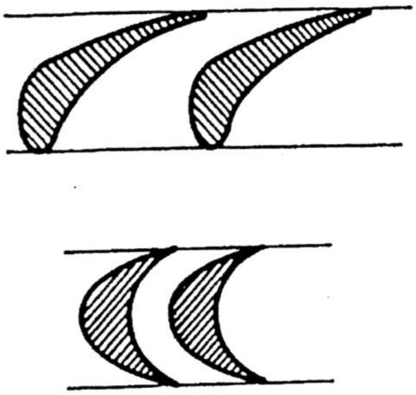

Fig.2. Real sections

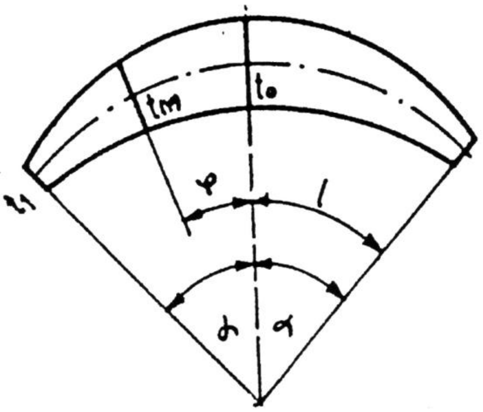

Fig.3. Approximated section

5.1 *Geometrical characteristics of the chosen cross section*

The expressions by which the geometrical characteristics which are needed for the calculation are derived starting from (5.1) and they are :

Area

$$A = Rt_0\alpha(1+t_1/t_0) \tag{5.2}$$

Torsional constant

$$I_t = (1/3)\int t(\varphi)^3 R d\varphi = (Rt_0^3\alpha/6)[1+t_1/t_0+(t_1/t_0)^2+(t_1/t_0)^3], \tag{5.3}$$

$$\begin{aligned}
&J_\omega / R^5 t_0 = \\
&= (\alpha^3/6)(1+3t_1/t_0)+4e/R\{(1/\alpha)(1-t_1/t_0)(\alpha.\sin\alpha+ \\
&+2\cos\alpha-2-(e/8R)\sin^2\alpha)+(t_1/t_0)(1+t_0^3/12t_1R^2) \\
&(\alpha\cos\alpha-\sin\alpha)+(e/8R)[(1+t_1/t_0).\alpha-(t_1/t_0).\sin 2\alpha]+ \\
&+(t_0/2R)^2[-(1/\alpha)(1-t_1/t_0)(\alpha^2.\cos\alpha-2\alpha.\sin\alpha-2.\cos\alpha+2)+ \\
&+(1/\alpha)^2(1-t_1/t_0)(\alpha^3\cos\alpha-3.\alpha^2\sin\alpha-6.\alpha.\cos\alpha+6.\sin\alpha)- \\
&-(4/3)(1/\alpha)^3(1-t_1/t_0)^3((\alpha^4/4)\cos\alpha-\alpha^3\cos\alpha-3.\alpha^2\sin\alpha+ \\
&+6.\alpha\sin\alpha+6.\cos\alpha-6)]\}+(t_0/2R)^2(e/R)^2[\alpha-\sin\alpha.\cos\alpha- \\
&-(3/2\alpha)(1-t_1/t_0)(\alpha^2-\alpha.\sin 2\alpha+\sin^2\alpha)+(3/2\alpha^2)(1-t_1/t_0)^2. \\
&.(2\alpha^3/3-\alpha^2\sin 2\alpha-\alpha+2\alpha.\sin^2\alpha+\sin\alpha.\cos\alpha)- \\
&-(3/4\alpha^3)(1-t_1/t_0)^3(\alpha^4/3-(2\alpha^3/3).\sin 2\alpha+2\alpha^2\sin^2\alpha-4\alpha^2+ \\
&+\alpha\sin 2\alpha-\sin^2\alpha)]
\end{aligned} \tag{5.4}$$

Besides them only the first and the third of the quantities defined by expressions (4.20) will be shown here

$$\begin{aligned}
&\beta_1/(R^2 t_0^3/48J_2)= \\
&=12.[1+4\rho_0^2(1+\eta^2)](1-\cos\alpha)-4\eta(1+12.\rho_0^2)\sin^2\alpha- \\
&-12.(1-t_1/t_0)(1/\alpha).[3+4\rho_0^2(1+\eta^2)(\alpha-\sin\alpha\cos\alpha-2\alpha\sin^2\alpha)]- \\
&-6.(1-t_1/t_0)^2(1/\alpha^2)[12(1-\alpha.\sin\alpha.-\cos\alpha+(\alpha^2/2).\cos\alpha)- \\
&-\eta.(\alpha-\alpha\sin 2\alpha+\sin^2\alpha-2\alpha^2\sin^2\alpha)]+ \\
&+3.(1-t_1/t_0)^3(1/\alpha^3)\{12.(2\sin\alpha-2\alpha\cos\alpha-\alpha^3\cos\alpha)+ \\
&+\eta.[\alpha(1-2.\sin^2\alpha)(3-\alpha^2)-3(1-2\alpha^2)\sin\alpha.\cos\alpha]\}
\end{aligned} \tag{5.5}$$

$$\begin{aligned}
&\beta_\omega/(R^3 t_0/120J_\omega)= \\
&=10(1+12\rho_0^2(1+\eta^2))\alpha^2+20(e/R)(1-12\rho_0^2(1+\eta^2))(1-\cos\alpha)+ \\
&+480\rho_0^2\eta(1-\alpha\sin\alpha-\cos\alpha)-20(e/R)\eta(1+12\rho_0^2)\sin^2\alpha- \\
&-5(1-t_1/t_0)(1/\alpha)[4+16\rho_0^2(1+\eta^2)\alpha^3-96\rho_0^2\eta(2\alpha\cos\alpha-\sin\alpha+ \\
&+\alpha^2\sin\alpha)+12(e/R)(1-4\rho_0^2(1+\eta^2))(\alpha\cos\alpha-\sin\alpha)+ \\
&+6(e/R)\eta(1+4\rho_0^2)(\alpha-2\alpha\sin\alpha-\sin\alpha\cos\alpha)]- \\
&-15(1-t_1/t_0)^2(1/\alpha)^2[8-8\alpha\sin\alpha-8\cos\alpha+4\alpha^2\cos\alpha)+ \\
&+2(e/R)\eta(2\alpha\sin\alpha-\alpha^2+2\alpha^2\sin^2\alpha-\sin^2\alpha)-\alpha^4]- \\
&-(1-t_1/t_0)^3(1/\alpha)^3\{20(e/R)(6\alpha\cos\alpha-6\sin\alpha+3\alpha^2\sin\alpha-\alpha^3\cos\alpha)+ \\
&+5(e/R)[[1-2\sin^2\alpha)(3\alpha-2\alpha^3)+3(1-2\alpha^2)\sin\alpha\cos\alpha]+4\alpha^5\}
\end{aligned} \tag{5.6}$$

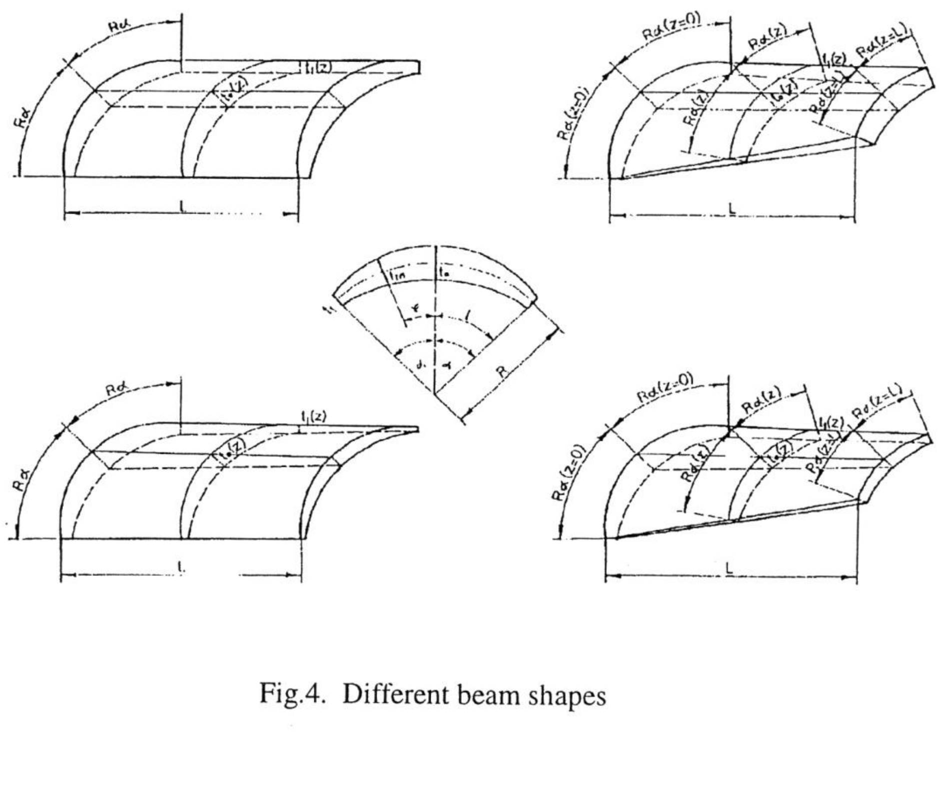

Fig.4. Different beam shapes

The stresses which appear as the consequences of the constrained torsion were calculated for the unit loads because the aim of the investigation was to find out if it is necessary or not to take into account the additional secondary effects and secondary sectorial coordinates. As the shear stresses and particularly those induced by the St. Venant torsion are not small the equivalent stresses were calculated using the Tresca approach.

Calculations were done for seversl straight beams (Fig.4) of the chosen section and for the dimensions L = 900 mm, R = 300 mm and $\alpha = 30°$.

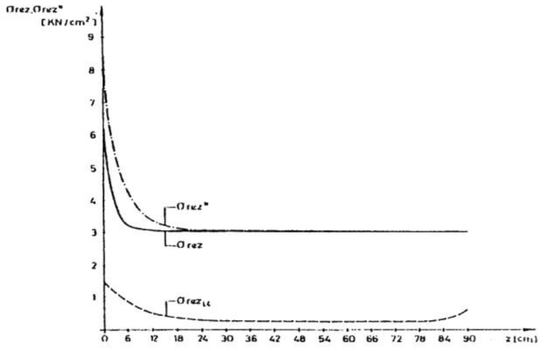

Fig. 5. Equivalent stresses (distribution of maximal values along the beam)

The distribution of maximal values of equivalent stresses is shown in Fig.6. The stress values connected with secondary effects are denoted by asterix.

6. CONCLUSION

From the obtained results it can be seen and concluded that when secondary effects are taken into account the differences between the stresses with and without those effects are quite remarquable at the clamped ends and decrease towards the middle of the span of the beam. The clamped ends are at the same time just the places where the blades are connected with the rotor body and which even because of that represent the critical points.

The explained problem is particularly evident in the cases of the cross sectional shapes similar to those considered in the chosen example.

BIBLIOGRAPHY

1. *Kollbruner, C. F., Hajdin, N.,* Dunnwandige Stabe, Bd.1, Springer-Verlag, Berlin-Heidelberg-New York, 1972.
2. *Hajdin, N.,* A contribution to the non-linear theory of thin-walled member with open cross section (in Serbian). Symposium on Contemporary problems of nonlinear continuum mechanics, Tara, Serbia, 1980.
3. *Vasiljević, N.,* Steam Turbines (in Serbian), Mašinski fakultet, Beograd, 190.
4. *Ružić, D.,* Strength of Structures (in Serbian), Mašinski fakultet, Beograd, 1995.
5. *Rnjak N.,* A contribution to the analysis of behaviour of thin-walled structural elements of turbine blade shape (in Serbian), Belgrade, 1993.