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ABSTRACT BOOK



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Influence of boundary conditions on natural frequencies of axially functionally graded Euler-Bernoulli cantilever beam with attached rigid body at the free end

Aleksandar Tomović

Faculty of Mechanical Engineering, University of Belgrade
Kraljice Marije 16, 11000 Belgrade, Serbia—
atomovic@mas.bg.ac.rs

Abstract

Functionally graded materials can be designed to satisfy a specific project conditions. Elastic beams made of functionally graded materials have been thoroughly analyzed in the last decades. The literature differs two types of beams of functionally graded materials, namely, sandwich beams and axially functionally graded (AFG) beams. The latter ones will be discussed in this presentation. AFG beams are designed in a manner that mass and/or material characteristics may change along the longitudinal axes of a beam, while the cross sectional area may be constant or may, also change along the axis. Modal analysis is of the utter importance in structural engineering in order to avoid critical behavior of a structure. The procedure for computation of natural frequencies using the Symbolic-Numeric method of initial parameters in differential form (SNMIP) is presented in [2] for the case of axial and transverse vibration of AFG beams.

The idea is to discuss the topic of the dependence of natural frequencies to various boundary conditions in the case of an AFG cantilever beam. Governing equations [1] are written in a form suitable for the application of the SNMIP. The Euler-Bernoulli beam theory is implemented in modeling elastic behavior of the beam. Boundary conditions can cause coupling in mode shapes which will be discussed through the example of an AFG cantilever beam with a heavy rigid body eccentrically displaced at the free end of the beam. Even though the equations of axial and transverse vibrations are not coupled the coupling can occur due to boundary conditions as it is presented. For different values of eccentricities coupling of mode shapes will be discussed and the changes in natural frequencies of a cantilever beam presented.

Keywords: Elastic beam, Vibration, SNMIP

References

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