Optimization of the bucket wheel boom length using structural reliability approach

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In this paper, structural reliability is applied to compare the original bucket wheel boom (BWB) of excavator SchRs 740 and the series of boom structures obtained by lengthening the original boom from 0 to 10 m. Stress field of BWE is acquired using finite element method. Then, stresses and yield criterion are modeled using probability density functions to account for uncertainties in their randomness. This stochastic approach is used to determine BWB's reliability, reliability index and probability of failure, with respect to various cases of optimization, namely, structure lengthening. Consequently, it enabled the evaluation, quantification and comparison of overall response of the similar structures taking into account range of stress rather than conventionally based single value approach. So, structural reliability approach can be used to evaluate, compare similar structures and thus, provide more sophisticated assessment than most of the structural analysis that are deterministic in their nature.

Keywords: *structural reliability, reliability index, bucket wheel boom, bucket wheel excavator (BWE) SchRs 740, finite element analysis, stress analysis.*

1. INTRODUCTION

In every day engineering practice, the structural response of large and complex steel structures is evaluated deterministically by comparing the single most extreme stress value to the single criterion. Criterion is generally defined thought the codes and regulations of the specific industry. It is mostly defined as a yield limit of the material. Furthermore, according to the regulations, the yield limit is often assumed as reduced to account for the safety of the structure. This is conventional and practical approach in terms of classifying the structure as failed or not failed in complying the criterion. However, it does not grade the structures that satisfied the criterion meaning that, for instance, all "not failed" structures would be labelled as equally safe in their response. This is not the case in practice. Some structures have the larger safety margin than others and hence, should be addressed. Moreover, such safety margin is not the same in all concerned locations within the structure.

In order to address the vast complexity of the response, the reliability methods try to introduce evaluation, quantification and comparison of similar structures while accounting their overall condition and uncertainties. Here, this stochastic approach is used on an example of the bucket wheel boom (BWB) structure with respect to its various cases of lengthening. Theoretical background and formulations used in structural reliability method can be found in literature, see [1, 2, 3].

Nonetheless, similar structures are investigated in terms of their reliability [4, 5]. However, most of analyses were based on age related data on failure of mechanical systems and not on the pure structural response as in this paper. Somewhat the analogous approach as in here is also applied in case of large steel hull structure, presented in [6].

This paper tries to separate structural from mechanical system failure evaluation. Authors mapped the BWB structure semi-arbitrarily while accounting for the various locations of interests on global structure that also included stress concentrations. Using finite element method, Von Mises stresses are acquired for all mapped locations and then treated as random values that represent the overall stress response. Stresses are compared to the yield criterion and code recommended share of the yield limit. Both stress and criterion are modeled via probability density functions. Such approach allowed the calculation of structure's corresponding reliability of failure, reliability and reliability index. The same is provided in the case of 10 cases of lengthening of the BWB structure along with various combinations of the criterion, in order to quantify the reliability of each case.

2. BWB COMPUTATIONAL MODEL

Bucket wheel boom (BWB) of the bucket wheel excavator SchRs 740 is used for the analysis. The same structure has been investigated in [7], in which the idea of lengthening was presented, but the assessment mechanism was simplified. In this paper, quantification and comparison of overall response of the original and

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elongated structures is taking into account range of stress rather than conventionally based single value approach.

Finite element analysis (FEA) is performed using custom made finite element method software KOMIPS developed at the Department of the Strength of Structures of the Faculty of the Mechanical Engineering (University of Belgrade) by [8]. Taking into account the appearance of the excavator SchRs740 boom structure, the structure was modelled by short beam elements, which means that the shear stress due to bending is taken into account. Classical beam theory is applied. Beside the truss steel structure of the boom, all the other elements that affect the rigidity of the boom structure were considered. That means that the model includes the following elements: transverse stiffeners, shafts of the wheel and return drum, torque leverage of both gearboxes, parts of the belt structure and stays. The boundary conditions represent how the boom is physicaly atached to the rest of the BWE structure.

Von Mises stresses are calculated for the single typical loading for which the structure is subjected most of the time and include steel weight (dead load) and working load. As workload is concerned, the overall digging force of 250 000 N was distributed in real terms in the three forces (vertical 1, lateral 0.3 and radial 0.15). Also weight of the wheel (200 000 N) and both gearboxes (by 80 000 N) are taken into calculation. Boundary conditions and loads are shown in Fig. 1.

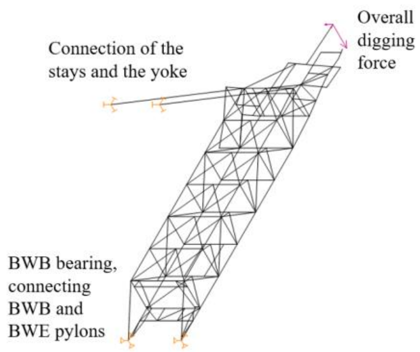

Figure 1. Bucket wheel boom, computational model, boundary conditions and loads

Analysis type is linear-static. BWB material of structure is steel S355J2G3 having modulus of elasticity of 210 GPa, poisson's ratio of 0.3 and a yield limit of 355 MPa.

Boom has been extended in a step of 1 m, without changing the cross-sections of beams, height and width of the truss. Beam was extended from 1 to 5 m, so that the extension was uniformly distributed on five segments in the middle. Total extension of 6 m involves a new segment, and the entire extension now is distributed to six (5 + 1) segments. Further extensions are done by retaining the inserted segment, and the total length is now distributed to six segments. The original stays of the existing boom were used for each additional extended boom. In addition, stays position was dictated by the geometry (length) of the yoke and stays, which means that it is not the same for the original and the extended booms.

The actual length of the basic boom is 34.93 m (hereinafter referred to as about 35 m long), and its weight is 64 869 kg.

As mentioned in previous section, the structure is mapped to identify the locations which would represent the overall geometry of the object. Locations include zones of high stress concentrations as well as ones important for the structural behavior independent of their actual structural response. Using this approach, more realistic response could be acquired rather than one based on just an extreme. Most of mapped locations are presented in Fig. 2. Location 23, 24, 25 and 26 are placed in the right wall, location 27 is a part of the truss near bucket wheel, location 28 and 29 represent the stays, and location 30 and 31 are spots in vertical tuss that is normal to boom direction placed near the boom bearing.

A corresponding FEA obtained Von Mises stresses in mapped locations are shown in Table 1. Stresses are obtained for the nominal case in which the lengthening of the object is not performed (Case 0). Case 1 represents the same structure lengthened by 1 m. Likewise, other cases are labeled according to their amount of lengthening.

3. STRUCTURAL RELIABILITY METHODOLOGY

Reliability theory and equations used in this paper is thoroughly explained and given in [1, 2, 3]. Structure's demand is labelled as D and is represented by the stress field (distribution). Capacity of the structure labelled as C and is represented by the criterion. Margin function, M, is a limit state function and is defined as in (1):

$$M = C - D \tag{1}$$

If $M > 0$ in all considered locations (or conditions), the capacity is larger than a demand (criterion larger than stresses) so that the structure can be considered as safe having reliability equal to one and probability of failure equal to zero. If $M < 0$, then a structure is failed to satisfy limit function and therefore, is considered as not safe. The corresponding reliability and probability that the limit function has failed depends on the number of locations and their stress values that failed to satisfy the criterion. Thus, choosing a limit state function is very sensitive since this significantly influences the safety.

3.1 Stresses

Histogram of Von Mises stresses for case 0 and case 1 is shown in Fig. 3, based on data from Table 1.

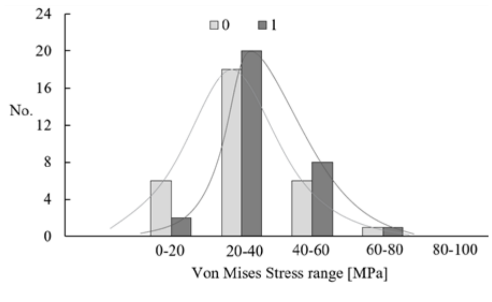

Figure 3. Von Mises stress histogram for case 0 and case 1

One could notice that the stresses are forming normal distribution where the mean value and most of the stresses are grouped in between 20-40 MPa. Remaining cases (2-10) are showing the similar distribution. Therefore, a probability density functions of normally distributed random values of each case from the Table 1 is produced, see Fig. 4 and formula (2) taken from [2], according to their own statistical parameters: mean value (μ), standard deviation (σ), random variable – stress (X). Statistical parameters for each of the case is given in Table 2.

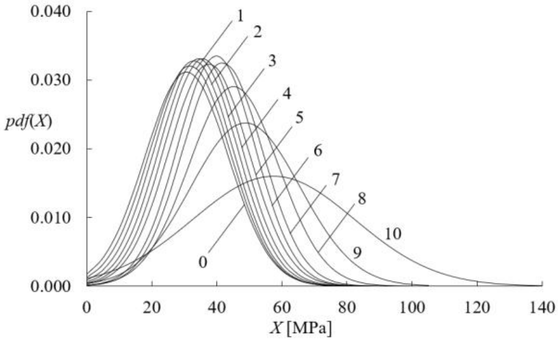

Figure 4. Probability density functions for cases 0-10

$$pdf(X) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-0.5 \cdot \left(\frac{X-\mu}{\sigma}\right)^2\right]$$

$$\mu = \frac{\sum_{i=1}^{n} X_i}{n} \quad (2)$$

$$\sigma^2 = \frac{\sum_{i=1}^{n} (X_i - \mu)^2}{n} \rightarrow \sigma = \sqrt{\sigma^2}$$

3.2 Criterion

Yield limit distribution is used as a criterion. In practice, such criterion is reduced by 1.5 or 2 times and such is called a safety factor. Despite, a reduction factor would be defined as ratio between a reduced yield criterion and a yield limit. For instance, if yield limit is reduced by 1.5 times (or 2 times), then the new criterion is equal to 355 MPa/1.5 (2) = 236.67 MPa (177.5 MPa). Consequently, a corresponding reduction factors would be defined as 235.67 MPa/355 MPa = 0.67 and 177.5 MPa/355 MPa = 0.5.

Mean value and standard distribution of the material used for the purpose of this analysis are taken from investigations given in [9] and presented in Table 3. Probability density function of criteria, assuming their normal distribution which is mostly used in literature, is also derived according to formula (2) and presented in Fig. 5 along with stress *pdfs*.

Table 3. Criteria statistical parameters

σ_y [MPa]	355	355	355
σ_y/σ_{all}	1	1.5	2
σ_{all} [MPa]	355.00	236.67	177.50
RF = σ_{all}/σ_y	1.00	0.67	0.50
μ	405.7	270.47	202.85
σ	69.1	69.1	69.1
$\sigma_{all,\,min}$ [MPa]	350		
$\sigma_{all,\,max}$ [MPa]	602		

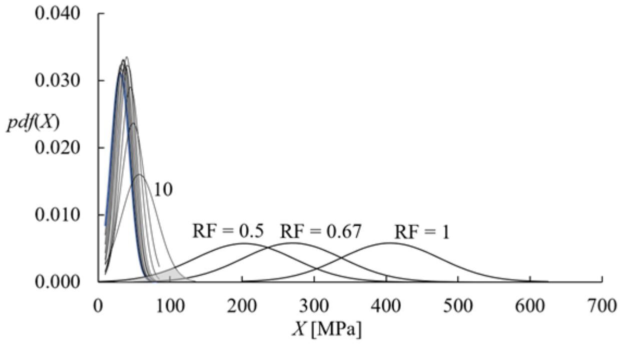

Figure 5. Probability density functions of stress (case 0-10) and criterion (RF = 1, 0.67, 0.5)

4. MARGIN FUNCTION AND MONTE CARLO METHOD

Shaded area in Fig. 5 represents the zone of overlapping of stress pdf (case 10) and criterion (*RF = 0.5*). This area is denoting to the failure criteria of limit state function M. The probability of failure P_f is calculated according to formula (3). Function $f(\sigma_{VM}, \sigma_{all})$ is a joint probability density function (jpdf) of the both Von Mises and the criterion domains.

$$P_f(M = C - D < 0) = \iint_{C-D<0} f(\sigma_{VM}, \sigma_{all})\,dx\,dx \quad (3)$$

Monte Carlo method is used here to calculate an area of *jpdf* in which the margin is negative. For the purpose of this analysis, a 10000 of random sets are produced counting the ones that satisfied and ones that did not satisfy $M < 0$ limit state function. Number of random number sets that satisfied $M < 0$ are divided by the total number of sets in order to calculate probability of failure (P_f). Reliability is then calculated as $1-P_f$. Summary of Monte Carlo method is given in (4):

$$P_f \approx \frac{n(M<0)}{N} \quad (4)$$
$$R = 1 - P_f$$

Additionally, according to (5), a margin function parameters are calculated for each of considered cases, namely: mean value, standard deviation, reliability index, probability of failure and reliability. Based on stress and criterion distributions, probability density function of M is derived and plotted for extreme cases (0 and 10) according to various reduction factors, see Fig. 6. Note that area of *pdf(M)* covering negative values of *X* is representing the probability of failure in which margin function is negative.

$$\mu(M) = \mu(\sigma_{VM}) - \mu(\sigma_{all})$$
$$\sigma(M) = \sqrt{\sigma^2(\sigma_{all}) + \sigma^2(\sigma_{VM}) - 2\rho\sigma(\sigma_{all})\sigma(\sigma_{VM})}$$
$$\beta = \frac{\mu(M)}{\sigma(M)} \tag{5}$$

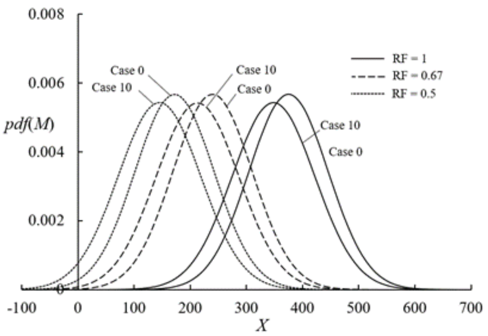

Figure 6. ***Pdf(M)***

Margin function statistical parameters for all variations are shown in Table 4 and illustrated in Fig. 7.

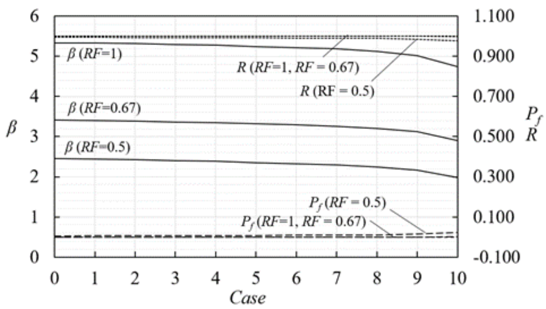

Figure 7. Statistical parameters

Taking into account random stresses and criteria used in analysis, reliability calculations show that lengthening of the BWB up to 8 m has relatively small effect on the probability of failure, reliability and reliability index. This negligible difference in reliability is also related to the variation of the criterion, i.e., reduction factor. Above the 8 m of lengthened structure, the reliability tends to drop more significantly, but the structure appears still in the somewhat safe zone, since in all considered cases the reliability is close to 1. Additionally, a selection of reduction factor with respect to the yield criterion, in this example, has a larger influence on the reliability than a lengthening variation.

5. CONCLUSION

The paper is introducing the pure stress based structural reliability assessment of the bucket wheel boom structure, performed for various cases. Apart from deterministic extreme value evaluation, here, a range of FEA obtained Von Mises stresses are compared to the criterion (yield limit and reduced yield limit), while both are modeled using normal distributions. This analysis takes into account safety margins of various locations on the objects to account the overall structural response. Moreover, limit state function is introduced in that regard in order to evaluate structural safety. Moreover, statistical parameters are derived for cases in which the structure is lengthened from 1 m to 10 m. Additionally, criterion is reduced to analyze its effect on reliability and probability of failure.

Structure's reliability is not so sensitive to lengthening, except for above 8 m. Moreover, reduction factor of 0.5 with respect to the yield limit has greater influence on reliability parameters: reliability, probability of failure, reliability index. Nevertheless, the structure exhibits high reliability indices in almost all cases which is understandable since the stresses are far lower than one denoted to the criteria.

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NOMENCLATURE

C	capacity of the structure (σ_{all})
D	demand of the structure (σ_{VM})
$f(\sigma_{VM},\sigma_{all})$	joint probability density function
jpdf	joint probability density function
M	margin function
n	numbers of variables (total or less than total)
N	total number of variables
$pdf(X)$	probability density function of a random variable X
P_f	probability of failure
R	reliability
RF	reduction factor of the yield stress limit
X	random variable (σ_{VM} or σ_{all})
β	reliability index
μ	mean value
ρ	correlation coefficient (taken as 0 here as two distributions are assumed as uncorrelated)
σ	standard deviation
σ_{all}	allowable stress (share of σ_y) range [MPa]
$\sigma_{all,\,max}$	max allowable stress according to the distribution range [MPa]
$\sigma_{all,\,min}$	min allowable stress according to the distribution range range [MPa]
σ_{VM}	Von Mises stress [MPa]
σ_y	yield limit stress [MPa]

Figure 2. Locations mapped in BWB model

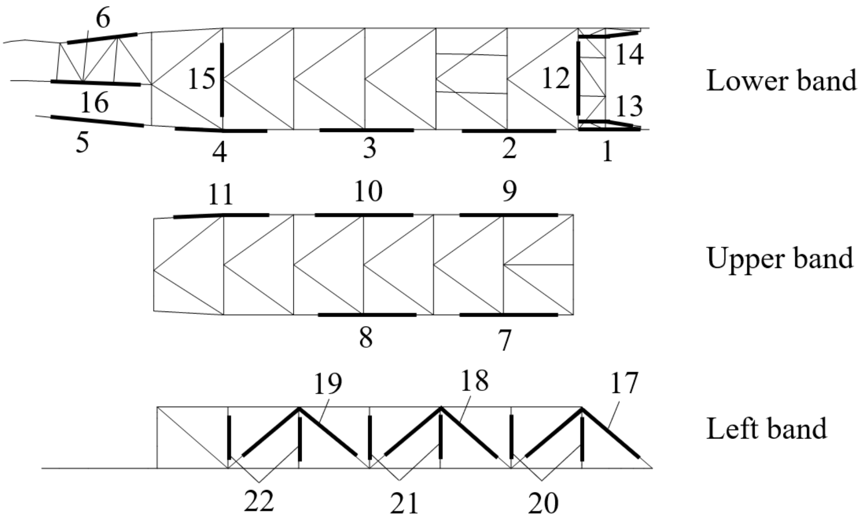

Table 1. Von Mises stress results at locations and by the lengthening case

Location	Cases - total lengthening [m]										
	0	1	2	3	4	5	6	7	8	9	10
	Von Mises stress [MPa]										
1	33.12	34.47	36.16	38.31	40.45	43.11	43.96	46.95	52.01	57.59	72.18
2	39.97	41.44	43.31	45.82	48.29	51.37	54.16	57.89	64.09	71.25	89.38
3	31.57	31.62	31.74	32.47	32.80	33.29	38.44	40.86	45.05	49.99	63.15
4	36.39	36.23	36.08	36.38	36.32	36.25	33.98	34.60	36.05	37.30	42.43
5	41.58	41.46	41.20	41.21	41.05	40.77	39.40	39.04	38.42	37.69	36.36
6	27.12	27.37	27.61	28.07	28.38	28.72	27.39	27.39	27.54	27.24	27.44
7	19.89	21.75	23.80	26.02	28.43	31.17	33.54	36.68	40.89	46.17	56.47
8	15.52	17.91	20.92	24.34	28.18	32.61	39.21	44.82	52.18	62.31	81.44
9	14.39	15.91	17.55	19.36	21.34	23.57	24.38	27.19	30.72	35.76	44.81

10	16.05	18.47	21.18	24.19	27.62	31.59	35.30	40.50	47.28	56.96	75.03
11	20.32	23.38	26.97	31.06	35.93	41.85	33.47	39.52	61.01	84.81	133.00
12	32.21	32.69	33.51	34.51	35.56	37.07	38.46	39.94	43.09	45.92	55.12
13	40.73	40.90	41.63	42.11	42.87	44.26	44.20	44.99	47.67	48.90	56.10
14	29.01	28.56	28.56	27.90	27.62	27.73	26.35	25.15	24.87	22.16	20.11
15	26.49	27.04	27.67	28.68	29.35	30.14	39.28	39.03	38.56	38.08	37.19
16	48.99	49.34	49.26	50.17	50.45	50.42	37.52	38.03	38.48	39.26	39.91
17	19.06	19.87	20.82	21.81	22.91	24.21	29.92	32.78	36.67	41.38	50.47
18	25.67	27.64	29.89	32.28	34.90	38.00	43.37	46.82	51.85	57.62	70.13
19	27.48	27.46	27.38	27.31	27.07	26.68	33.49	33.59	33.69	33.88	34.78
20	31.96	33.99	36.27	38.58	41.07	43.92	40.34	42.37	45.28	47.23	51.88
21	36.79	39.52	40.01	45.73	49.07	52.88	49.95	52.92	57.24	60.39	67.98
22	34.58	36.52	38.61	40.78	42.93	45.25	34.83	35.19	35.74	35.07	38.31
23	18.78	20.33	22.08	23.84	25.62	27.65	27.20	29.37	32.69	35.00	41.18
24	17.92	19.28	20.80	22.36	23.94	25.70	26.98	27.94	29.50	30.38	36.04
25	17.95	18.33	18.70	18.95	19.07	19.11	22.48	21.90	21.25	18.92	16.55
26	25.78	26.18	26.59	26.99	27.30	27.57	49.83	50.73	51.36	53.20	56.59
27	23.44	23.64	23.91	24.21	24.56	25.03	49.19	50.20	51.88	53.41	57.46
28	41.44	42.66	44.04	45.78	47.84	50.31	53.47	57.86	63.87	73.57	92.96
29	37.96	39.21	40.69	42.47	44.59	47.24	51.28	55.68	62.07	71.50	91.49
30	77.59	77.40	77.83	77.86	78.29	79.40	82.25	82.39	84.54	83.88	86.72
31	41.82	41.97	42.76	43.18	44.05	45.65	48.87	49.75	52.69	53.49	58.29

Table 2. Stress statistical parameters

	0	1	2	3	4	5	6	7	8	9	10
μ	30.70	31.69	32.82	34.28	35.74	37.50	39.76	41.68	45.10	48.72	57.45
σ	12.79	12.44	12.14	12.04	12.04	12.31	11.90	12.27	13.75	16.81	24.91

Table 4. Margin statistical parameters

RF = 1											
Case	0	1	2	3	4	5	6	7	8	9	10
μ(M)	375.00	374.01	372.88	371.42	369.96	368.20	365.94	364.02	360.60	356.98	348.25
σ(M)	70.27	70.21	70.16	70.14	70.14	70.19	70.12	70.18	70.45	71.11	73.45
P_f	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
R	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
β	5.34	5.33	5.31	5.30	5.27	5.25	5.22	5.19	5.12	5.02	4.74
RF = 0.67											
Case	0	1	2	3	4	5	6	7	8	9	10
μ(M)	239.77	238.77	237.64	236.19	234.73	232.97	230.71	228.79	225.36	221.75	213.02
σ(M)	70.27	70.21	70.16	70.14	70.14	70.19	70.12	70.18	70.45	71.11	73.45
P_f	0.0002	0.0002	0.0002	0.0003	0.0003	0.0005	0.0005	0.0004	0.0008	0.0006	0.0022
R	0.9998	0.9998	0.9998	0.9997	0.9997	0.9995	0.9995	0.9996	0.9992	0.9994	0.9978
β	3.41	3.40	3.39	3.37	3.35	3.32	3.29	3.26	3.20	3.12	2.90
RF = 0.5											
Case	0	1	2	3	4	5	6	7	8	9	10
μ(M)	172.15	171.16	170.03	168.57	167.11	165.35	163.09	161.17	157.75	154.13	145.40
σ(M)	70.27	70.21	70.16	70.14	70.14	70.19	70.12	70.18	70.45	71.11	73.45
P_f	0.0063	0.0067	0.0075	0.0072	0.0082	0.0075	0.0090	0.0092	0.0116	0.0139	0.0235
R	0.9937	0.9933	0.9925	0.9928	0.9918	0.9925	0.9910	0.9908	0.9884	0.9861	0.9765
β	2.45	2.44	2.42	2.40	2.38	2.36	2.33	2.30	2.24	2.17	1.98