STRESS ANALYSIS OF THIN-WALLED STRUCTURAL ELEMENTS OF TURBINE BLADE SHAPE

Nina Anđelić, Vesna Milošević-Mitić, Taško Maneski, Ana Petrović

Faculty of Mechanical Engineering,
The University of Belgrade, Kraljice Marije 16, 11120 Belgrade 35
e-mail: nandjelic@mas.bg.ac.rs

Abstract. Structural elements of turbine blade shapes are considered from the point of view of the Theory of thin-walled beams. A particular attention is paid to the influence of secondary sectorial coordinates on the values of total normal stresses which are calculated with and without them into account. The expressions for geometrical characteristics are derived and a numerical example is given. All geometrical characteristics are calculated for one structural element having the modified cross-section shape and it is shown that it is not recommendable to neglect the influence of the secondary effects.

1. Introduction

Investigations of the behaviour of thin-walled members with open cross-sections were carried out extensively since the early works of Timoshenko [1] and Vlasov [2]. Besides mentioned authors, the theory of thin-walled structures was later developed by Kollbruner and Hajdin [3] and others and all of them showed that the cross-sections of thin-walled beams exhibit significant out-of-plane warping as a response to torsion.

The linear theory of thin-walled members with open cross-sections differs from the conventional form by including the change of longitudinal normal stresses along the wall thickness. The linear "classical" theory of thin-walled open section beams was extended in [3] by including the secondary sectorial coordinates. That procedure is still linear and later in [4] the second order theory was extended to thin-walled members with an open cross-section having an arbitrary polygonal middle line.

A similar approach is applied in this paper to the sections with a middle line of an arbitrary curvilinear shape and with constantly varying thickness. The main purpose of this paper is to present an approach of stress analysis of beam-type structures with a thin-walled open cross-section. The authors tried to investigate the influence of the introduction of second order terms on the results.

2. Calculation model

According to the classical linear theory of thin-walled open section beams [1, 2, 3], normal stresses in cross-sections are assumed to be constant across the wall thickness and proportional to the sectorial coordinates. In [3] the secondary sectorial coordinate is introduced and the normal stresses are assumed to have the distribution that is not constant but linear across the wall thickness.

Basic assumptions of the theory [2] of thin-walled beams are assumed to be valid:
(a) The cross-sections do not change their shape and their projections on the initial planes behave as rigid plates, (b) The shear deformation in the middle surface is neglected, (c) The line elements that are initially perpendicular to the middle surface remain straight and perpendicular during the whole deformation.

The real steam turbine blades [5] usually have the cross-sections of the shapes shown in Fig. 1 and they have to be treated as thin-walled sections of non-constant thickness. The real cross-sections were approximated for the calculations by the cross-section having one axis of symmetry (Fig. 1). Its symmetry obviously induced some simplifications in the used expressions, and the whole element was considered as clamped at one end and loaded in a complex way by continuously distributed lateral load and torque along its length.

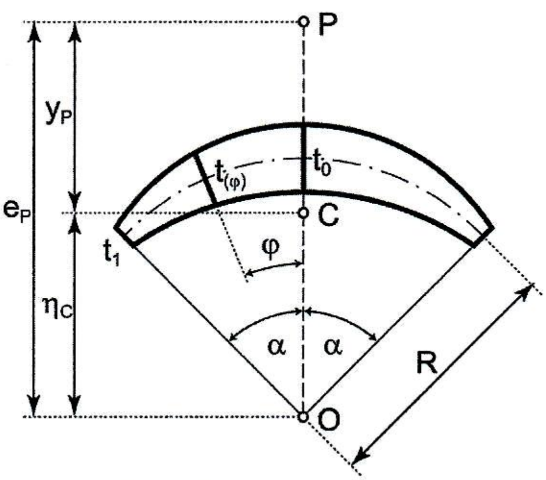

Figure 1. Cross-section

In Fig. 1:
- R is the radius of curvature of the middle line, e_P is the distance between the center of curvature O and the pole P (shear center), η_C is the distance between the center of curvature O and the centroid of the cross-section C, $y_P = e_P - \eta_C$.

The chosen cross-section having non-constant thickness $t(\varphi)$ (Fig.1) was defined by the following expression [4]

$$t(\varphi) = t_0 - (t_0 - t_1)\varphi / \alpha \tag{1}$$

where: $t_0 = t(\varphi = 0)$, $t_1 = t(\varphi = \alpha)$.

2. Calculation of normal stress

The expression for total normal stress σ [3, 6, 7] is

$$\sigma = \frac{N}{A} + \frac{M_x}{I_x} y - \frac{M_y}{I_y} x + \frac{B}{I_\omega} \omega = \sigma_N + \sigma_x + \sigma_y + \sigma_{\omega rez} \tag{2}$$

where:
- A is area of the cross-section, N is axial forse, B is bimoment, I_x, I_y are moments of inertia of the cross-sectional area about the centroidal axes x and y, I_ω is sectorial moment of inertia, M_x, M_y are bending moments around the centroidal axes x and y, σ_N, σ_x, σ_y, $\sigma_{\omega rez}$

are normal stresses caused by the axial forse, bending moments and the bimoment, respectively.

2.1. Geometrical characteristics

The expressions defining the geometrical characteristics needed for the calculation were derived in this chapter:

- Cross-sectional area

$$A = Rt_0\alpha(1+t_1/t_0) \tag{3}$$

- Moment of inertia of the cross-sectional area about the centroidal axes x and y,

$$I_x = R^3 t_0/2\alpha^3[1/(1+t_1/t_o)]\cdot\{\alpha^4(1+t_1/t_o)^2 + 2(t_1/t_o)\alpha^3\sin\alpha\cos\alpha(1+t_1/t_o)+$$
$$+\alpha^2\sin^2\alpha\left[1-9(t_1/t_o)^2\right]-8(1-t_1/t_o)(1-\cos\alpha)[2(t_1/t_o)\alpha\sin\alpha+(1-t_1/t_o)(1-\cos\alpha)]\}$$

$$I_y = R^3 t_0/2\alpha\left[\alpha^2/(1+t_1/t_o)-\sin^2\alpha(1-t_1/t_o)-2(t_1/t_o)\alpha\sin\alpha\cos\alpha\right] \quad (4), (5)$$

- Flexural-torsion constant [3, 7, 8] is

$$k=\sqrt{GI_t/EI_\omega}\,,\ k^*=\sqrt{GI_t/\overline{E}I_\omega} \tag{6}$$

where:

- E,G are modulus of elasticity and shear modulus, $\overline{E}=E/(1-\nu^2)$ is reduced modulus of elasticity, ν is Poisson coefficient, I_t is torsion constant

$$I_t=(1/3)\int_\varphi t(\varphi)^3R\,d\varphi=(Rt_0^3\alpha/6)[1+t_1/t_0+(t_1/t_0)^2+(t_1/t_0)^3]\,. \tag{7}$$

- Sectorial moment of inertia

$$I_\omega = 2R^5t_0\{(\alpha^3/12)(1+3t_1/t_0)+2(e_P/R)(t_1/t_0)(\alpha\cos\alpha-\sin\alpha)$$
$$+2(e_P/R)(1-t_1/t_0)(1/\alpha)[\alpha\sin\alpha+2\cos\alpha-2-(e_P/8R)\sin^2\alpha]+ \tag{8}$$
$$+(e_P/R)^2[(\alpha/4)(1+t_1/t_0)-(t_1/2t_0)\sin\alpha\cos\alpha]$$

Except normal sectorial coordinate (classical) the secondary sectorial coordinates were taken into account. The generalized sectorial coordinate, equal to the sum of the "classical'' and the previously mentioned secondary sectorial coordinate (ω_P^*) is

$$\omega_P^* = R^2[\varphi-(e/R)\sin\varphi]+(1/2)et(\varphi)\sin\varphi\,, \tag{9}$$

and then

- Generalized sectorial moment of inertia

$$J_\omega=\int_A \omega_P^{*2}dA \tag{10}$$

$$J_\omega / R^5 t_0 = (\alpha^3/6)(1+3t_1/t_0)+4e_P/R\{(1/\alpha)(1-t_1/t_0)(\alpha\sin\alpha+2\cos\alpha-2-(e_P/8R)\sin^2\alpha)+$$
$$+(t_1/t_0)(1+t_0^3/12t_1R^2)(\alpha\cos\alpha-\sin\alpha)+(e_P/8R)[(1+t_1/t_0)\alpha-(t_1/t_0)\sin 2\alpha]+$$
$$+(t_0/2R)^2[-(1/\alpha)(1-t_1/t_0)(\alpha^2\cos\alpha-2\alpha\sin\alpha-2\cos\alpha+2)+$$
$$+(1/\alpha^2)(1-t_1/t_0)(\alpha^3\cos\alpha-3\alpha^2\sin\alpha-6\alpha\cos\alpha+6\sin\alpha)-$$
$$-(4/3)(1/\alpha^3)(1-t_1/t_0)^3((\alpha^4/4)\cos\alpha-\alpha^3\sin\alpha-3\alpha^2\cos\alpha+6\alpha\sin\alpha+6\cos\alpha-6)]\}+$$
$$+(t_0/2R)^2(e_P/R)^2[\alpha-\sin\alpha\cos\alpha-3/2\alpha)(1-t_1/t_0)(\alpha^2-\alpha\sin 2\alpha+\sin^2\alpha)+$$
$$+(3/2\alpha^2)(1-t_1/t_0)^2\cdot(2\alpha^3/3-\alpha^2\sin 2\alpha-\alpha+2\alpha\sin^2\alpha+\sin\alpha\cos\alpha)-$$
$$-(3/4\alpha^3)(1-t_1/t_0)^3(\alpha^4/3-(2\alpha^3/3)\sin 2\alpha+2\alpha^2\sin^2\alpha-\alpha^2+\alpha\sin 2\alpha-\sin^2\alpha)]. \quad (11)$$

2.2. Influence of centrifugal force

Normal force and the corresponding normal stress occur as in the present case, as a consequence of centrifugal force:

$$N(z)=F_c(z)=\frac{\rho\omega^2AL^2}{2}\left[1-\left(\frac{z}{L}\right)^2\right],\ \sigma_N=\frac{N(z)}{A}=\frac{\rho\omega^2L^2}{2}\left[1-\left(\frac{z}{L}\right)^2\right]. \quad (12), (13)$$

Maximum values of these quantities are at the clamped end, for $z = 0$.

$$N_{\max}=\frac{\rho\omega^2AL^2}{2},\ \sigma_{N\max}=\frac{\rho\omega^2L^2}{2}. \quad (14), (15)$$

2.3. Influence of bending moments and constrained torsion

Bending moments are caused by distributions of transverse pressures which in this example is adopted as uniformly distributed along the length of the blades. Maximum values of bending moments are at the clamped end.

Normal stresses and shear stresses are caused by the consrained torsion:

$$\sigma_\omega=\frac{B}{I_\omega}\omega,\ \sigma_\omega^*=\frac{B}{I_\omega}\omega^*,\ \tau=\frac{M_t}{I_t}t \quad (16)$$

where the bimoment B and Saint Venant's torsion moment M_t is

$$B(z)=-\frac{M^*}{k}\frac{Sh[kL(1-z/L)]}{Ch(kL)},\ M_t(z)=M^*\left[1-\frac{Ch[kL(1-z/L)]}{Ch(kL)}\right], \quad (17), (18)$$

where M^* is the given value of twisting torque.

Then the equivalent (resultant) stress is calculated according to Hypothesis of maximum shear stress (Tresca)

$$\sigma_{\omega rez}=\sqrt{\sigma_\omega^2+4\tau^2}. \quad (19)$$

3. Numerical example and analysis of results

The calculations were done for the straight beam having the chosen cross-section (Fig. 1), and for the dimensions: length of the turbine blades $L = 900$ mm, $R = 300$ mm, $\alpha = 30^0$,

t_0=26 mm, t_1=13 mm, where: Poisson coefficient: $\nu = 0.3$, modulus of elasticity $E = 21000$ kN/cm^2, reduced modulus of elasticity $\overline{E} = 23077$ kN/cm^2, shear modulus $G = 8077$ kN/cm^2, specific mass of material $\rho = 7.8*10^3$ kg/m^3, bending moments $M_{xmax} = M_{ymax} =$ 100 kNcm, concentrated torque $M^* = 100$ kNcm, normal force $N = 100$ kN.

Attention is particularly paid to the constrained torsion effects as well as to the stress components which are their consequences because in that case the secondary effects become evident.

Normal stresses are proportional to the sectorial coordinate and it is necessary to point out that the secondary sectorial coordinate will be taken into consideration together with the generalized sectorial moment of inertia calculated using (11).

As the shear stresses and particularly stresses induced by the Saint Venant's torques should not be neglected, it was necessary to calculate the equivalent stresses $\sigma_{\omega rez}$, and it was done using the maximum shear stress failure (Tresca) criterion (19).

The distribution of maximal stresses along the beam is presented in Fig. 2 for normal stresses and in Fig. 3 for equivalent stresses. Numerical calculations were done separately for the "classical" case when the secondary sectorial coordinate was not taken into account.

The stress values connected with secondary effects are denoted by asterisk.

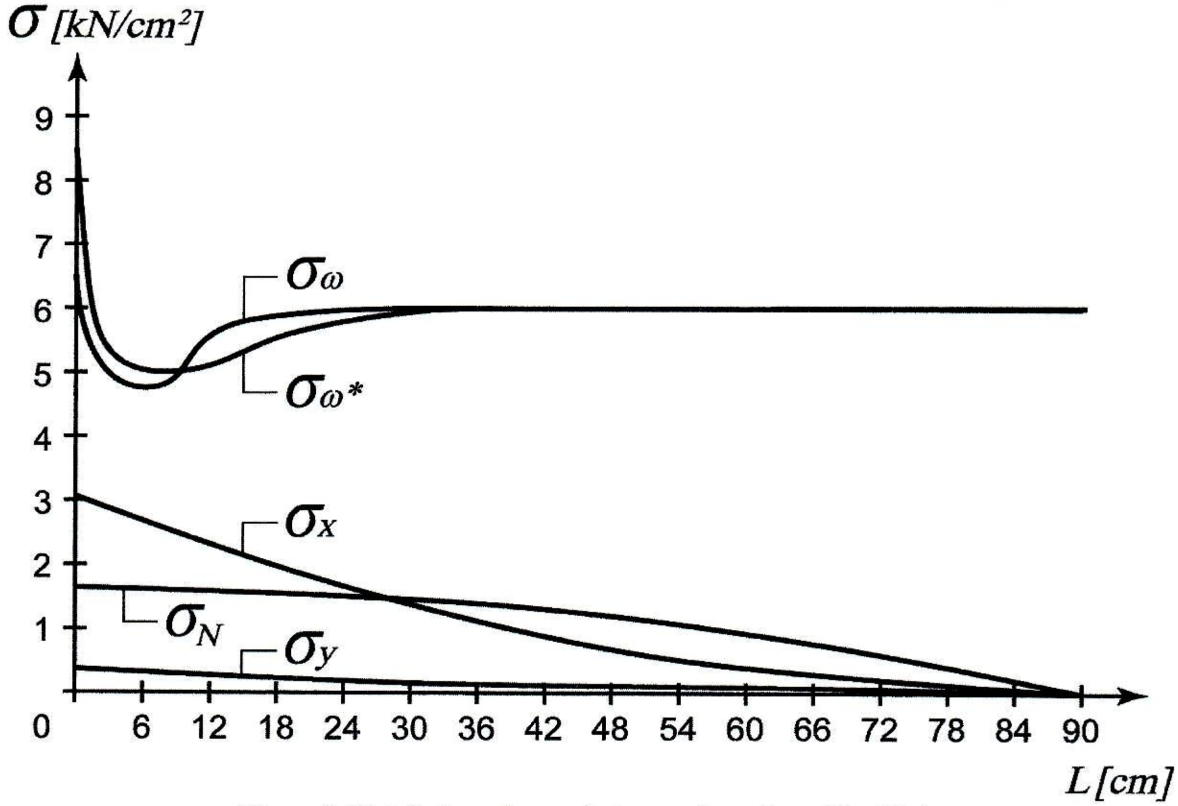

Figure 2. Distribution of normal stresses along the turbine blades

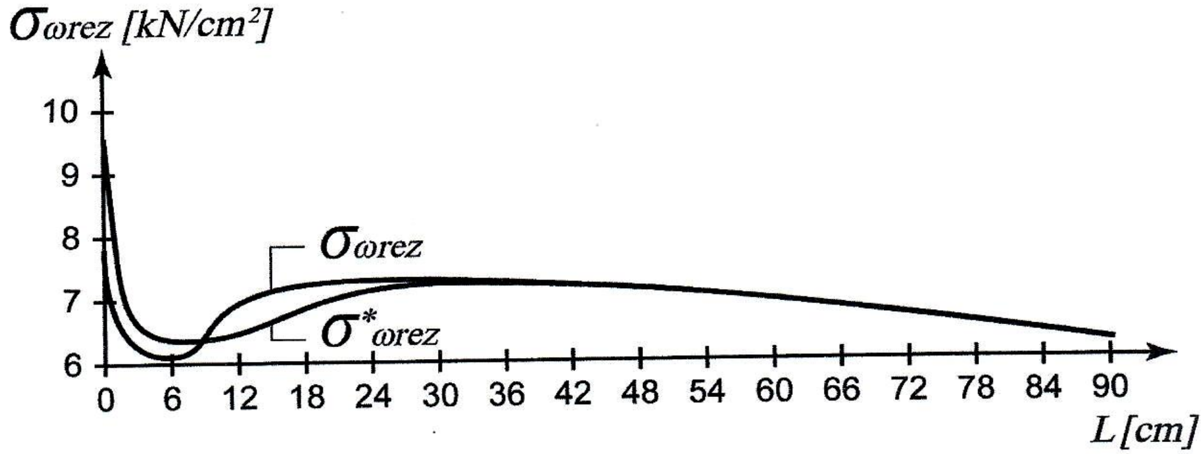

Figure 3. Maximum values of equivalent stresses

4. Conclusions

All geometrical characteristics for the chosen cross-section and, after that, all stress components for the considered structural element having the shape of the turbine blade were calculated in the way explained in [3, 4]. The geometrical quantities obtained with and without the secondary sectorial coordinates were mutually compared as well as the stress components in the case of complex loads. Their distributions over the cross-section and along the element for the applied unit loads were determined. The dependence of the secondary effects on the cross-sectional dimensions was shown.

From the obtained results it can be seen and concluded that if secondary effects are taken into account the differences between the stresses with and without mentioned effects are quite remarkable at the clamped ends. If the additional terms are taken into account they have non negligible effects on the level of stress components, particularly in some cases of the cross-sectional shapes similar to those considered in the numerical example.

Acknowledgement. This work was supported by the Ministry of Science and Technological Development of Serbia funded projects TR 35011 and TR 35040.

5. References

[1] Timoshenko S P (1945) Theory of bending, torsion and buckling of thin-walled members of open cross section, Reprinted from the *Journal of the Franklin Institute*, 239 (3), pp. 201-219, (4), pp. 249-268, (5), pp. 343-361.
[2] Vlasov V Z (1959) *Thin-Walled Elastic Beams*, 2nd ed., Moscow, 1959. (English translation, Israel Program for Scientific Translation, Jerusalem, 1961)
[3] Kollbruner C F and Hajdin N (1972) *Dunnwandige Stabe*, Band 1, Springer Verlag, Berlin.
[4] Andjelić N (2011) Nonlinear Approach to Thin-Walled Beams with a Symmetrical Open Section, *Strojniški vestnik - Journal of Mechanical Engineering*, 57, pp. 69-77.
[5] Vasiljevic N (1990) *Steam Turbines*, University of Belgrade, Faculty of Mechanical Engineering, Belgrade (in Serbian).
[6] Anđelić N, Miloševic-Mitić V and Maneski T (2009) An approach to the optimization of Thin-walled Z-beam", *Strojniški vestnik - Journal of Mechanical Engineering*, 55, pp. 742-748.
[7] Anđelić N (2005) *Optimization of thin-walled structural elements*, monographs, Andrejević Foundation, Belgrade (in Serbian).
[8] Anđelić N (2007) One View to the Optimization of Thin-Walled Open Sections Subjected to Constrained Torsion, *FME Transactions*, 35, pp. 23-28.