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Modelling the Container Yard as an Operational System in a Port: The Case Studies

The paper presents the two Case Studies of the modelling process at the Container Yard (CY) as an Operational System based on a particular batch arrival multi-server $M^X/M/c$ queue described and analysed in [4], where the batch size (X) has the shifted-Poisson and Poisson-like distributions. Using a more general formula for such queue models, here it is deduced the expressions for the specific cost ratio involving the state probabilities, the utilization factor, the mean and the variance of the group size. Applying this expression, the various results are presented when the number of yard cranes at container yard in port is 1, 2 or 3.

Keywords: *Container yard, Operational system, $M^X/M/c$ queue, Specific cost ratio, The Case Studies.*

1. INTRODUCTION

Our attentiom in the paper is fosused on solving or at least simplifying the problems concerning container yard (CY) in the port by applying $M^X/M/c$ queue which is studied and analysed in [4]. The objective is to derived exact solution for specific cost ratio at the CY.

We consider here batch arrivals of containers at a CY which is a multi-channel system with c yard cranes for the service. It is modelled as a multi-server $M^X/M/c$ queue which the size X of arriving group is distributed by the shifted-Poisson and Poisson-like distributions. For this queue, we is deduced the explicit expressions for the specific cost ratio, R_c. Using these expressions, the various graphical results are presented.

Notice that the derived expressions for specific cost ratio is closely related to the average number of containers at the CY. The obtained results are applied for determination of specific cost ratio in a port with $c=1,2,3$. Related results show that the values of R_c increase with respect to any of the following parameters of a considered queue model: the number of yard cranes, the utilization factor, ρ and the mean of batch size. Our analytical approach can be adopted for performance analysis of other container terminal subsystem with large number of shared resources. We confirm here the two special cases can be solved exactly, while some others have been done in [14].

This approach is also based to identifying specific knowledge by [1], [2], [6]-[8] and [11]. Also, some other approaches were given in [5], [11] and [12].

The CY operations are considered by Case Study I in Section 2, whilst Section 3 presents operations modelling at CY by Case Study II. The results with apprropriate discussions are presented in Section 4. Conclusions are presented in Section 5.

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2. THE CASE STUDY I

We consider containers arrivals at a port container yard as a particular batch arrival multi-server queue $M^X/M/c$ described in [3], [4], [9] and [10], , where the batch size (X) has a shifted-Poisson distribution (cf. [14], where it was assumed that X has a constant or a geometric distribution). A container yard is a single or multi-channel system with c yard cranes for the service ($c=1,2,3$). The number of containers that arrive for service at the same time is a shifted-Poisson distribution X given by (14) from [4] with mean $\bar{a}=2$ or $\bar{a}=5$. For related discussion it is supposed that the value of utilisation factor ρ varies from 0.2 to 0.8. Daily yard crane–container cost ratio is equal to $r_{cy}=0.4$.

For these purposes, by using the formulae (5) - (7), (14) and (18) from [4], here we express the formulae for the specific cost ratio R_c with $c=1,2,3$ related to the $M^X/M/c$ queue in which a group of containers that arrive at yard has the shifted-Poisson distribution X given by (14) from [4].

Case 1: $c=1$. Then from (4) we have $P_0=1-\rho$, which together with $A_1=1$ putting into (5) with $c=1$ yields $P_1=\rho(1-\rho)$. Substituting these values for P_0 and P_1 into (18) with $c=1$ yields

$$R_1=r_{cy}+\frac{(a^2+4a+2)\rho}{2(a+1)(1-\rho)}. \tag{1}$$

Case 2: $c=2$. Then from (4) [4] we have $P_0=1-\rho-P_1/2$, while from (5) - (7) [4] we have $P_1=\theta P_0 A_1=2\rho P_0/(a+1)$. The previous two equalities yield

$$P_0 = \frac{(1-\rho)(a+1)}{a+1+\rho}$$
and $P_1 = \dfrac{2\rho(1-\rho)}{a+1+\rho}$. (2)

Substituting the values for P_0 and P_1 given by (2) into (18) [4] with $c=2$ gives

$$R_2 = 2r_{cy} + \frac{(a^2+4a+2)\rho}{2(a+1)(1-\rho)} + \frac{\rho}{a+1+\rho}. \quad (3)$$

Case 3: $c=3$. Then from (4) [4] we have $P_0 = 1-\rho-(2P_1+P_2)/3$, while from (5) - (7) and (14) [4] we have $P_1 = \theta P_0 A_1 = 3\rho P_0/(a+1)$ and $P_2 = (\theta/2)(P_0A_2 + P_1A_1) = (3\rho/2(a+1))(P_0(1-e^{-a})+P_1)$. The previous system of three linear equations in variables P_0, P_1 and P_2 has the following solution:

$$P_0 = \frac{2(1-\rho)(a+1)^2 e^a}{2e^a + 4ae^a + 2a^2e^a + 5\rho e^a + 5a\rho e^a + 3\rho^2 e^a - \rho - a\rho}, \quad (4)$$

$$P_1 = \frac{6(1-\rho)(a+1)\rho e^a}{2e^a + 4ae^a + 2a^2e^a + 5\rho e^a + 5a\rho e^a + 3\rho^2 e^a - \rho - a\rho}, \quad (5)$$

and

$$P_2 = \frac{3(1-\rho)\rho(-1-a+e^a+ae^a+3\rho e^a)}{2e^a + 4ae^a + 2a^2e^a + 5\rho e^a + 5a\rho e^a + 3\rho^2 e^a - \rho - a\rho}. \quad (6)$$

Substituting the values for P_0, P_1 and P_2 given by (4) - (6) into (18) from [4] with $c=3$ gives

$$R_3 = 3r_{cy} + \frac{(a^2+4a+2)\rho}{2(a+1)(1-\rho)} + \frac{2\rho(3e^a+3ae^a+3\rho e^a-1-a)}{2e^a + 4ae^a + 2a^2e^a + 5\rho e^a + 5a\rho e^a + 3\rho^2 e^a - \rho - a\rho}. \quad (7)$$

3. THE CASE STUDY II

We consider containers arrivals at a port container yard as a particular batch arrival multi-server queue $M^X/M/c$ described in [3], [4], [9] and [10], where the batch size (X) has a Poisson-like distribution (cf. [14], where it was assumed that X has a constant or a geometric distribution, and in [3], where it was supposed that X has a shifted-Poisson distribution). A container yard is a single or multi-channel system with c yard cranes for the service ($c=1,2,3$). The number of containers that arrive for service at the same time is a Poisson-like distribution X given by (20) from [4] with mean $\bar{a}=2$ or $\bar{a}=5$. For related discussion it is assumed that the value of utilization factor ρ varies between 0.2 to 0.8. It is assumed that daily yard crane–container cost ratio is equal to $r_{cy}=0.4$. For these purposes, by using the formulae (4) - (7), (20) and (22) from [4], here we deduce the expressions for the specific cost ratio R_c with $c=1,2,3$ related to the $M^X/M/c$ queue in which a group of containers that arrive at yard has the Poisson-like distribution X defined by (20) from [4].

Case 1: $c=1$. Then from (25) with $c=1$ we immediately have

$$R_1 = r_{cy} + \frac{(a+2)\rho}{2(1-\rho)}. \quad (8)$$

Case 2: $c=2$. Then from (4) [4] we have $P_0 = 1-\rho-P_1/2$, while from (5) - (7) [4] we have $P_1 = \theta P_0 A_1 = 2\rho P_0/a$. The previous two equalities yield

$$P_0 = \frac{(1-\rho)a}{a+\rho} \quad \text{and} \quad P_1 = \frac{2\rho(1-\rho)}{a+\rho}.$$

Substituting the the above values for P_0 and P_1 into (25) from [4] with $c=2$ gives

$$R_2 = 2r_{cy} + \frac{(a+2)\rho}{2(1-\rho)} + \frac{\rho}{a+\rho}. \quad (9)$$

Case 3: $c=3$. Then from (4) [4] we have $P_0 = 1-\rho-(2P_1+P_2)/3$, while from (5) - (7) and (20) [4] we find that $P_1 = \theta P_0 A_1 = 3\rho P_0/a$ and $P_2 = (\theta/2)(P_0A_2+P_1A_1)$
$= 3\rho(P_0(1-e^{-a}-ae^{-a})+P_1)/(2a)$.

The previous system of three linear equations in variables P_0, P_1 and P_2 has the following solutions:

$$P_0 = \frac{2a^2(1-\rho)e^a}{2a^2e^a + 5a\rho e^a + 3\rho^2e^a - a\rho - a^2\rho}$$

$$P_1 = \frac{6a\rho(1-\rho)e^a}{2a^2e^a + 5a\rho e^a + 3\rho^2e^a - a\rho - a^2\rho},$$

$$P_2 = \frac{3(1-\rho)\rho(ae^a + 3\rho e^a - a - a^2)}{2a^2e^a + 5a\rho e^a + 3\rho^2e^a - a\rho - a^2\rho}.$$

Substituting the above values for P_0, P_1 and P_2 into (25) from [ ] with $c=3$ gives

$$R_3 = 3r_{cy} + \frac{(a+2)\rho}{2(1-\rho)} + \frac{2\rho(3ae^a + 3\rho e^a - a - a^2)}{2a^2e^a + 5a\rho e^a + 3\rho^2e^a - a\rho - a^2\rho}. \quad (10)$$

4. RESULTS AND DISSCUSIONS

Applying the expressions (1), (3), (7), (8), (9) and (10), we obtain the graphical results given in Fig. 1 and Fig. 2, respectively, as well as in Fig A1, Appendix A. It can be seen from all figures that the values R_c (Specific Cost Ratio =Ttotal annual cost for queuing system with c yard cranes / Total annual cost of containers) increase with respect to each of the variables (parameters) c (the number of yard cranes) and ρ (the utilization factor). Moreover, the same fact is true with respect to the parameter a (a is the mean of batch size) when $a>1$. Accordingly, in order to decrease the total annual cost for queuing system with c yard cranes with respect to the total annual cost of containers, it is necessary to decrease at least one of these three parameters.

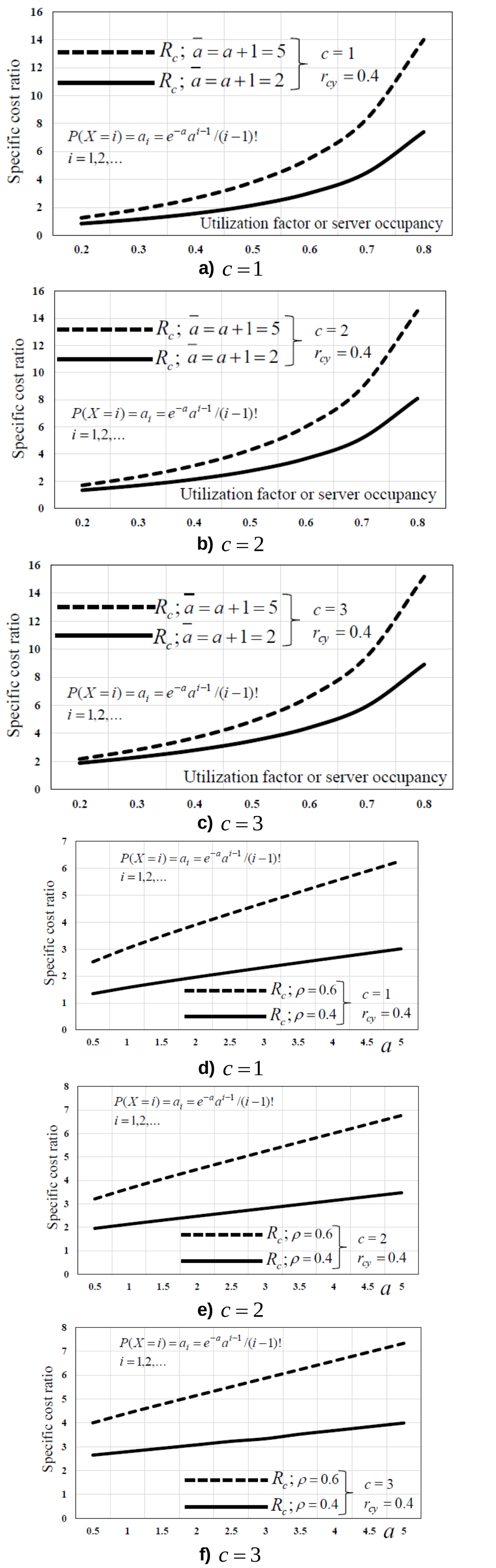

Figure 1. Specific cost ratio R_c related to $M^X / M / c$ queue with $P(X = i) = a_i = e^{-a}a^{i-1}/(i-1)!$, $i = 1,2,...$

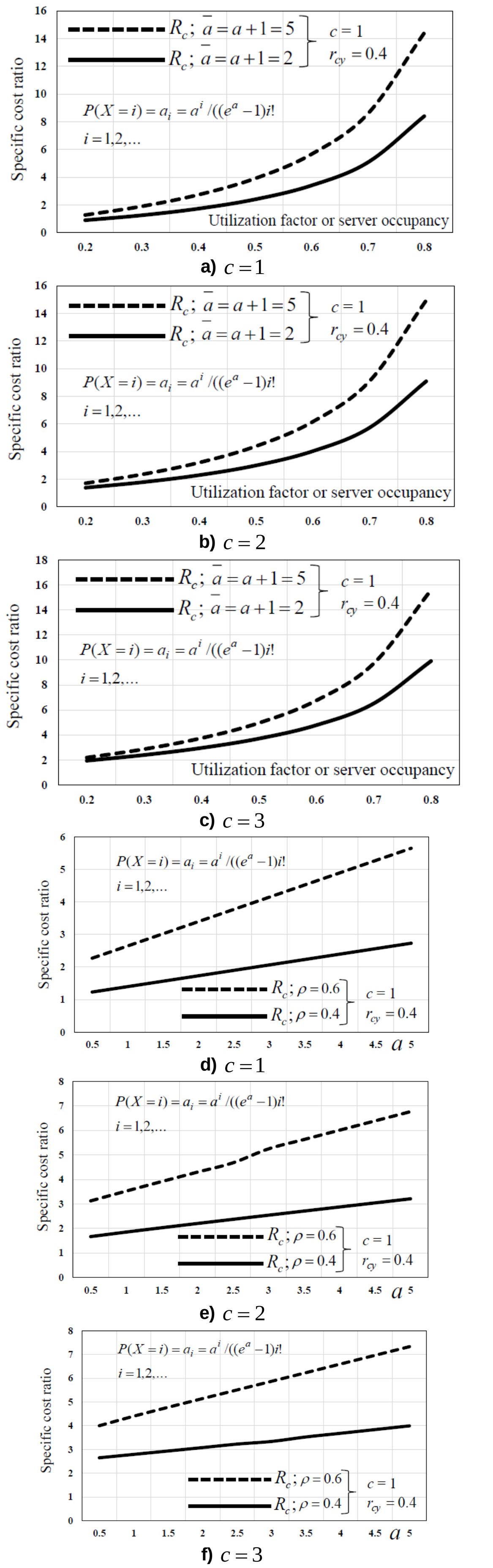

Figure 2. Specific cost ratio R_c related to $M^X / M / c$ queue with $P(X = i) = a_i = a^i/((e^a - 1)i!$, $i = 1,2,...$

5. CONCLUSIONS

We show that $M^X/M/c$ queue applying to the analysis of specific cost ratio can be important for understanding various kinds of CY operations and determining the optimal values of their performances. One straightforward application of our modelling approach is sizing of arriving group is distributed by the shifted-Poisson and Poisson-like distributions based on the handling technology choice for a given throughput level of CY.

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Appendix A

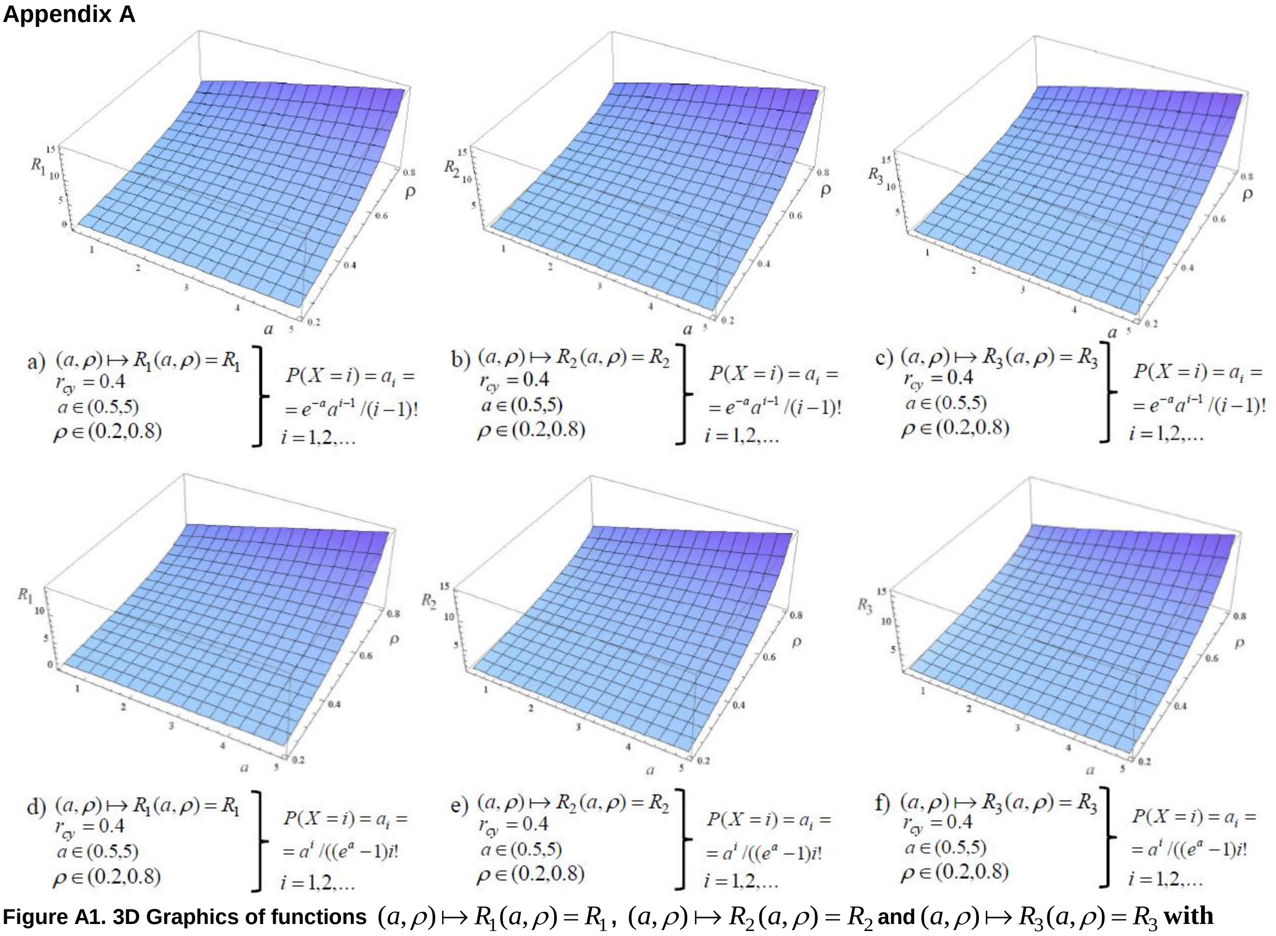

Figure A1. 3D Graphics of functions $(a,\rho)\mapsto R_1(a,\rho)=R_1$**,** $(a,\rho)\mapsto R_2(a,\rho)=R_2$ **and** $(a,\rho)\mapsto R_3(a,\rho)=R_3$ **with** $P(X=i)=a_i=e^{-a}a^{i-1}/(i-1)!$**,** $i=1,2,\ldots$ **for a), b) and c), and with** $P(X=i)=a_i=a^i/((e^a-1)i!$**,** $i=1,2,\ldots$ **for d), e) and f)** [3], [9] and [10]