12th INTERNATIONAL SCIENTIFIC CONFERENCE
NOVI SAD, SERBIA, SEPTEMBER 25-26, 2015

Original scientific paper

Jakovljević, Ž., Marković, V., Živanović, S.

RECOGNITION OF QUADRICS FROM 3D POINT CLOUDS GENERATED BY SCANNING OF ROTATIONAL PARTS

Abstract: *This paper presents a method for recognition of second order surfaces (quadrics) from point clouds containing information about scanned rotational parts. The method is region growing method that exploits the scatter of data during least squares fitting of quadrics as a region growing criterion. The presented procedure is convenient for segmentation of regions with high (G1 or higher) continuity. Besides, the region seed point is automatically selected which is its comparative advantage to a number of existing methods. The applicability of the proposed method is evaluated using two case studies; the first case study refers to a synthesized signal, and the second presents the applicability of the method on a real world example.*

Key words: *3D point cloud, surface recognition, quadrics segmentation, reverse engineering*

1. INTRODUCTION

Research efforts and practical applications in the field of implementation of 3D scanning devices in on-line manufacturing process control are expanding over the years. Typical examples are assembly automation and robots navigation. The hardware of contemporary 3D scanning devices, and especially optical ones, is characterized by high resolution, speed, and accuracy, and it is suitable for on-line applications. The same holds for algorithms for raw data preprocessing (3D data registration, integration and meshing). However, there is a lack of efficient real-time applicable algorithms for automatic recognition (segmentation and fitting) of geometric primitives from point cloud. During reverse engineering in CAD systems, geometric primitives from point clouds are recognized interactively by user [1], where user recognizes geometric primitives and then annotates a number of points on them. Afterwards, software is capable of fitting the best surfaces through selected points. On the other hand, implementation of 3D scanning devices in control applications requires fully automatic recognition of geometric primitives from point cloud.

Motivated by applications in a number of fields, such as mobile robots navigation [2] and seam pose detection in robotized welding [3], automatic recognition of planes has attracted many research efforts [4, 5]. Besides planar surfaces, in mechanical engineering rotational surfaces are most frequently met. These surfaces usually consist of second order surfaces (quadrics), planes, and general tori. In the focus of this paper is recognition of quadrics from scanned rotational surfaces.

There are a number of algorithms for detection of quadrics from point cloud, and a survey of these methods can be found in [6]. Generally, the methods belong to one of the two classes: 1) edge based, and 2) region based methods. Edge based techniques can be employed for G0 continuous surfaces where there exists an abrupt change between adjacent regions. Region based methods employ split and/or merge or region growing approach. Region growing approaches start from a seed point and grow region around it using different criteria such as local surface normal [7], average curvature [8], or principle curvature [9].

Recent methods combine edge based approaches for initial segmentation and region based approaches for surface fitting and segmentation of regions with higher continuity. For example, procedure proposed in [10] detects feature edges using dihedral angle; afterwards variational shape approximation is employed for region partition and general shape quadrics are fitted through obtained regions; finally, over-segmented adjacent regions are merged using combination of Euclidean distance and normal deviation as metrics. Another approach from [11] employs tensor voting based method for edges detection and combination of RANSAC, slipping motion analysis, Gauss map and principle curvature for recognition of shapes. An interesting research in the field of recognition of planes, spheres, cones and cylinders from sparse 3D meshes obtained from CAD is presented in [1]. The presented method recognizes feature edges using dihedral angle, and employs region growing based on principal curvature for primitives' extraction.

In this paper we propose a method for automatic recognition of quadrics from point cloud obtained by scanning of rotational parts. The proposed method is in its essence region growing method based on scatter of data during direct least squares fitting of second order surfaces. The presented method is applicable for recognition of G1 (or higher) continuous surfaces.

The remainder of the paper is structured as follows. In Section 2 we provide some theoretical background and present the proposed recognition method and corresponding algorithm. Section 3 presents the implementation of the algorithm on a synthesized point cloud, as well as on a real-world example. Finally, in Section 4 we give some concluding remarks.

2. METHOD FOR RECOGNITION OF QUADRICS FROM SCANNED ROTATIONAL PARTS

The method for recognition of quadrics from scanned rotational parts that is presented in this paper is based on the properties of scatter matrix during least squares fitting of second order surfaces (quadrics). General quadric can be represented by the following equation:

$$a_1x^2 + a_2y^2 + a_3z^2 + a_4xy + a_5yz + a_6xz + a_7x + a_8y + a_9z + a_{10} = 0 \qquad (1)$$

where a_i represent surface parameters and $[x\ y\ z]$ are the coordinates of a point on quadric. Equation (1) can be represented in matrix form:

$$\mathbf{x}\cdot\mathbf{a} = 0 \qquad (2)$$

where $\mathbf{x}=[x^2\ y^2\ z^2\ xy\ yz\ xz\ x\ y\ z\ 1]$, and $\mathbf{a}=[a_1\ a_2 \dots\ a_{10}]^T$ represents the vector of surface parameters.

Coefficients **a** can be estimated by solving the following minimization problem [12, 13]:

$$\begin{aligned}&\min\|\mathbf{Da}\|^2\\ &\text{subject to } \mathbf{a}^T\mathbf{Ca} = 1\end{aligned} \qquad (3)$$

where **C** is 10×10 matrix with all elements equal zero except **C**(10,10) =1, and **D** is design matrix in the form:

$$\mathbf{D} = \begin{bmatrix} x_1^2 & x_2^2 & x_3^2 & \dots & x_N^2 \\ y_1^2 & y_2^2 & y_3^2 & \dots & y_N^2 \\ z_1^2 & z_2^2 & z_3^2 & \dots & z_N^2 \\ x_1y_1 & x_2y_2 & x_3y_3 & \dots & x_Ny_N \\ y_1z_1 & y_2z_2 & y_3z_3 & \dots & y_Nz_N \\ x_1z_1 & x_2z_2 & x_3z_3 & \dots & x_Nz_N \\ x_1 & x_2 & x_3 & \dots & x_N \\ y_1 & y_2 & y_3 & \dots & y_N \\ z_1 & z_2 & z_3 & \dots & z_N \\ 1 & 1 & 1 & \dots & 1 \end{bmatrix}^T \qquad (4)$$

Solution of minimization problem (3) is positive eigen value of

$$\mathbf{Sa} = \lambda\mathbf{Ca} \qquad (5)$$

where

$$\mathbf{S} = \mathbf{D}^T\mathbf{D} \qquad (6)$$

represents scatter matrix, and λ is Lagrange multiplier.

When all data in design matrix are sampled from one exact quadric, scatter matrix will be singular [12]. On the other hand, when data are sampled from approximate quadric, such as scanned quadric surface, the scatter matrix will be close to singular. As data that do not belong to the particular quadric enter design matrix, matrix S will be farther from singular matrix. We have exploited this property of scatter matrix to create the method for segmentation of quadrics from considered class of point clouds.

The proposed method is in its essence region growing method. Region growing procedure starts from the first point in the point cloud and adds point by point to the region using reciprocal condition number of scatter matrix as region growing criterion. When reciprocal condition number of **S** passes predefined threshold, a point that does not belong to quadric is detected. In this situation region growing is stopped and the coefficients **a** of recognized quadric are estimated. The region is considered a quadric if it contains more than 30 points from the cloud. Otherwise, it is assumed that points do not belong to quadric, but to another type of surface.

When region growing procedure stops, all points that belong to recognized quadric within predefined threshold are excluded from point cloud; for detection of distance between points from the cloud and surface we exploit estimated surface parameters. The parameters estimation and point exclusion procedure is iteratively repeated until no more points are recognized to belong to given quadric. The pseudo-code of the algorithm for implementation of the procedure is presented in Fig. 1.

Since we are dealing with rotational parts, to accelerate and facilitate computational efforts, the algorithm starts with alignment of scanned part's rotation axis with z axis of point cloud, and with sorting of points from point cloud in descending order along z axis. The alignment of rotation axis along z direction is carried out using moments of inertia for all three axes of coordinate system, center of mass, and elementary computer graphics transforms.

```
INPUT: x, y, z – points from the cloud
       thres1, thres2 – segmentation thresholds
put axis of rotation along z axis
cloud=[x, y, z];
sortrows(cloud,-3);
m=0; trials=0;
while trials<30
 m=m+1;
 surfaces(m).surf(1,:)=cloud(1,:)
 for i=1:length(cloud)
  surf_aux= [surfaces(m).surf; cloud(i,:)];
  calculate rcond(S) using surf_aux
  if rcond(S)<thres1
   add cloud(i,:) to surfaces(m).surf; end
 end
 if length(surfaces(m).surf)>30
  l1=0; l2=1000;
  while l1<l2
   l1=length(surfaces(m).surf)
   calculate A
   for i=1:length(cloud)
    if distance<thres2
     add cloud(i,:) to surfaces(m).surf
    else add cloud(i, :) to new_cloud;  end
   end
   l2=length(surfaces(m).surf)
  end
 cloud=new_cloud;
 else trials=trials+1; end
 if length(cloud)<2 trials=31; end
end
OUTPUT: surfaces – structure containing segmented quadrics
```

Fig. 1. Pseudo-code of the algorithm for implementation of the method for recognition of quadrics from scanned rotational parts

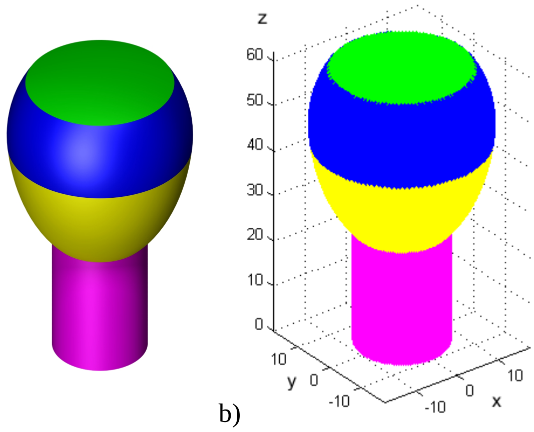
3. IMPLEMENTATION OF THE PROPOSED METHOD

To illustrate and experimentally verify the proposed procedure, we have implemented it in recognition of second order surfaces in two case studies. The first case study refers to a synthesized point cloud, while the second considers a real world example.

3.1 Case study 1: synthesized point cloud

The point cloud that we use in this case study consists of four second order surfaces, i.e. of one sphere, two ellipsoids (spheroids in particular), and one cylinder. Sphere and cylinder are G0 continuous with adjacent surfaces, while between ellipsoids G1 continuity is achieved. The CAD model of synthesized part is presented in Fig. 2.a, and parameters of synthesized surfaces in Table 1. Synthesized point cloud is noised by white noise with signal to noise ratio of 90dB.

After application of the proposed method, all four surfaces were adequately recognized. Segmentation results are graphically presented in Fig. 2.b, and estimated parameters in Table 1. From Table 1 it can be observed that the algorithm was able to adequately recognize surfaces. The discrepancy between estimated surface parameters and generated surface parameters is less than 0.3%. It should be noted that the parameters of ellipsoids are close and that accurate distinction between these surfaces is a challenging task.



a) b)

Fig. 2. Case study 1: a) synthesized part; b) segmentation results

	Seg. no	Parameters of quadric[a_i×10^2], i=1,…10
generetaed	1	[0.04 0.04 0.04 0 0 0 0 0 -0.96 -94.24]
	2	[0.33 0.33 0.25 0 0 0 0 0 -24 476]
	3	[0.32 0.32 0.11 0 0 0 0 0 -11.29 186.76]
	4	[1.23 1.23 0 0 0 0 0 -100]
estimated	1	[0.04 0.04 0.04 0.0 -0.0 -0.0 0.0 -0.0 -0.96 -94.28]
	2	[0.33 0.33 0.25 -0.0 -0.0 -0.0 0.0 0.0 -23.98 474.61]
	3	[0.32 0.32 0.11 -0.0 0.0 -0.0 0.0 -0.0 -11.29 186.76]
	4	[1.23 1.23 -0.0 0.0 -0.0 -0.0 0.0 0.0 0.0 -100.0]

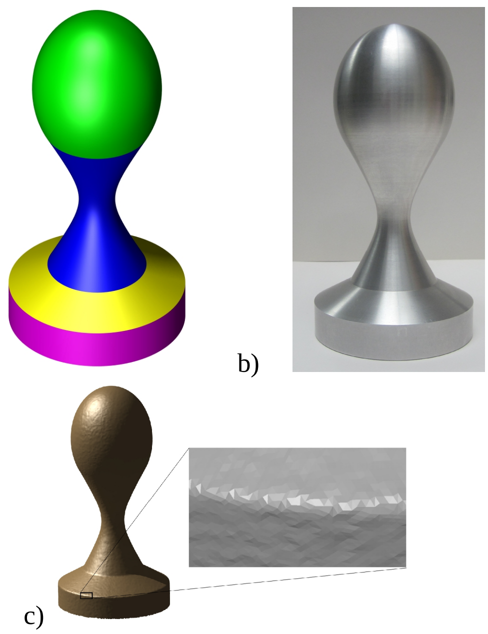
Table 1. Generated and estimated surface parameters in case study 1

3.1 Case study 2: real-world example

In second case study we consider the part presented in Fig. 3. The part consists of ellipsoid, hyperboloid, cone and cylinder. The parameters of the designed part are presented in Table 2. The part was made on CNC lathe TCN410 – echoENG. The point cloud (Fig. 3.c) is obtained by 3D digitization of the part using ZScanner® 700 [14] handheld laser scanner with resolution of 0.1mm.

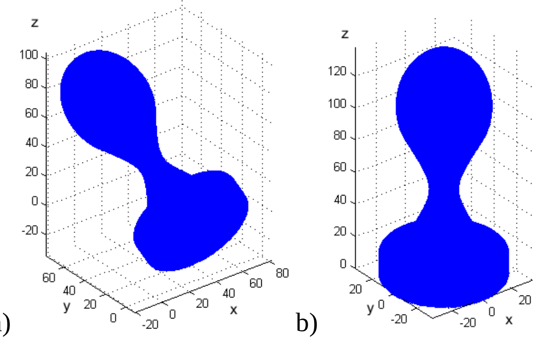
As presented in pseudo-code from Fig. 1, the recognition procedure starts by alignment of part's rotation axis with z axis of the coordinate system (Fig. 4). After applying the whole procedure, four quadric surfaces were adequately recognized. Segmentation results are presented in Fig. 5, and estimated parameters in Table 2.

From estimated parameters it can be observed that for all four surfaces adequate surface types (ellipsoid, hyperboloid, cone and cylinder) were recognized. The discrepancy between designed and estimated parameters for all coefficients except for a_3 in cone segment is less than 6%. This discrepancy includes all errors introduced during machining and scanning process, such as interpolation error (ellipse and hyperbola are approximated by circular and linear segments), machining error, scanner accuracy and resolution. It should be noted that the scanner has shown poor performance as presented in Fig. 3c. It has very low accuracy in vicinity of sharp edges and cone segment is located between two sharp edges.



a) b) c)

Fig. 3. Part from case study 2: a) designed part; b) photo of real-world part; c) scanned part



a) b)

Fig. 4. Point cloud from case study 2: a) before graphic

transforms; b) after alignment

	Seg no	Parameters of quadric[$a_i \times 10^2$], i=1,…10
designed	1	[0.16 0.16 0.09 0 0 0 0 0 -19.78 933.87]
	2	[2.04 2.04 -0.68 0 0 0 0 0 71.46 -1976.42]
	3	[0.33 0.33 1.0 0 0 0 0 0 66.64 1.11e+3]*100
	4	[0.086 0.086 0 0 0 0 0 0 0 -100]
estimated	1	[0.17 0.17 0.09 -0.0 0.0 -0.0 0.04 -0.02 -19.81 930.61]
	2	[2.04 2.04 -0.68 -0.0 0.01 -0.0 0.34 -1.11 72.68 -2033.5]
	3	[0.34 0.34 -0.89 -0.0 0.0 0.0 0.04 -0.29 64.4 -1.13e+3]*100
	4	[0.086 0.086 0.0 0.0 0.0 0.0 0.01 -0.04 0.0 -99.91]

Table 2. Designed and estimated surface parameters in case study 2

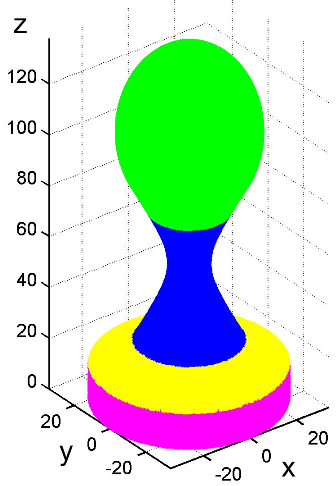


Fig. 5. Segmentation results in case study 2

4. CONCLUSION

In this paper we have presented a method for recognition of quadrics from point clouds generated by scanning of rotational parts. It is a region growing method based on properties of scatter matrix calculated during least squares fitting of second order surfaces. The method is convenient for segmentation of adjacent segments with G1 or higher continuity, and this is its comparative advantage to edge based procedures. Another important property of this method is that seed point is automatically selected as the highest point (point with maximum z coordinate) on the part.

The performances of the method were presented using a synthesized and a real-world point cloud, and the method has shown good recognition results.

To increase the robustness of the method to poor scanner performances, the future work will address the combination of the presented region growing method with edge based segmentation during preprocessing. In addition, recognition of tori, which are also frequently met in rotational parts, will be a part of future research efforts.

4. REFERENCES

[1] Bénière, R., Subsol, G., Gesquière, G., Le Breton, F., Puech, W.: *A comprehensive process of reverse engineering from 3D meshes to CAD models*, Computer-Aided Design, Vol. 45, p.p. 1382–1393, 2013.

[2] Fu, G., Corradi, P., Menciassi, A., Dario, P.: *An Integrated Triangulation Laser Scanner for Obstacle Detection of Miniature Mobile Robots in Indoor Environment*, IEEE/ASME Transactions on Mechatronics, Vol.16, No.4, p.p.778 – 783, 2011.

[3] Fang, Z., Xu, D., Tan, M.: *A Vision-Based Self-Tuning Fuzzy Controller for Fillet Weld Seam Tracking*, IEEE/ASME Transactions on Mechatronics, Vol.16, No.3, p.p.540 – 550, 2011.

[4] Wang, L., Cao, J, Han, C.: *Multidimensional particle swarm optimization-based unsupervised planar segmentation algorithm of unorganized point clouds*, Pattern Recognition, Vol.45, No.11, p.p. 4034–4043, 2012

[5] Jakovljevic, Z., Puzovic, R., Pajic, M.: *Recognition of Planar Segments in Point Cloud based on Wavelet Transform*, IEEE Trans. on Industrial Informatics, Vol. 11, No. 2 , p.p. 342-352, 2015.

[6] Petitjean, S.: *A Survey of Methods for Recovering Quadrics in Triangle Meshes*, ACM Computing Surveys, Vol.2, No.34, p.p.1-61, 2002.

[7] Rabbani, T., Van den Heuvel, F.A., Vosselman, G.: *Segmentation of point clouds using smoothness constraint*, International Archives of Photogrammetry, Remote Sensing and Spatial Information Sciences, Vol.36, No.5, p.p.248-253, 2006.

[8] Lai, H.C., Chang, Y.H., Lai, J.Y., *Development of feature segmentation algorithms for quadratic surfaces*, Advances in Engineering Software, Vol.40, No.10, p.p.1011–1022, 2009.

[9] Lavoue, G., Dupont F., Baskurt A.: *Curvature Tensor Based Triangle Mesh Segmentation with Boundary Rectification*, Proceedings of the Computer Graphics International, p.p. 10-17, IEEE, 2004.

[10] Yan, D., M., Wang, W., Liu, Y., Yang, Z.: *Variational mesh segmentation via quadric surface fitting*, Computer-Aided Design Vol. 44, pp. 1072–1082, 2012.

[11] Yi, B., Liu, Z., Tan, J., Cheng, F., Duan, G., Liu, L.: *Shape recognition of CAD models via iterative slippage analysis*, Computer-Aided Design Vol. 55, pp. 13–25, 2014.

[12] Fitzgibbon, A., Pilu, M., Fisher R.B.: *Direct Least Square Fitting of Ellipses*, IEEE Transactions on Pattern Analysis, Vol.21, No.5, pp.476-480, 1999.

[13] Rosin, P., L.: *A note on the least squares fitting of ellipses*, Pattern Recognition Letters, Vol. 14, No. 10, p.p. 799–808, 1993.

[14] http://www.zcorp.com/documents/380_ZScanner700_PX_SpecSheet_HiRes.pdf

Authors: **Dr. Živana Jakovljević, Assistant Professor, M.Sc. Veljko Marković, Dr. Sasa Zivanović, Assistant Professor,** University of Belgrade, Faculty of Mechanical Engineering, Kraljice Marije 16, 11000 Belgrade, Serbia
E-mail: zjakovljevic@mas.bg.ac.rs; vmarkovic@mas.bg.ac.rs; szivanovic@mas.bg.ac.rs;

Acknowledgements. This research was partially supported by Serbian Ministry of Education, Science and Technological Development under research grants TR35007 and TR35020.