Second Harmonics Excitation of Electron Longitudinal Electric Wave Modes in the Suddenly Created Weakly Nonlinear Magnetized Plasma. Transversal Propagation

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Abstract. The nonlinear transformation of the linearly polarized source electromagnetic wave into electron andelectromagnetic plasma oscillations, stationary (rectified and space-varying) modes and traveling electron and electromagnetic plasma waves, due to a weak nonlinearity, has been analyzed by second order perturbation theory in a radio approximation. The efficiency of the excitation of electron longitudinal second-harmonic electric wave modes, with double wave number with respect to the wave number modes obtained in the linear transformation, have been studied for different values of source wave frequency and taking electron cyclotron frequency as a parameter.

INTRODUCTION

Rapidly created plasmas appear practically in all pulse gas discharges, laser created plasmas, lightning, and plasmas created by nuclear explosions. If the rise time of plasma is much smaller then the decay time, it is possible to approximate the time variation of plasma parameters with the Heaviside step function. The basic results of the transformation of electromagnetic (EM) waves in such time varying linear media has been summarized by Kalluri [1].

In this article we have assumed that, for t<0 , the linearly polarized (LP) source electromagnetic wave (EMW) with angular frequency ω_0 and wave number $\vec{k}_0$ is propagating in free space in the positive z direction. The static magnetic field is assumed to be along positive y direction, $\vec{B}_0 = \hat{y}\, B_0$, where $\hat{y}$ is unit vector in positive y-direction. At t=0 the entire free space is ionized with an electron plasma density increase from zero to some constant value N_0. In the linear theory, analyzed by the first order perturbation theory, two transmitted and two reflected wave modes are generated due to interaction between LP source EMW and suddenly created magnetoplasma medium, with following frequencies [2]:

$$\omega_{1,2} = \sqrt{a \pm \sqrt{a^2 - b}}\ ; \quad a = \omega_P^2 + (\omega_0^2 + \omega_B^2)/2, \quad b = \omega_P^4 + (\omega_P^2 + \omega_B^2)\cdot\omega_0^2, \tag{1}$$

where ω_0, $\omega_P = \sqrt{q\, N_0 / \varepsilon_o m}$, $\omega_B = q\, B_0 / \varepsilon_o m$ are angular frequency of the source wave, electron plasma and electron cyclotron angular frequencies, respectively. Angular frequencies of transmitted and reflected waves are: ω_1, ω_2 ($\omega_1, \omega_2 > 0$) and $-\omega_1, -\omega_2$, respectively.

Taking into consideration interaction between newly created wave modes, obtained in the linear theory, using the second order perturbation theory one obtains terms for the nonlinear EM and electron modes in the magnetized plasma medium.

PROBLEM FORMULATION AND THE CLOSED FORM SOLUTION

Electric and magnetic fields of the linearly polarized source EMW for t<o are given by

$$\vec{e}_o(z,t) = E_o\cos(\omega_0 t - k_0 z)\cdot\vec{x}\,, \tag{2}$$

$$\vec{h}_0(z,t) = H_0\cos(\omega_0 t - k_0 z)\cdot\vec{y}, \tag{3}$$

where $\vec{x}$, $\vec{y}$ and $\vec{z}$ are the unit vectors in positive direction of x, y and z axis, with $E_0 = \sqrt{\mu_0/\varepsilon_0}\,H_0$, where μ_0 and ε_0 are permeability and permittivity of free space.

Implementing, for $t \geq 0$, the second order perturbation theory into equation of continuity, Maxwell equations and equation of electron fluid motion one obtaines linearized system of equations for nonlinear fields, in suddenly created magnetoplasma

$$rot\,\vec{e}_2(z,t) + \mu_0\frac{\partial\vec{h}_2(z,t)}{\partial t} = 0\,, \tag{4}$$

$$rot\,\vec{h}_2(z,t) + q\,N_0\vec{u}_2(z,t) - \varepsilon_0\frac{\partial\vec{e}_2(z,t)}{\partial t} = -q\,n_1(z,t)\vec{u}_1(z,t)\,, \tag{5}$$

$$\begin{aligned}\frac{\partial\vec{u}_2(z,t)}{\partial t} + \frac{q}{m}\vec{e}_2(z,t) + \frac{q}{m}\vec{u}_2(z,t)\times\vec{B}_0 &= -\frac{q}{m}\vec{u}_1(z,t)\times\left(\mu_0\vec{h}_1(z,t) + \frac{n_1(z,t)}{N_0}\vec{B}_0\right)\\ &-(\vec{u}_1(z,t)\nabla)\vec{u}_1(z,t) - \frac{n_1(z,t)}{N_0}\cdot\frac{\partial\vec{u}_1(z,t)}{\partial t} - \frac{q\,n_1(z,t)}{m\,N_0}\vec{e}_1(z,t)\end{aligned}. \tag{6}$$

Expresions for the linear fields $\vec{e}_1, \vec{h}_1$ and n_1 are given in [2] and [3], respectively.

The corresponding initial conditions that follow from the continuity of EM and velocity fields at t=0 and an assumption that newly created electrons in plasma are at rest at
t=0, are presented as

$$\vec{e}_2(z,t=0^+) = \vec{h}_2(z,t=0^+) = \vec{u}_2(z,t=0^+) = 0\,. \tag{7}$$

In order to solve the system of partial differential equations (4)-(6), with the prescribed initial conditions given by Eqs.(7), we have applied Laplace transform in time, and
as the plasma is unbound, Fourier transform in space

$$L(f(z,t)) = \int_0^\infty f(z,t)\exp(-st)dt\,; \qquad F(f(z,s)) = \int_{-\infty}^{+\infty} f(z,s)\exp(-jk)dz\,. \tag{8}$$

In a domain of complex frequency $s = j\omega$ and wave number k the EM and velocity fields are obtained by solving the system of linear algebraic equations, obtained from (4)-(6). Then, performing inverse Laplace and Fourier transforms one obtains EM and velocity fields in space-time domain in the following form:

$$e_{2x}(z,t) = E_{2x} + \sum_{i=1}^{8} e_{2x}(\varphi_i t) + \sum_{i=1}^{2} e_{2x}^{t,r}(\pm\,\omega_{i\beta}, 2\,k_0) + \sum_{i=1}^{6} e_{2x}^{t,r}(\pm\,\varphi_i, 2\,k_0)\,, \tag{9}$$

$$h_{2y}(z,t) = h_{2y}(2\,k_0\,z) + \sum_{i=1}^{2} h_{2y}^{t,r}(\pm\,\omega_{i\beta}, 2\,k_0) + \sum_{i=1}^{6} h_{2y}^{t,r}(\pm\varphi_i, 2\,k_0)\,, \tag{10}$$

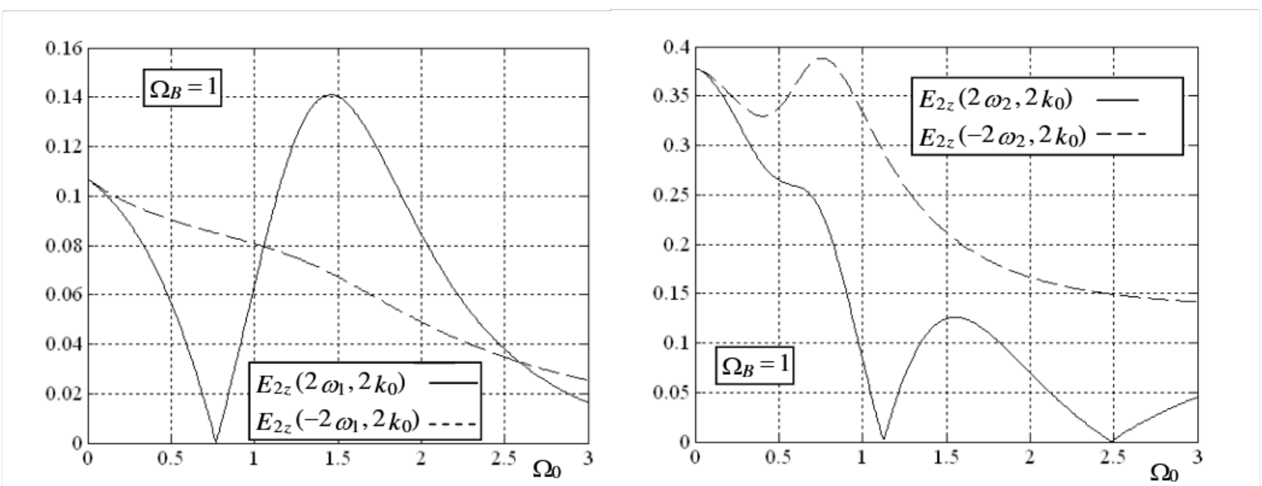
$$e_{2z}(z,t) = e_{2z}(2\,k_0\,z) + \sum_{i=1}^{8} e_{2z}(\omega_{i\alpha}) + \sum_{i=1}^{2} e_{2z}^{t,r}(\pm\,\omega_{i\beta}.2\,k_0) + \sum_{i=1}^{6} e_{2z}^{t,r}(\pm\,\varphi_i, 2\,k_0)\,, \tag{11}$$

where $\varphi_1=\omega_1$, $\varphi_2=\omega_2$, $\varphi_3=\omega_1-\omega_2$, $\varphi_4=\omega_1+\omega_2$, $\varphi_5=2\omega_1$, $\varphi_6=2\omega_2$, $\varphi_{7,8}=\omega_{1,2}(\omega_0\to 0)$ and $\omega_{i\beta}=\omega_i(\omega_0\to 2\omega_0)$; $\mathrm{i}=1{,}2$. In above Eqs. (9)-(11) t,r in superscripts refer to transmitted and reflected wave modes, respectively. Upper sign in «$\mp$» refers to transmitted wave mode and lower to reflected one.

Numerical Results

The effect of a suddenly switching nonlinear magnetoplasma medium is the creation of EM field with following components: rectification electric mode E_{2x}^0, stationary space-varying magnetic mode with wave number $2k_0$, eight oscillating electric wave modes with angular frequencies $\varphi_i, \mathrm{i}\in(1{,}8)$, eight electric and eight magnetic transmitted and reflected wave modes with angular frequencies $\omega_{i\beta}, \mathrm{i}=1{,}2$ and $\varphi_i, \mathrm{i}\in(1{,}6)$ with wave number $2k_0$, and creation of electric electron plasma field with following components: stationary space-varying mode with wave number $2k_0$, eight oscillating modes with angular frequencies $\varphi_i, \mathrm{i}\in(1{,}8)$, eight transmited and reflected wave modes with angular frequencies $\omega_{i\beta}, \mathrm{i}=1{,}2$ and $\varphi_i, \mathrm{i}=1{,}2{,}3{,}4{,}5{,}6$ with wave number $2k_0$.

In this article the efficiency of the excitation of second harmonics $(\pm 2\omega_{1,2}, 2k_0)$ of electric electron plasma field is analyzed. Distribution of amplitudes of these newly created wave modes in plasma, normalized to $E_{20}=qE_0^2/(mc\omega_P)=586E_0^2/\omega_p=5.86\ V/m$, (for $E_0=100\,V/m$ and $\omega_p=10^6 1/s$) versus angular frequency of the source wave, normalized on electron plasma angular frequency ω_P, are presented in corresponding diagrams (see Figs.1(a) and 1(b)), where electron cyclotron angular frequency is taken as a parameter.



(a) **(b)**

Figure 1. Amplitude distribution of electron longitudinal nonlinear electric wave modes with angular frequencies: $\mp 2\omega_1$ (Fig.1(a)) and $\mp 2\omega_2$ (Fig.1(b)) , normalized to the $E_{20}=qE_0^2/(mc\omega_p)=586\,E_0^2/\omega_P\,[V/m]$, versus normalized angular frequency of the source EMW, $\Omega_0=\omega_0/\omega_P$.

The peak value of amplitude of mode $e_{2x}(2\omega_1, 2k_0)\approx 0.14\,E_{20}=0.82\,V/m$ is obtained for a value $\Omega_0=1.47$. The peak value of amplitude of mode $e_{2xz}(-2\omega_2, 2k_0)\approx 0.38\,E_{20}=2.23\,V/m$ is obtained for a value $\Omega_0=0.75$.

CONCLUSION

The nonlinear interaction of linearly polarized source EMW with suddenly created magnetoplasma in the case of transversal propagation is solved in the closed form. The nonlinearities caused by the force $-\mu_0 q\vec{v}_1 \times \vec{h}_1$ and convection derivative term $(\vec{u}_1\nabla)\vec{u}_1$ are taken into account in the equation of electron fluid motion. The existence of electron density perturbation along direction of propagation is taken into account in the Maxwell equation for magnetic field rotor and in the equation of electron fluid motion. The efficiency of the excitation of electron longitudinal electric field modes $(\pm 2\omega_{1,2}, 2k_0)$ is influenced by the magnitude of an external static magnetic field and the angular frequency of source EMW.

REFERENCES

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