Excitation of Longitudinal Stationary Space-varying Electric Mode in the Suddenly Created Weakly Nonlinear Magnetized Plasma. Transversal Propagation

Z.M. Trifković, B.V. Stanić*

Faculty of Mechanical Engineering, University of Belgrade, 27 Marta 80
****Faculty of Electrical Engineering, University of Belgrade, Kralja Aleksandra 73***

Abstract. The transformation of the linearly polarized (LP) source electromagnetic wave (EMW) into the electron and electromagnetic (EM) plasma oscillations and stationary and traveling electron and EM plasma waves, due to a weak nonlinearity, have been analyzed by the use of second order perturbation theory. The efficiency of excitation of the longitudinal stationary space-varying electric mode has been studied for different values of source wave and electron cyclotron frequencies.

1. INTRODUCTION

Suddenly created plasmas appear practically in all pulse gas discharges, laser created plasmas, lightning and nuclear explosions. In the special case where the rise time of plasma is much smaller then the decay time, it is possible to approximate the time variation of plasma parameters with the Heaviside step function.

In the linear theory, analyzed by the first order perturbation theory, two transmitted and two reflected wave modes are generated due to interaction between LP source EMW and suddenly created magnetoplasma medium, with following frequencies [1]:

$$\omega_{1,2} = \sqrt{a \pm \sqrt{a^2 - b}}\ ; \quad a = \omega_P^2 + (\omega_0^2 + \omega_B^2)/2, \quad b = \omega_P^4 + (\omega_P^2 + \omega_B^2)\cdot\omega_0^2, \tag{1}$$

where ω_0, $\omega_P = \sqrt{\dfrac{q\,N_0}{\varepsilon_o m}}$, $\omega_B = \dfrac{q\,N_0}{m}$ are angular frequency of the source wave, electron plasma and electron cyclotron angular frequencies, respectively. Transmitted waves have angular frequencies ω_1, ω_2 ($\omega_1, \omega_2 > 0$) and reflected waves have angular frequencies $-\omega_1, -\omega_2$.

In this paper we have taken into account interaction between new created wave modes, obtained in the liner theory. We assumed that the linearly polarized source EMW, with angular frequency ω_0 and wave number $\vec{k}_0 = k_0 \cdot \vec{z}$ is propagating in the free space, in the presence of static magnetic field $\vec{B}_0 = B_0 \cdot \vec{y}$, for t<0.

At t=0 the entire medium is ionized with electron plasma angular frequency ω_P. Distribution of the new created EM and velocity fields in magnetoplasma will be analyzed by the second order perturbation theory in the radio approximation.

2. PROBLEM FORMULATION AND THE CLOSED FORM SOLUTION

Electric and magnetic fields of the linearly polarized source EMW for t<o are given by

$$\vec{e}_o(z,t) = E_o \cos(\omega_0 t - k_0 z) \cdot \vec{x}\,, \tag{2}$$

$$\vec{h}_0(z,t) = H_0 \cos(\omega_0 t - k_0 z) \cdot \vec{y}, \tag{3}$$

where $\vec{x}$, $\vec{y}$ and $\vec{z}$ are the unit vectors in positive direction of x, y and z axis, with $E_0 = \sqrt{\mu_0/\varepsilon_0}\, H_0$, where μ_0 and ε_0 are permeability and permittivity of free space.

Implementing, for $t \geq 0$, the second order perturbation theory into equation of continuity, Maxwell equations and equation of electron fluid motion one obtaines linear system of equations which nonlinear fields, in suddenly created magnetoplasma, have to satisfy:

$$rot\, \vec{e}_2(z,t) + \mu_0 \frac{\partial \vec{h}_2(z,t)}{\partial t} = 0\,, \tag{5a}$$

$$rot\, \vec{h}_2(z,t) + q N_0 \vec{u}_2(z,t) - \varepsilon_0 \frac{\partial \vec{e}_2(z,t)}{\partial t} = -q\, n_1(z,t) \vec{u}_1(z,t)\,, \tag{6a}$$

$$\begin{aligned} \frac{\partial \vec{u}_2(z,t)}{\partial t} + \frac{q}{m}\vec{e}_2(z,t) + \frac{q}{m}\vec{u}_2(z,t) \times \vec{B}_0 &= -\frac{q}{m}\vec{u}_1(z,t) \times \left(\mu_0 \vec{h}_1(z,t) + \frac{n_1(z,t)}{N_0}\vec{B}_0 \right) \\ &- (\vec{u}_1(z,t)\nabla)\vec{u}_1(z,t) - \frac{n_1(z,t)}{N_0} \cdot \frac{\partial \vec{u}_1(z,t)}{\partial t} - \frac{q\, n_1(z,t)}{m N_0}\vec{e}_1(z,t) \end{aligned} . \tag{7a}$$

Expresions for the linear fields $\vec{e}_1, \vec{h}_1$ and n_1 are given in [1] and [2], respectively.

The corresponding initial conditions that follow from the continuity of EM and velocity fields at t=0 and an assumption that newly created electrons in plasma are at rest at t=0, i.e.

$$\vec{e}_2(z, t = 0^+) = \vec{h}_2(z, t = 0^+) = \vec{u}_2(z, t = 0^+) = 0\,. \tag{8}$$

In order to solve the system of partial differential equations (5a)-(7a), with the prescribed initial conditions given by Eqs. (8), we have applied Laplace transform in time

$$L(f(z,t))=\int_0^{\infty} f(z,t)\exp(-st)dt = F(z,s), \qquad (9)$$

and, as the plasma is unbound, Fourier transform in space

$$F(f(z,s))=\int_{-\infty}^{+\infty} f(z,s)\exp(-jk)dz = F(k,s) \ . \qquad (10)$$

In a domain of complex frequency $s = j\omega$ and wave number k the EM and velocity fields one solves obtained system of linear algebraic equations, obtained from (5a)-(7a), and performing inverse Laplace and Fourier transforms. After these steps nonlinear EM and velocity fields are obtained in the following form:

$$\begin{aligned} e_{2x}(z,t) = E_{2x}^{0} + \sum_{i=1}^{8} E_{2x}^{1i}\cos(\omega_{i\alpha}t) + \sum_{i=1}^{6} E_{2x}^{2i}\cos(\varphi_i t) \\ + \sum_{i=1}^{2} E_{2x}^{3i\pm}\cos(\omega_{i\beta}t \pm 2k_0 z) + \sum_{i=1}^{6} E_{2x}^{4i\pm}\cos(\varphi_i t \pm 2k_0 z) \end{aligned}, \qquad (11)$$

$$\begin{aligned} e_{2z}(z,t) = -E_{2z}^{0}\sin(2k_0 z) + \sum_{i=1}^{2} E_{2z}^{1i}\sin(\omega_{i\alpha}t) + \sum_{i=1}^{6} E_{2z}^{2i}\sin(\varphi_i t) \\ + \sum_{i=1}^{2} E_{2z}^{3i\pm}\sin(\omega_{i\beta}t \pm 2k_0 z) + \sum_{i=1}^{6} E_{2z}^{4i\pm}\sin(\varphi_i t \pm 2k_0 z) \end{aligned}, \qquad (12)$$

$$h_{2y}(z,t) = H_{2y}^{0}\cos(2k_0 z) + \sum_{i=1}^{2} H_{2y}^{1i\pm}\cos(\omega_{i\beta}t \pm 2k_0 z) + \sum_{i=1}^{6} H_{2y}^{2i\pm}\cos(\varphi_i t \pm 2k_0 z), \qquad (13)$$

where $\varphi_1 = \omega_1; \varphi_2 = \omega_2; \varphi_3 = \omega_1 - \omega_2; \varphi_4 = \omega_1 + \omega_2; \varphi_5 = 2\omega_1; \varphi_6 = 2\omega_2;$ $\omega_{i\alpha} = \omega_i(\omega_0 \to 0);$ $\omega_{i\beta} = \omega_i(\omega_0 \to 2\omega_0);$ $\mathrm{i} = 1,2$.

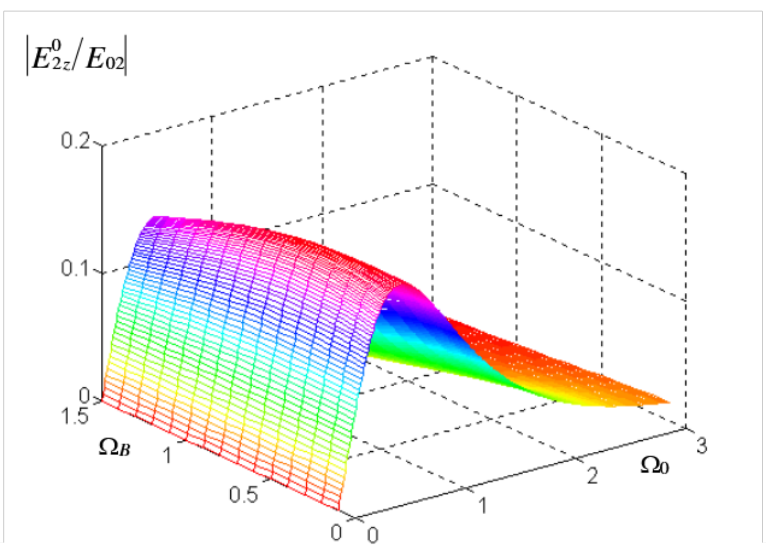

Fig.1 Amlitude distribution of the nonlinear longitudinal stationary space- varying electric field, normalized to the $E_{02} = qE_0^2/(mc\omega_p)$ $= 586E_0^2/\omega_P\,[V/m]$, versus normalized angular frequency of the source EMW, $\Omega_0 = \omega_0/\omega_P$, and electron cyclotron angular frequency $\Omega_B = \omega_B/\omega_P$.

Distribution of the longitudinal stationary space-varying electric field versus source wave and electron cyclotron frequencies is presented in Fig.1. The efficiency of excitation of this mode is negligible for $\omega_0 > 3\omega_P$. The best efficiency is achieved for $\omega_0 = 0.48\omega_P$ and $\omega_B = 0.5\omega_P$. In this case $E_{2z}^0 = 0.98\ V/m$ for $E_0 = 100\ V/m$ and $\omega_P = 10^6\ Hz$.

3. CONCLUSION

The nonlinear interaction of linearly polarized source EMW with suddenly created magnetoplasma in the case of transversal propagation is solved in the closed form. The nonlinearities caused by the force $-\mu_0 q\vec{v}_1 \times \vec{h}_1$ and convection derivative term $(\vec{u}_1\nabla)\vec{u}_1$ are taken into account in the equation of electron fluid motion. The existence of electron density perturbation along direction of propagation is taken into account in the Maxwell equation for magnetic field rotor and in the equation of electron fluid motion. Angular frequencies of new generated oscillating and travelling nonlinear modes, obtained by the second order perturbation theory, are the linear combinations of the angular frequencies obtained by the first order perturbation theory [1].

The efficiency of excitation of longitudinal stationary space-varying electric field mode ($\omega = 0$ and $k = 2k_0$) is strongly influenced by the magnitude of an external static magnetic field and the angular frequency of source EMW (see Fig.1).

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