Excitacion of Electron Plasma Oscillations in Suddenly Created Weakly Nonlinear Magnetized Plasma

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1. INTRODUCTION

Suddenly created plasmas appear practically in all pulse gas discharges, laser created plasmas, lightning and nuclear explosions. In the special case where the rise time of plasma is much smaller then the decay time, it is possible to approximate the time variation of plasma parameters with the Heaviside step function.

The transformation of the left hand circularly polarized source EM wave in the suddenly created cold magnetized plasma is analyzed by the first order perturbation theory. The three wave modes in plasma are then created with following frequencies [1]:

$$\omega_i = \left(-\frac{1}{3}p\right)^{1/3} \cos\left[\frac{\phi}{3} + 2(i-1)\frac{\pi}{3}\right] - \frac{\omega_B}{3},\ i = 1,2,3$$

with

$$p = -\left(\frac{\omega_B^2}{3} + \omega_0^2 + \omega_P^2\right),\ \phi = \arccos\left(\frac{q}{2\rho}\right),\ q = \frac{\omega_B}{27}\left(-2\,\omega_B^2 + 18\,\omega_0^2 - 9\,\omega_P^2\right),\ \rho = \left(-\frac{1}{3}p\right)^{3/2},$$

where ω_0, $\omega_P = \sqrt{\dfrac{q\,N_0}{\varepsilon_o m}}$, $\omega_B = \dfrac{q\,N_0}{m}$ are angular frequency of the source wave, electron plasma angular frequency and electron cyclotron angular frequency respectively. Transmitted wave has angular frequency ω_1 (ω_1>0) and reflected waves have angular frequencies ω_2, ω_3 (ω_2, ω_3<0).

The generation of new wave modes in suddenly created cold magnetized plasma have been investigated also in [1]-[3] for the some specific orientation of the external static magnetic field in the respect of the wave number $\vec{k}_0$ and when ion motion was taken into account. However, the interaction between new created wave modes in plasma were not considered.

In this paper we assumed that the left-hand circularly polarized source EM plane wave, with angular frequency ω_0 and wave number $\vec{k}_0$ is propagating in the free space in positive z direction, in the presence of static magnetic field $\vec{B}_0$, for t<0.

The external static magnetic field is also assumed to be along positive z direction. At t=0 the entire medium is ionized with electron plasma angular frequency ω_P. Distribution of the electric fields of new created modes in plasma will be analyzed by the second order perturbation theory in the radio approximation.

2. PROBLEM FORMULATION AND THE CLOSED FORM SOLUTION

Electric and magnetic fields of the left-hand circularly polarized source EM wave for t<o are given by

$$\vec{e}_o(z,t) = E_o \cos(\omega_0 t - k_0 z)\cdot \vec{x} - E_0 \sin(\omega_0 t - k_0 z)\cdot \vec{y}\,, \tag{1}$$

$$\vec{h}_0(z,t) = H_0 \sin(\omega_0 t - k_0 z)\cdot \vec{x} + H_0 \cos(\omega_0 t - k_0 z)\cdot \vec{y}, \tag{2}$$

where $\vec{x}$ and $\vec{y}$ are the unit vectors in positive direction of x and y axis, with $E_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} H_0$,

where μ_0 and ε_0 are permeability and permittivity of free space.

The EM and velocity fields in the magnetoplasma medium satisfy the following equations:

$$rot\, \vec{e}(z,t) = -\mu_0 \frac{\partial \vec{h}(z,t)}{\partial t}\,, \tag{3}$$

$$rot\, \vec{h}(z,t) = -q N_0 \vec{v}(z,t) + \varepsilon_0 \frac{\partial \vec{e}(z,t)}{\partial t}\,, \tag{4}$$

$$\frac{d\vec{v}(z,t)}{dt} = -\frac{q}{m}\vec{e}(z,t) - \frac{q}{m}\vec{v}(z,t)\times \vec{B}_0 - \frac{q\mu_0}{m}\vec{v}(z,t)\times \vec{h}(z,t)\,. \tag{5}$$

The system of equations (3)-(5) is nonlinear. EM and velocity fields for the weakly nonlinear plasma could be expressed in the following form:

$$\vec{e}(z,t) = \vec{e}_1(z,t) + \vec{e}_2(z,t)\,, \tag{6}$$

$$\vec{h}(z,t) = \vec{h}_1(z,t) + \vec{h}_2(z,t)\,, \tag{7}$$

$$\vec{v}(z,t) = \vec{v}_1(z,t) + \vec{v}_2(z,t)\,. \tag{8}$$

Taking complex representation for the fields and substituting (6)-(8) in (3)-(5) the following system of equations is obtained:

$$rot\, \vec{E}_1 + \mu_0 \frac{\partial \vec{H}_1}{\partial t} = 0\,, \qquad rot\, \vec{E}_2 + \mu_0 \frac{\partial \vec{H}_2}{\partial t} = 0\,, \tag{9}$$

$$rot\,\vec{H}_1 + q\,N_0\,\vec{V}_1 - \varepsilon_0 \frac{\partial \vec{E}_1}{\partial t} = 0, \qquad rot\,\vec{H}_2 + q\,N_0\,\vec{V}_2 - \varepsilon_0 \frac{\partial \vec{E}_2}{\partial t} = 0, \tag{10}$$

$$\frac{\partial \vec{V}_1}{\partial t} + \frac{q}{m}\vec{V}_1 \times \vec{B}_0 + \frac{q\,\mu_0}{m}\vec{E}_1 = 0, \qquad \frac{\partial \vec{V}_2}{\partial t} + \frac{q}{m}\vec{E}_2 + \frac{q}{m}\vec{V}_2 \times \vec{B}_0 = -\left(\vec{V}_1\vec{\nabla}\right)\vec{V}_1 - \frac{q\,\mu_0}{m}\vec{V}_1 \times \vec{H}_1. \tag{11}$$

Term $\left(\vec{V}_1\vec{\nabla}\right)\vec{V}_1$ is equal to zero. EM and velocity fields $\vec{E}_1, \vec{H}_1, \vec{V}_1$ are determined in [1]. In (9)-(11) the complex field vectors have space and time dependence. The variation of the EM and velocity fields can be written in the form [1]

$$f(z,t) = f(t)\exp(-j\,k_0\,z). \tag{12}$$

We apply the Laplace transform $L(s) = \int_0^\infty l(t)\exp(-st)dt$, to (9)-(11) with initial conditions:

$$\vec{E}_2\left(z,t=0^-\right) = \vec{E}_2\left(z,t=0^+\right) = \vec{H}_2\left(z,t=0^-\right) = \vec{H}_2\left(z,t=0^+\right) = \vec{V}_2\left(z,t=0^-\right) = \vec{V}_2\left(z,t=0^+\right) = 0. \tag{13}$$

Solving obtained system of linear algebraic equations in the domain of complex frequency s=jω, the EM and velocity fields are obtained in the following form:

$$E_{2x}(z,s) = E_{2y}(z,s) = 0, \qquad E_{2z}(z,s) = -\frac{\mu_0\omega_P^2}{s^2 + \omega_P^2}\,I(z,s), \tag{14}$$

$$V_{2x}(z,s) = V_{2y}(z,s) = 0, \qquad V_{2z}(z,s) = \frac{\varepsilon_0}{q\,N_0}\,s\,E_{2z}(z,s), \tag{15}$$

$$H_{2x}(z,s) = H_{2y}(z,s) = H_{2z}(z,s) = 0, \tag{16}$$

with

$$I(z,s) = V_{1x}(z,s) \otimes H_{1y}(z,s) - V_{1y}(z,s) \otimes H_{1x}(z,s), \tag{17}$$

where $\otimes$ is convolution operator in the complex s domain.

After performing inverse Laplace transform and taking the real parts of the obtained solutions, the time-varing electric and magnetic fields have the form:

$$e_{2x}(z,t) = e_{2y}(z,t) = 0, \qquad e_{2z}(z,t) = \sum_{i=1}^{3} E_{2z}\frac{P_i\hat{\varphi}_i}{\hat{\varphi}_i^2 - 1}\sin(\omega_P t) - \sum_{i=1}^{3} E_{2z}\frac{P_i}{\hat{\varphi}_i^2 - 1}\sin(\varphi_i t), \tag{18}$$

$$h_{2x}(z,t) = h_{2y}(z,t) = h_{2z}(z,t) = 0, \tag{19}$$

where

$$P_1 = V_2 H_1 - V_1 H_2,\ \varphi_1 = \omega_2 - \omega_1, \tag{20}$$

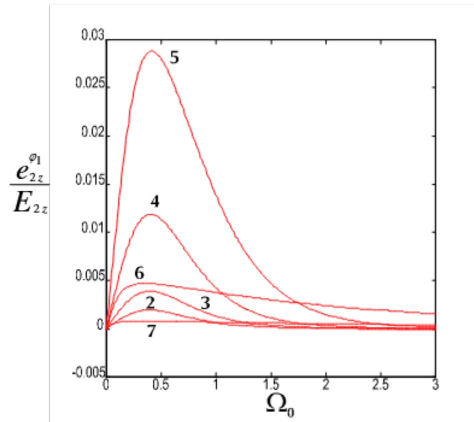
$$P_2 = V_3 H_1 - V_1 H_3,\ \varphi_2 = \omega_3 - \omega_1, \tag{21}$$

$$P_3 = V_3 H_2 - V_2 H_3,\ \varphi_3 = \omega_3 - \omega_2, \tag{22}$$

$$V_i = \frac{q\,E_0}{2m}\cdot\frac{\Omega_i + \Omega_0}{3\Omega_i^2 + \Omega_B\Omega_i - \Omega_0^2 - 1}, \qquad H_i = \frac{H_0}{2}\cdot\frac{(\Omega_i + \Omega_0)(\Omega_i + \Omega_B) - 1}{3\Omega_i^2 + \Omega_i\Omega_B - \Omega_0^2 - 1}, \tag{23}$$

where $\hat{\varphi}_i, \Omega_i, \Omega_0, \Omega_B$ are normalized angular frequencies $\frac{\varphi_i}{\omega_P}, \frac{\omega_i}{\omega_P}, \frac{\omega_0}{\omega_P}, \frac{\omega_B}{\omega_P}$, $E_{2z} = \frac{q E_0^2}{4mc\,\omega_P}$.

Normalized amplitude of electric fields of two oscillating modes versus Ω_0 are presented on Figs.1 and 2.



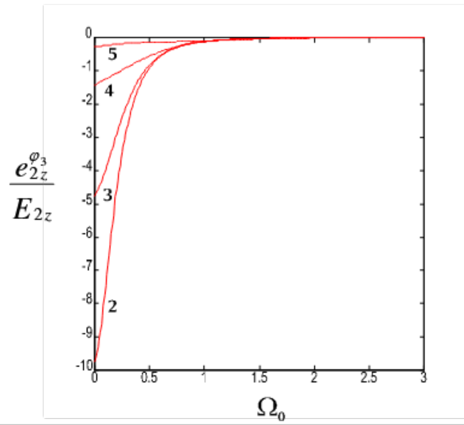


Fig. 1 Normalized amplitude of electric oscillating mode with angular frequency $\omega_2 - \omega_1$ versus Ω_0. Branches marked with n (2-7) have the values of parameter Ω_B=0.05, 0.1, 0.3, 1, 5, 10 respectively.

Fig. 2 Normalized amplitude of electric oscillating mode with angular frequency $\omega_3 - \omega_2$ versus Ω_0. Branches marked with n (2-5) have the values of parameter Ω_B=0.05, 0.1, 0.3, 1 respectively.

3. CONCLUSION

The nonlinear interaction of circularly polarized EM source wave with suddenly created magneto plasma in the case of longitudinal propagation is solved in the closed form. The nonlinearities caused by the force $-\mu_0 q \vec{v}_1 \times \vec{h}_1$ are taken into account in the equation of electron fluid motion. Angular frequencies of excited oscillating longitudinal modes obtained by the second order perturbation theories are: $\omega_2 - \omega_1, \omega_3 - \omega_1, \omega_3 - \omega_2$, where $\omega_1, \omega_2, \omega_3$ are angular frequencies of the first order transverse wave modes obtained in [1].

Efficiency of the excitation of new created oscillating modes could be controlled by either changing the angular frequency of source wave ω_0 or changing magnitude of external static magnetic field B_0.

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