FURTHER RESULTS ON $PI^{\alpha}D^{\beta}$ TYPE CONTROL OF EXPANSION TURBINE IN THE AIR PRODUCTION CRYOGENIC LIQUID

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Abstract. Here, it suggests and obtains a new algorithms of PID control based on fractional calculus (FC) in the producing of technical gases, i.e air production cryogenic liquid. Production liquid air low pressure was first introduced by P. L. Kapica and includes production liquor air pressure $p_2 = 6 \div 7\left[bar\right]$ and expansion in the gas turbine. For application in the synthesis of control input temperature and the flow of air expansion turbine, it is necessary to determine the appropriate differential equations linear's part of the building guidance as well as the procedural object. The paper presents a new robust control algorithms of $PI^{\alpha}D^{\beta}$ type which based on using fractional calculus. The objective of this work is to find out suitable settings for a fractional $PI^{\alpha}D^{\beta}$ controller in order to fulfill different design specifications for the closed-loop system, taking advantage of the fractional orders, α and β . Last, problem of discretization of proposed $PI^{\alpha}D^{\beta}$ will be treated as a key step in digital implementation.

1. Introduction

In classical control theory, state feedback and output feedback are two important techniques in system control. While it is not satisfied in most cases, the former technique requires that all variables are obtained directly. Although output feedback may avoid the restriction of state feedback, rather strong conditions such as the strict positive real condition, output feedback passivity and minimum phase, etc., are often on the system. Specially, the PID controller is by far the most dominating form of feedback in use today. Due to its functional simplicity and performance robustness, the proportional-integral-derivative controller has been widely used in the process industries. Design and tuning of PID controllers have been a large research area ever since Ziegler and Nichols presented their methods in 1942, [1]. Specifications, stability, design, applications and performance of the PID controller have been widely treated since then [2,3].On the other hand, fractional calculus is a mathematical topic with more than 300 years old history, but its application to physics and engineering

has been reported only in the recent years. Moreover, it is remarkable the increasing number of studies related with the application of fractional controllers in many areas of science and engineering, where specially fractional-order systems are of interest for both modeling and controller design purposes. It has been found that in interdisciplinary fields, many systems can be described by the fractional differential equations i.e. in the fields of continuous-time modeling, fractional derivatives have proved useful in linear viscoelasticity, acoustics, rheology, polymeric chemistry, biophysics,*etc.* [4-6]. However, in the recent years, emergence of effective methods to solve differentiation and integration of noninteger order equations makes fractional-order systems more and more attractive for the systems control community. The fractional PD^{α} controller [7], the fractional PI^{α} controller [8], the fractional controller $PI^{\beta}D^{\alpha}$ [6], the CRONE controllers [9,10], and the fractional lead-lag compensator [11] are some of the well-known fractional order controllers. In some of these works, it is verified that the fractional-order controllers can have better disturbance rejection ratios and less sensitivity to plant parameter variations compared to the traditional controllers. The fractional controllers have been used in many practical applications such as lateral and longitudinal control of autonomous vehicles [12], control of power electronic buck converters [13], control of robotic time delay systems [7], control of hexapod robots [14], and etc.
In this paper, we suggest and obtain a new algorithms of PID control based on fractional calculus (FC) in the producing of technical gases, i.e air production cryogenic liquid. The objective of this work is to find out suitable settings for a fractional $PI^{\alpha}D^{\beta}$ controller in order to fulfill different design specifications for the closed-loop system, taking advantage of the fractional orders, α and β.

2. Fundamentals of fractional calculus

Fractional calculus (FC) as an extension of ordinary calculus has a 300 years old history. FC was initiated by Leibniz and L`Hospital as a result of a correspondence which lasted several months in 1695. Both Leibniz and L`Hospital, aware of ordinary calculus, raised the question of a noninteger differentiation (order $n = 1/2$) for simple functions. It had always attracted the interest of many famous ancient mathematicians, including L'Hospital, Leibniz,Liouville, Riemann, Grünward, and Letnikov [4-6]. Futher, the theory of fractional-order derivative was developed mainly in the 19th century. In his 700 pages long book on Calculus, 1819 Lacroix [15] developed the formula for the n-th derivative of $y = x^m$, m – is a positive integer, $D^n x^m = \frac{m!}{(m-n)!} x^{m-n}$ where $n\ (\le m)$ is an integer. Replacing the factorial symbol by the Gamma function, he further obtained the formula for the fractional derivative

$$D^{\alpha} x^{\beta} = \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)} x^{\beta-\alpha} \tag{1}$$

where α and β are fractional numbers and Gamma function $\Gamma(z)$ is defined for $z > 0$ as:

$$\Gamma(z) = \int_0^{\infty} e^{-x} x^{z-1} dx, \quad \Gamma(z+1) = z\Gamma(z)\ . \tag{2}$$

On the other hand, Liouville (1809-1882) formally extended the formula for the derivative of integral order *n*

$$D^n e^{ax} = a^n e^{ax} \Rightarrow D^\alpha e^{ax} = a^\alpha e^{ax}, \ \alpha - arbitrary\,order \tag{3}$$

Using the series expansion of a function, he derived the formula known as *Liouville`s first formula for fractional derivative,* where α may be rational, irrational or complex.

$$D^\alpha f(x) = \sum_{n=0}^{\infty} c_n a_n^\alpha e^{a_n x} \tag{4}$$

where $f(x) = \sum_{n=0}^{\infty} c_n \exp(a_n x), \ \mathrm{Re}\, a_n > 0$. However, it can be only used for functions of the previous form. Also, it was J. B. J. Fourier,[16] who derived the functional representation of function

$$f(t) = \frac{1}{2\pi} \int_R \int_R f(\zeta) \cos\big(\xi(x-\zeta)\big) d\zeta d\xi \tag{5}$$

where he also formally introduced the fractional derivative version. Since from 19th century as a foundation of fractional geometry and fractional dynamics, the theory of FO, in particular, the theory of FC and FDEs and researches of application have been developed rapidly in the world. The modern epoch started in 1974 when a consistent formalism of the fractional calculus has been developed by Oldham and Spanier,[4], and later Podlubny,[6]. Applications of FC are very wide nowadays, in rheology, viscoelasticity, acoustics, optics, chemical physics, robotics, control theory of dynamical systems, electrical engineering, bioengineering and so on, [4-12]. In fact, real world processes generally or most likely are fractional order systems. The main reason for the success of applications FC is that these new fractional-order models are more accurate than integer-order models, i.e. there are more degrees of freedom in the fractional order model. Furthermore, fractional derivatives provide an excellent instrument for the description of memory and hereditary properties of various materials and processes due to the existence of a "memory" term in a model. This memory term insure the history and its impact to the present and future. A typical example of a non-integer (fractional) order system is the voltage-current relation of a semi-infinite lossy transmission line [17] or diffusion of the heat through a semi-infinite solid, where heat flow is equal to the half-derivative of the temperature [6].

2.1 Definition of fractional differintegral

As an essential preliminary consider some definitions concerning fractional derivatives. Fractional derivatives are typically treated as a particular case of pseudo-differential operators. Since they are nonlocal and have weakly singular kernels, the study of fractional differential equations seems to be more difficult and less theories have been established than for classical differential equations. Now, it is well known that, one may generalize the differential and integral operators into one fundamental D_t^α operator t which is known as fractional calculus:

$$ {}_aD_t^{\alpha} = \begin{cases} \dfrac{d^{\alpha}}{dt^{\alpha}} & \Re(\alpha) > 0, \\ 1 & \Re(\alpha) = 0 , \\ \int_a^t (d\tau)^{-\alpha} & \Re(\alpha) < 0. \end{cases} \tag{6} $$

The definition of fractional integral is described by

$$ {}_aD_t^{-\alpha} x(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-s)^{\alpha-1} x(s)ds, \quad \alpha > 0. \tag{7} $$

where $\Gamma(.)$ is the well known Euler's gamma function. There are three kinds of widely used fractional derivatives, namely the Grunwald-Letnikov (GL) derivative, the Riemann-Liouville (RL) derivative, and the Caputo (C) derivative. The GL derivative and RL derivative are equivalent if the functions they act on are sufficiently smooth. Besides, the RL derivative is meaningful under weaker smoothness requirements. The G-L definition is given by

$$ {}_a^{GL}D_t^{\alpha} f(t) = \lim_{h \to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{[(t-a)/h]} (-1)^j \binom{\alpha}{j} f(t - jh) \tag{8} $$

where a, t are the limits of operator and [x] means the integer part of $\mathbf{x}$. As indicated above, the previous definition of GL is valid for α > 0 (fractional derivative) and for α < 0 (fractional integral) and, commonly, these two notions are grouped into one single operator called *differintegral*. The RL derivative is given as:

$$ {}_a^{RL}D_t^{\alpha} x(t) = D^n \, {}_aD_t^{\alpha-n} x(t), \quad \alpha \in [n-1, n), \tag{9} $$

and the Caputo derivative

$$ {}_a^{C}D_t^{\alpha} x(t) = {}_aD_t^{\alpha-n} D^n x(t), \quad \alpha \in n(n-1, n), \tag{10} $$

where $n \in Z^+, D^n$ is the classical n-order derivative. Moreover, previous expressions show that the fractional-order operators are *global* operators having a memory of all past events, making them adequate for modeling hereditary and memory effects in most materials and systems. Also, for the RL derivative, we have

$$ \lim_{\alpha \to (n-1)^+} {}_a^{RL}D_t^{\alpha} x(t) = \frac{d^{n-1}x(t)}{dt^{n-1}} \quad \text{and} \quad \lim_{\alpha \to n^-} {}_a^{RL}D_t^{\alpha} x(t) = \frac{d^n x(t)}{dt^n} \tag{11} $$

But for the Caputo derivative, we have

$$ \lim_{\alpha \to (n-1)^+} {}_a^{C}D_t^{\alpha} x(t) = \frac{d^{n-1}x(t)}{dt^{n-1}} - D^{(n-1)}x(a) \quad \text{and} \quad \lim_{\alpha \to n^-} {}_a^{C}D_t^{\alpha} x(t) = \frac{d^n x(t)}{dt^n} \tag{12} $$

Obviously, ${}_a^{RL}D$, $n \in (-\infty, +\infty)$ varies continuously with n, but the Caputo derivative cannot do this. On the other side, initial conditions of fractional differential equations with Caputo derivative have a clear physical meaning and Caputo derivative is extensively used in real applications. For numerical calculation of fractional–order differ-integral operator one can use relation derived from the GL definition. This relation has the following form:

$$ {}_{((t-L)}D_t^{\pm\alpha} f(t) \approx h^{\mp\alpha} \sum_{j=0}^{N(t)} b_j^{(\pm\alpha)} f(t-jh) \tag{13}$$

where L is the "memory length", h is the step size of the calculation,

$$ N(t) = \min\left\{\left[\frac{t}{h}\right],\left[\frac{L}{h}\right]\right\}, \tag{14}$$

$[x]$ is the integer part of x and $b_j^{(\pm\alpha)}$ is the binomial coefficient given by

$$ b_0^{(\pm\alpha)} = 1, \quad b_j^{(\pm\alpha)} = \left(1-\frac{1\pm\alpha}{j}\right) b_{j-1}^{(\pm\alpha)} \tag{15}$$

For convenience, Laplace domain is usually used to describe the fractional integro-differential operation for solving engineering problems. The formula for the Laplace transform of the RL fractional derivative has the form:

$$ \int_0^\infty e^{-st}\, {}_0D_t^{\alpha} f(t)dt = s^\alpha F(s) - \sum_{k=0}^{n-1} s^k\, {}_0D_t^{\alpha-k-1} f(t)\big|_{t=0} \tag{16}$$

For $\alpha < 0$ (*i.e*., for the case of a fractional integral) the sum in the right-hand side must be omitted). A geometric and physical interpretation of fractional integration and fractional differentiation can be found in Podlubny's work [18].

3. Basic ideas of $PI^{\alpha}D^{\beta}$ feedack type control

In recent years, fractional calculus has been applied in the modeling and control of various kinds of physical systems, as it is well known and documented in many control theories or in the literature data. In what concerns the area of automatic control, the fractional-order algorithms are extensively investigated. Thanks to the widespread industrial use of PID controllers, even a small improvement in PID features, achieved by using $PI^{\alpha}D^{\beta}$, could have a relevant impact. Recently, published results [8-12] indicate that the use of a fractional-order PID controller can improve both the stability and performance robustness of feedback control systems. In [6],Podluny proposed a generalization of the PID controller namely fractional PID ($PI^{\alpha}D^{\beta}$) where α and β are the order of integration and derivation respectively that can be real numbers. In fact, in principle, they provide more flexibility in the controller design, with respect to the standard PID controllers, because they have five parameters to select (instead of three). However, this also implies that the tuning of the controller can be much more complex. Therefore classical design method may not be applied directly to adjust all fractional controller parameters. In order to address this problem, different methods for the design of a fractional order PID (FOPID) controller have been proposed in the literature. Further research activities are running in order to develop new tuning rules for fractional controllers, studying previously the effects of the non integer order of the derivative and integral parts to design a more effective controller to be used in real-life models. Some of these technics are based on an extension of the classical PID control theory. To this respect, in [19] the extension of derivation and integration order from integer to non integer numbers provides a more flexible tuning strategy and therefore an easier achieving of control requirements with respect to classical controllers. In [20] an optimal fractional order PID controller based on specified gain margin and phase margin with a minimum ISE criterion has been designed by using a differential evolution algorithm. In [21] a tuning method for fractional PID controller based on Ziegler-Nichols-type rules was proposed. Monje et al., [22] present a frequency domain approach based on the expected crossover frequency and phase margin. A state-space tuning method based on pole placement was also used (see [23]). Recent tuning method based on Quantitative Feedback Theory (QFT) are presented in [24]. In this paper, a fractional order PID controller ($PI^{\alpha}D^{\beta}$) is used to control the production process of technical gases as follows:

$$u(t) = K_p e(t) + K_d \, {}_0D_t^{\alpha} e(t) + K_i \, {}_0D_t^{-\beta} e(t) \tag{17}$$

The most common form of a fractional order PID controller is the $PI^{\alpha}D^{\beta}$ controller [6], involving an integrator of order α and a differentiator of order β where α, β can be any real numbers. The transfer function of such a controller has the form

$$G_c(s) = K_p + K_I s^{-\alpha} + K_D s^{\beta}, \quad (\alpha, \beta > 0) \tag{18}$$

The integrator term is s^{α} , that is to say, on a semi-logarithmic plane, there is a line having slope −20αdB. /dec. Clearly, selecting $\alpha = \beta = 1$, a classical PID controller can be recovered. The selections of $\alpha = 1, \beta = 0$, $\alpha = 0, \beta = 1$, respectively corresponds conventional PI & PD controllers. All these classical types of PID controllers are the special cases of the fractional $PI^{\alpha}D^{\beta}$ controller given by (17), see Fig. 1. It can be expected that the controller $PI^{\alpha}D^{\beta}$ may enhance the systems control performance. One of the most important advantages of the $PI^{\alpha}D^{\beta}$ controller is the better control of dynamical systems, which are described by fractional order mathematical models. Another advantage lies in the fact that the $PI^{\alpha}D^{\beta}$ controllers are less sensitive to changes of parameters of a controlled system [6-12]. This is due to the two extra degrees of freedom to better adjust the dynamical properties of a fractional order control system. However, in theory, $PI^{\alpha}D^{\beta}$ itself is an infinite dimensional linear filter due to the fractional order in differentiator or integrator.

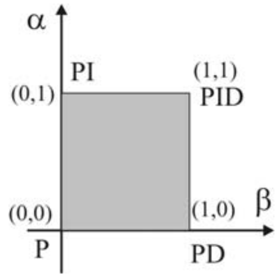


Figure 1. Generalization of the FOPID Controller: From point to plane

Fractional order controllers such as CRONE controller, TID controller, fractional PID controller and lead-lag compensator, [25] have so far been implemented to improve the performance and robustness in the closed loop control systems. As compared to conventional PID compensators, the TID compensator allows for simpler tuning, better disturbance rejection, and smaller effects of plant parameter variations on the closed-loop response. Feedback control system compensator of the PID type is provided, wherein the proportional component of the compensator is replaced with a tilted component having a transfer function $s^{-1/n}$. The resulting transfer function of the entire compensator more closely approximates an optimal loop transfer function, thereby achieving improved feedback control performance. On the other hand, the CRONE control was proposed by Oustaloup in pursuing *fractal robustness* [9], [10] where "fractal robustness" is used to describe the following two characteristics: the iso-damping and the vertical sliding form of frequency template in the Nichols chart. Also, it is possible to extend the classical lead-lag compensator to the fractional-order case which was studied in [26]. The fractional lead-lag compensator is given by

$$C_r(s) = C_0\left(\frac{1+s/\omega_b}{1+s/\omega_h}\right)^r \tag{19}$$

where $0<\omega_b<\omega_h, C_0>0$ and $r\in(0,1)$.Transfer functions such as (18) are not easy to implement for computational purposes. Simulations are usually carried out with software prepared to deal with integer powers of *s* only. Hardware implementations of controllers are nowadays usually achieved with electronic components allowing implementation of usual integer transfer functions easily, while how fractional transfer functions can be achieved with them is not clear at all. This makes the task of finding integer order approximations of fractional transfer functions a most important one where fractional transfer functions are usually replaced by integer transfer functions, with a behavior close enough to the one desired, but much easier to handle. Approximations are available both in the *s*-domain and in the *z*-domain. Moreover, one may find that many discretization schemes reported in literature which can be classified as either *direct* or *indirect*. The distinction is made based on whether an auxiliary continuous-time (*s* domain) approximation is constructed in the process. With direct methods, an intermediate continuous time approximation is not necessary, while with indirect methods such analogue approximation must be created. Most of the direct methods start with a suitable discrete approximation of the first order derivative or integral. Discretization scheme is then obtained by truncating some expansion of an appropriate non-integer power of the selected approximation. For example, a method based on power series expansion (PSE) of Euler operator,[27], or continued fraction expansion (CFE) is applied to Tustin operator,[28]. Further direct schemes are reported in [29-30]. Indirect methods are constructed in two steps where in the first step, a finite dimensional, continuous time approximation of the target fractional order system is found such as Oustaloup's rational approximation (ORA), [31] or sub-optimum H_2 rational approximation ,[32]. Once a satisfactory continuous-time approximation has been found, the second step of each indirect method is to find its discrete-time equivalent,as follows: approximations of Euler and Tustin, response invariant transformations (impulse-invariant and step invariant) and others, see[33].

4. Mathematical model of cryogenic process of mixing of two gaseous airs flows at different temperatures before entrance of expansion turbine

Cryogenics is the science and technology dealing with temperatures less than about 120 K, although this historical summary does not adhere to a strict 120K definition. The techniques used to produce cryogenic temperatures differ in several ways from those dealing with conventional refrigeration. Also, liquid air is air that has been cooled to very low temperatures(cryogenic temperatures) so that it has condensed to a pale blue mobile liquid. To protect it from room temperature, it must be kept in a vacuum flask. In practice, these two areas often overlap and the boundary between conventional and cryogenic refrigeration is often indistinct. Significant reductions in temperature often have very pronounced effects on the properties of materials and the behavior of systems. New way to technical production liquid air work is obtained by C. Linde at the end of the nineteenth century. On with the help of the reversing heat exchangers, slightly used cooling air, which appears in the damping of the higher of the lower pressure, the successful simple and economical production liquor large amounts of air. Production liquid air low pressure was first introduced in 1938. by the Russian academician P. L. Kapitsa, and includes production liquor air pressure $p_2 = 6 \div 7\left[bar\right]$ and expansion in the gas turbine. So, expansion turbine in the air production liquid used for expansion air with thermodynamics state $P\left(p_p, T_p\right)$ to state $K\left(p_k, T_k\right)$ lowering when the air temperature with T_p at T_k and the pressure with p_p at p_k. Expansion of cold air after the start of equipment and waste heat arising due to exchange heat with the environment during the work. The amount of air that expansion in the gas turbine, according to [34], does not 25% exceed the amount of usable air. The air from the compressed state 1 turbocompressor, Fig. 2,(b) and cool to the state of the 2nd compressed air with pressure p_2 are in the reverse exchangers heat, where the cold to the state of 3rd Part of the air with the environment reverse heat state 3* and part of the state 3, which consists $m_e\left[kg / kg\right]$ of compressed air, are in expansion turbine where the expansion achieved by the state 8, where pressure p_1. Because of loss coefficient and other non-reverse expansion is not adiabatic line to state 8_{ad}, but to state 8, which is right. Place for removal of air state 4 elected to state 8, at the end of expansion, is in the area near the upper border curve (in the TS diagram $x = 1,0$ on $1 \div 3\left[K\right]$ above the temperature saturated steam. Basic devices of the plant are (TK –turbocompressor, H - refrigerator air, RR – reverse exchangers heat, ET – expansion turbine, RK - exchangers heat i.e. air condenser, PV-damping valve)

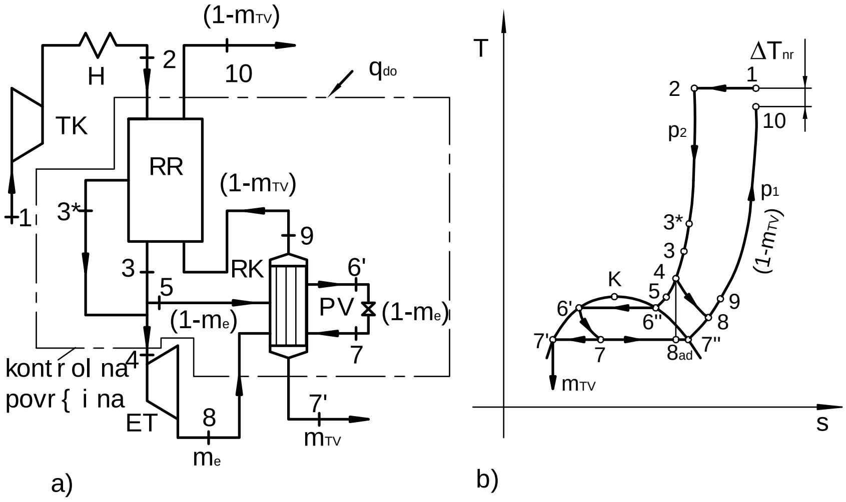


Figure 2. Scheme for plant liquid air flow pressure (a) and TS diagram of the process (b)

Liquid air quantity m_{TV} can be determined on the basis of heat balance,[34]:

$$h_2 = m_{TV} h_7^{'} + \left(1 - m_{TV}\right) i_{10} + m_e \left(h_4 - h_8\right) - q_{do}, \ \left[kJ/kg\right] \tag{20}$$

where are $q_{do}\left[kJ/kg\right]$ - heat from the environment brought by kg air, $m_{TV}\left[kg/kg\right]$ - mass liquid air, $m_e\left[kg/kg\right]$ - mass air which expansion in expansion turbine. In the ideal case when the $q_{do} \approx 0$ and $\Delta T_{nr} = T_1 - T_{10} \approx 0$ liquid air mass is

$$m_{TV} = \frac{h_{10} - h_2}{h_{10} - h_7^{'}} + m_e \frac{h_4 - h_8}{h_{10} - h_7^{'}} \le \frac{h_1 - h_2}{h_{10} - h_7^{'}} + m_e \frac{h_4 - h_8}{h_{10} - h_7^{'}} \tag{21}$$

The main advantage of the procedure Kapic's according to the toe cap in relation to other procedures production liquid air [35] to be in the low pressure p_2 still does not have to spend inordinate work for production liquid $1\left[kg\right]$ air. Since the turbine is capable of much greater bandwidth than the reciprocating compressor is adapted to this process for large plants such as the face in practice. For qualitative assessment of gas turbine, with thermodynamics' point of view is used isentropic (internal) level of utility which is determined by the following terms:

$$\eta_T = \frac{\Delta h}{\Delta h'} = \frac{h_p - h_k}{h_p - h_{k'}} \approx \frac{T_p - T_k}{T_p - T_{k'}}, \ \eta_T = 0,80 \div 0,85 \tag{22}$$

For the development of the expansion works after refrigeration air, in the *" Factory of technical gas*" in Bor, built two expansion turbines, one gas turbine is always in operation, the other in the reserves, and the factory in preparation for the start after longer delays both turbine running in parallel. Energy received in expansion turbine in the work spent to drive fan that absorptive air from the atmosphere, regardless of air flow in the gas turbine. Ventilator compressed air and thus prevent an unlimited increase in the number of turbine rotor speed, a compressed air is emissive in the atmosphere which is not justified from the energy aspect. Technical adiabatic the work of expansion of air in gas turbine is:

$$l_{t,ad} = 10^{-3}\left(k/k-1\right)R\cdot T_p\left[1-\left(p_k/p_p\right)^{\frac{\kappa-1}{\kappa}}\right];\left[kJ/kg\right] \tag{23}$$

Power returned to the turbine - effective power, the expansion $\dot{m}\left[kg/s\right]$ of the pressure p_p to p_k have the value:

$$N_{eT} = N_i\cdot\eta_m = \dot{m}\cdot l_{t,ad}\cdot\eta_T\cdot\eta_m;\left[kW\right] \tag{24}$$

where are: $N_i\left[kW\right]$ - the internal (isentropic) gas turbine power, η_m - mechanical degree of utility gas turbines (due to friction in the bearings and stuffing box). $x_{i1}\left(t\right) = g_5\left(t\right)\left[m_N^3/h\right]$ - deviation values flow from the nominal value of gas's air flow at the entrance to the expansion turbine: $G_{56N} = 7600\left[m_N^3/h\right]$, $x_{i2}\left(t\right) = \theta_5\left(t\right)\left[K\right]$ -value of temperature deviation from the nominal value of gas's air temperature at the entrance to the expansion turbine, $T_{5N} = 124\left[K\right]$, $z_1(t) = \theta_1\left(t\right)\left[K\right]$ - value of temperature deviation from the nominal value of temperature gas's air environment with exchangers,heat $T_{1N} = 153\left[K\right]$, $z_2(t) = \theta_3\left(t\right)\left[K\right]$ -value of temperature deviation from the nominal value of temperature air with the end of the cold heat exchangers $T_{3N} = 101\left[K\right]$, $y_A\left(t\right)\left[mm\right]$ - deviation value position of the nominal value of the position control valves $TV946A\ Y_{AN} = 14,7\left[mm\right]$, $y_B\left(t\right)\left[mm\right]$ - deviation value position of the nominal value of the position control valves, $TV946B\ Y_{BN} = 30,2\left[mm\right]$ .On Fig. 3 it is presented diagram of process and symbolic-functional scheme with relevant variables.

4.1. *System description of mechatronic system in state space*

For application in the synthesis of proposed control input temperature and the flow of air expansion turbine, it is necessary to determine the appropriate differential equations linear's part of the cryogenic process of mixing of two gaseous airs flows at different temperatures before entrance of expansion turbine. Linear's differential equations that describe the work process are given as appropriate equation of state and output as follows

$$\begin{aligned}\dot{x}\left(t\right) &= \begin{bmatrix}-0,2 & 0\\ 0 & -0,2\end{bmatrix}x\left(t\right)+\begin{bmatrix}45,736 & 28,07\\ 0,174 & -0,085\end{bmatrix}u\left(t\right)+\begin{bmatrix}0 & 0\\ 0,088 & 0,112\end{bmatrix}z(t)\\ x_{iz}\left(t\right) &= \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}x\left(t\right)\end{aligned} \tag{25}$$

or, in condensed form is

$$\dot{x}\left(t\right) = Ax\left(t\right)+B_u u\left(t\right)+B_z z\left(t\right),\qquad x_i\left(t\right) = Cx\left(t\right) \tag{26}$$

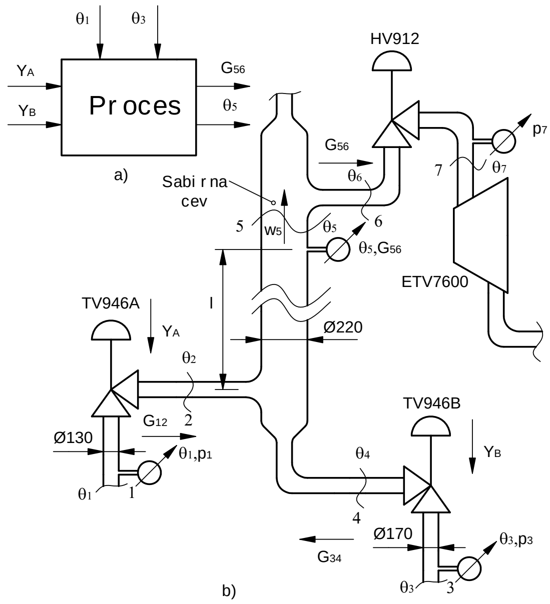


Figure 3. Diagram of the process (a) symbolic-functional scheme (b)

where corresponding vectors are, $u(t)=\left[y_A(t)\ y_B(t)\right]^T$, $z(t)=\left[z_1(t)\ z_2(t)\right]^T$, and A, B_u, B_Z, C are matrices with appropriate dimensions. From the above it is clear that model presents a MIMO system(multiple input, multiple output) where the number of inputs being to equal to that of outputs, system us square and it is possible to apply a control strategy uncoupling, whereby each of the inputs is made affect presented by one output only.In that way, one may obtain so called non-interactive system where is transfer function $W(s)$ of given system is decoupled, diagonal, and nonsingular matrix. To decouple the system, a new input $u(t)$ is introduced by means of feedback

$$u(t)=-K_c x(t)+F_c v(t) \tag{27}$$

where are $c_i - i$ -th row of matrix C and

$$N=\begin{bmatrix} c_1 A^{p_1} B_u \\ c_2 A^{p_2} B_u \\ \cdots\cdots \\ c_m A^{p_m} B_u \end{bmatrix}, \det N \neq 0 \quad p_i=\begin{cases} \min\left(j, c_i A^j B_u \neq 0\right) \\ n-1, c_i A^j B_u = 0,\ \forall j \end{cases}, \forall j = 0,1,2,\ldots,n-1, \tag{28}$$

So, one can obtain

$$F_c = N^{-1},\ K_c = N^{-1}\left[c_1 A^{p_1+1} \quad c_2 A^{p_2+1} \quad \ldots \quad c_m A^{p_m+1}\right]^T \tag{29}$$

The transfer function $W(s)$ of the original system is

$$W(s) = C(sI - A)^{-1} B_u = \begin{bmatrix} \dfrac{45.736}{s+0.2} & \dfrac{28.07}{s+0.2} \\ \dfrac{0.174}{s+0.2} & \dfrac{-0.85}{s+0.2} \end{bmatrix} \tag{30}$$

and after applying new control is $W(s) = C(sI - A + B_u K_c)^{-1} B_u F_c$. Taking into account the proposed procedure for F_c, K_c it follows

$$\begin{aligned} &c_1 = [1 \;\; 0],\; c_2 = [0 \;\; 1],\; p_1 = 0,\; p_2 = 0, \\ &N = B_u,\; F_c = B_u^{-1},\; K_c = B_u^{-1} A \end{aligned} \tag{31}$$

and

$$W(s) = C(sI)^{-1} = \begin{bmatrix} 1/s & 0 \\ 0 & 1/s \end{bmatrix} \tag{32}$$

Now, decoupling system is

$$\begin{aligned} \dot{x}_1 &= v_1 \\ \dot{x}_2 &= v_2 + 0.088 z_1 + 0.112 z_2 \end{aligned} \tag{33}$$

5. The proposed fractional PIDs

Unlike conventional PID controller, there is no systematic and rigor design or tuning method existing for $PI^{\alpha}D^{\beta}$ controller. Here, design goals are choosing suitable α and β as well as load disturbance rejection.The controller parameters are the proportional gain K_p, the derivative gain K_d, the integral gain K_i, as well as noninteger order of derivative β and integrator α ,Eq. 34. Load disturbances are typically low frequency signals and their attenuation is a very important characteristic of a controller. It is shown [1], that by maximizing the integral gain K_i, the effect of load disturbance at output will be minimum.

$$G_c(s) = \frac{K_p s^{\alpha} + K_I + K_D s^{\beta+\alpha}}{s^{\alpha}}, \quad (\alpha, \beta > 0) \tag{34}$$

Here, in order to obtain step response, simulation model has been developed using Simulink Library of MATLAB by using a special toolbox for non-integer control. For the simulation purpose, here we present the Crone approximation algorithm. It is based on the approximation of a function of the form:

$$C(s) = ks^{\nu},\; \nu \in R \tag{35}$$

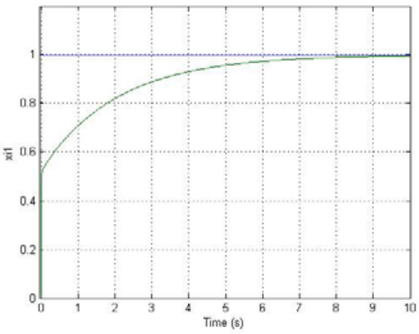
which uses a recursive distribution of N poles and N zeros:

$$C(s) = k' \prod_{n=1}^{N} \frac{1 + \dfrac{s}{\omega_{z,n}}}{1 + \dfrac{s}{\omega_{p,n}}}, \tag{36}$$

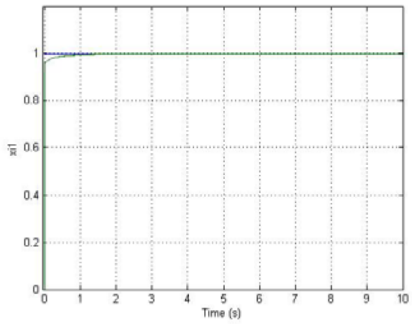
Gain k' is adjusted so that if k is 1 then $|C(s)| = 0\,dB$ at $1\,rad/s$. Zeros and poles are found inside a frequency interval $[\omega_l, \omega_h]$ and are given, for a positive ν, by

$$\eta = (\omega_h / \omega_l)^{\frac{1-\nu}{N}}, \alpha = (\omega_h / \omega_l)^{\frac{\nu}{N}}, \quad \omega_{p,n} = \omega_{z,n-1}\alpha,\ n = 1,2,...,N$$
$$\omega_{z,n} = \omega_{p,n-1}\eta,\ n = 2,...,N, \quad \omega_{z,1} = \omega_l\sqrt{\eta} \qquad (37)$$

For a negative ν the role of zeros and poles is interchanged. The controller is reckoned from k,ν,ω_l,ω_h and N. Here, they are presented simulation results for x_{i1}, Fig.4
$K_p = 1, K_d = 1, K_i = 0.1, \alpha = 0.99, \beta = 0.99$ $K_p = 50, K_d = 20, K_i = 10, \alpha = 0.9, \beta = 0.8$



a)

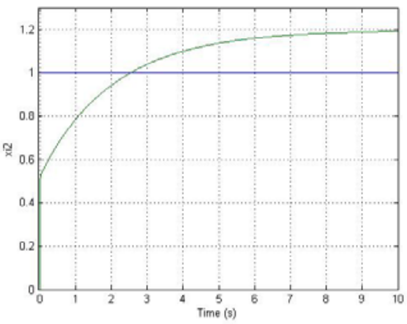


b)

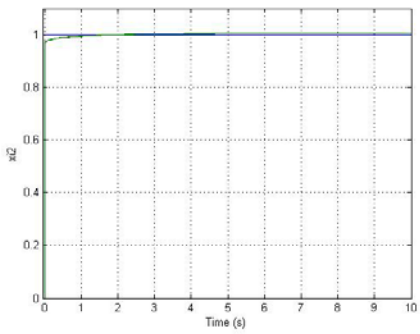
Figure 4. a) step response of x_{i1}, for $K_p = 1, K_d = 1, K_i = 0.1, \alpha = 0.99, \beta = 0.99$

b) **)** step response of x_{i1}, for $K_p = 50, K_d = 20, K_i = 10, \alpha = 0.9, \beta = 0.8$

and for x_{i2} as follows, Fig.5:
$K_p = 1, K_d = 1, K_i = 0.1, \alpha = 0.99, \beta = 0.99$, $K_p = 30, K_d = 10, K_i = 40, \alpha = 0.9, \beta = 0.5$



a)



b)

Figure 5. a) step response of x_{i2}, for $K_p = 1, K_d = 1, K_i = 0.1, \alpha = 0.99, \beta = 0.99$

b) **)** step response of x_{i2}, for $K_p = 30, K_d = 10, K_i = 40, \alpha = 0.9, \beta = 0.5$

6. Discussion

Here, in this paper it is proposed new robust control algorithms of $PI^{\alpha}D^{\beta}$ type which based on using fractional calculus in the control of producing of technical gases, i.e air production cryogenic liquid. Design goals are suitable setting the controller parameters

K_i, K_d, K_p, noninteger order of derivative β and integrator α to fulfill different design specifications for the closed-loop system, for example, load disturbance rejection. Also, the problem of discretization of proposed $PI^{\alpha}D^{\beta}$ is considered as a one of important steps in digital implementation. In order to obtain step response, simulation model has been developed using Simulink Library of MATLAB by using a special toolbox for non-integer control.

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