Some applications of fractional calculus on control problems in robotics and system stability

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In recent years, there have been extensive research activities related to applications of fractional calculus (FC), [5] in nonlinear dynamics, mechatronics as well as control theory. In this paper, they are presented recently obtained results which are related to applications of fractional calculus in mechanics - specially stability and control issues. Some of these results [1-4] are presented at the *Fifth symposium of fractional differentiation and its applications FDA2012*, was held at the Hohai University, Nanjing, China in the period of May 14-May 17, 2012. Also, fractional order dynamic systems and controllers have been increasing in interest in many areas of science and engineering in the last few years.In that way, our objective of using fractional calculus is to apply the fractional order controller to enhance the system control performance as well as it has better disturbance rejection ratios and less sensitivity to plant parameter variations.

First, they are introduced and obtained the new algorithms of fractional order PID control based on genetic algorithms in the position control of a 3 DOF's robotic system driven by DC motors. Then, the main task is to find out the optimal settings for a fractional $PI^{\alpha}D^{\beta}$ controller in order to fulfill the proposed design specifications for the closed-loop system. In addition, this method allows the optimal design of all major parameters of a fractional PID controller and then enhances the flexibility and capability of the PID controller. Last,in simulations, they are compared step responses of these two optimal controllers where it will be shown that fractional order PID controller improves transient response as well as provides more robustness in than conventional PID.

Second, we propose sufficient conditions for finite time stability for the (non)homogeneous fractional order systems with time delay. Specially, the problem of finite time stability with respect to some of the variables (partial stability) is considered. Namely, along with the formulation of the problem of stability to all variables, Lyapunov also formulated a more general problem on the stability to a given part of variables (but not all variables) determining the state of a system,[6].The problem of the stability of motion with respect to some of the variables also known as partial stability arises naturally in applications. So, in this presentation, it will be proposed finite time partial stability test procedure of perturbed (non) linear (non)autonomous time varying delay fractional order systems. Time-delay is assumed to be varying with time but its upper bound is assumed to be known over given time interval. New stability criteria for this class of fractional order systems will be derived using “classical” Bellman-Gronwall inequality,as well as another another suitable inequality, [7]. Last,a numerical example is provided to illustrate the application of the proposed stability procedure.

Third, some attention is devoted to the problem of stability of linear discrete-time fractional order systems is addressed, [8]. It was shown that some stability criteria for discrete time-delay systems could be applied with small changes to discrete fractional order state-space systems. Accordingly, simple conditions for the stability and robust stability of a particular class of linear discrete time-delay systems are derived. These results are modified and used for checking the stability of discrete-time fractional order systems. The systems under consideration involve time delays in the state and parameter uncertainties. The parameter uncertainties are assumed to be time-varying and norm bounded.

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