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KINEMATIC MODELING OF RECONFIGURABLE PARALLEL ROBOTS BASED ON DELTA CONCEPT

Abstract: *In order to develop reconfigurable DELTA robot with rotary and translatory actuated joints the generalized modeling approach is discussed. The results of a study on generalization of modeling approach without any use of non-actuated variables has been reported in this paper.*
Key words: *Delta robot, reconfigurability, kinematic modeling*

1. INTRODUCTION

The 3-DOF DELTA structure [1,2] is one of the most famous parallel mechanisms, Fig. 1a. The 4-DOF DELTA robots based on this structure, Figs. 1b and 1c, have been the first real commercial success for parallel robots. The 4-DOF DELTA robot comprise 3-DOF DELTA parallel mechanism and 1-DOF serial wrist for end-effector orientation. Parallel mechanism consist of three kinematic chains with identical topology so that the platform in its motion through the space retains constant orientation. The motors of parallel part of DELTA robot are mounted on fixed base while motor for end-effector orientation may also be on fixed base, Fig. 1b or on the movable platform, Fig. 1c.

The technological structure and capacities of DELTA robot (velocity up to 10 m/s, acceleration over 10g) make it ideal for handling tasks in multitude of sectors such as in food and agriculture, the hygiene sector, beauty care, health care or electronic components. It is also important to mention that DELTA "linear" version is now the base of the fastest machine tools ever produced by industry (acceleration 3.5 - 5g) [4].

The concept of reconfigurable DELTA robots and small machine tools with rotary or translatory actuated joints, Fig. 2, is planed [5,6] with the idea to enable static and dynamic reconfiguration [7]. Several different approaches in kinematic modeling of DELTA robot with rotary actuated joints have already been published [8-10].

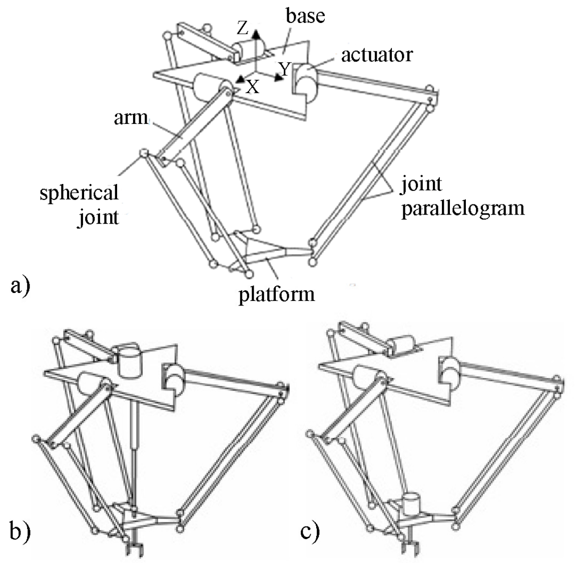

Fig. 1. DELTA robot with rotary actuated joints [3]

Approach presented in [8] uses non-actuated variables while approaches presented in [9,10] does not use any non-actuated variables but lacks generality. One of the most important prerequisites for such widely adopt

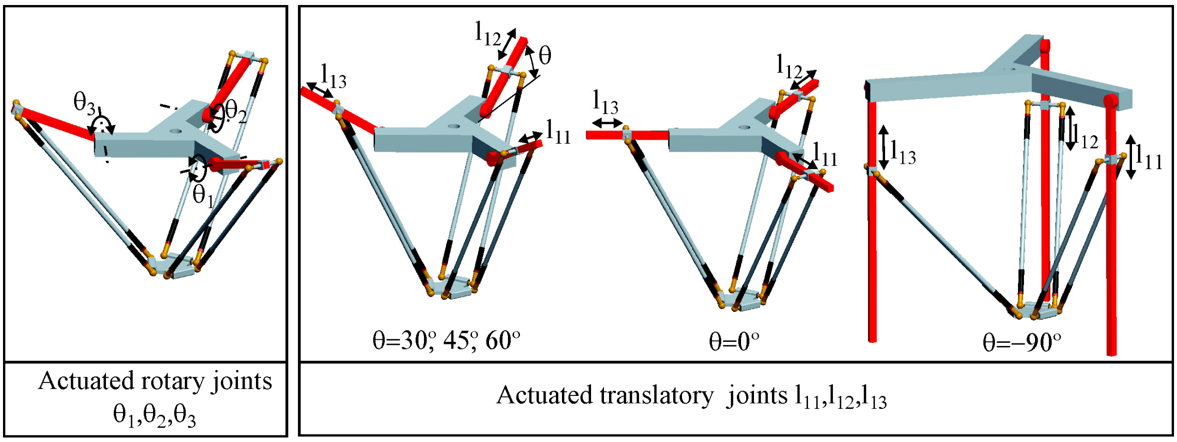

Fig. 2. Concept of reconfigurable DELTA robots and small machine tools with rotary or translatory actuated joints

approach of reconfigurability is generalized modeling approach without any use of non-actuated variables which is reported in this paper.

2. GENERALIAZATION OF MODELING APPROACH OF RECONFIGURABLE DELTA ROBOT

Figures 3a and 3b represent geometric models of DELTA robots with rotary and translatory actuated joints from Fig.2, where each parallelogram is represented as a unique rod.

As can be seen from Fig. 3, DELTA robots comprise 3-DOF DELTA spatial parallel mechanisms with rotary or translatory actuated joints and 1-DOF serial wrist for end-effector orientation.

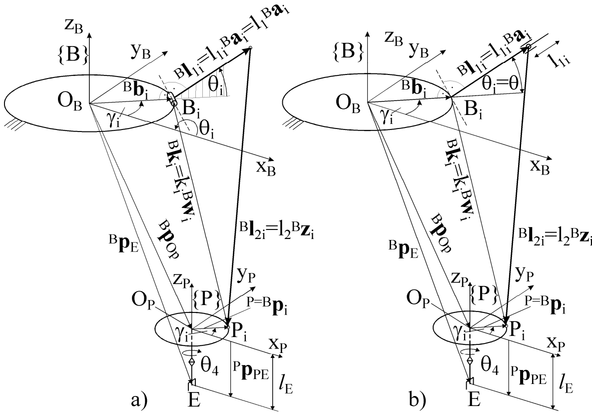

Fig. 3. Geometric models of DELTA robots with rotary and translatory actuated joints

These efficient geometric descriptions of DELTA kinematic structures provide generalized modeling approach without any use of non-actuated variables and implementation of control algorithms on low-cost hardware. Coordinate frames {B} and {P} attached to the base and movable platform are always mutually parallel due to the mechanism's nature. Vectors $\mathbf{v}$ referenced in frames {B} and {P} are denoted by ${}^{B}\mathbf{v}$ and ${}^{P}\mathbf{v}$.

Vectors defined by the robot parameters:

- ${}^{P}\mathbf{p}_i = [r\cdot c\gamma_i \quad r\cdot s\gamma_i \quad 0]^T$, i=1,2,3 (1)
  are the position vectors of the midpoints P_i between joint centers at the platform located on the circle of radius r with angular position $\gamma_i = 2\pi\cdot(i-1)/3$ and are defined in frame {P}. "c" and "s" refer to *cosine* and *sine* functions;
- ${}^{P}\mathbf{p}_{PE} = [0 \quad 0 \quad l_E]^T$ (2)
  is the position vector of the end-effector tip defined in frame {P} where l_E is length of end-effector;
- ${}^{B}\mathbf{b}_i = [R\cdot c\gamma_i \quad R\cdot s\gamma_i \quad 0]^T$ (3)
  are position vectors of the points B_i at the base located on the circle of radius R with angular position $\gamma_i = 2\pi\cdot(i-1)/3$ defined in the base frame {B}.
  For DELTA robot with rotary actuators points B_i, i=1,2,3 represent centers of rotary joints while for DELTA robot with translatory actuators points B_i represent references points of driving axes;

World coordinates vectors:

- $\mathbf{x}_E = \begin{bmatrix} {}^{B}\mathbf{p}_E \\ \phi \end{bmatrix}$ represents the position and orientation of end-effector for all cases of 4-DOF DELTA robots from Fig. 3, where ${}^{B}\mathbf{p}_E = [x_E \quad y_E \quad z_E]^T$ is the position vector of end-effector tip E in the base frames {B} while rotational angle ϕ around axis z_B defines end-effector orientations;
- ${}^{B}\mathbf{p}_{OP} = [x_P \quad y_P \quad z_P]^T = \mathbf{x}_P$ represents location of the platform i.e. origin O_P of the coordinate frame {P} attached to it. The relationship between vectors ${}^{B}\mathbf{p}_{OP}$ and ${}^{B}\mathbf{p}_E$ is obvious since coordinate frames {B} and {P} are always mutually parallel i.e.

$$
{}^{B}\mathbf{p}_{OP} = \begin{bmatrix} x_P \\ y_P \\ z_P \end{bmatrix} = {}^{B}\mathbf{p}_E - {}^{P=B}\mathbf{p}_{PE} = \begin{bmatrix} x_E \\ y_E \\ z_E - l_E \end{bmatrix} \tag{4}
$$

Vector ${}^{B}\mathbf{p}_{Op}$ is further considered as world coordinates vector for all cases of 3-DOF DELTA mechanisms.

Joint coordinates vectors:

- $\boldsymbol{\theta}_E = [\theta_1 \quad \theta_2 \quad \theta_3 \quad \theta_4]^T$ is joint coordinates vector for 4-DOF DELTA robot with rotary actuated joints;
- $\mathbf{L}_E = [l_{11} \quad l_{12} \quad l_{13} \quad \theta_4]^T$ is joint coordinates vector for all cases of 4-DOF DELTA with translatory actuated joints.

The relationship between joint angle θ_4 and end-effector orientation angle ϕ, in all cases of 4-DOF DELTA robots, is obvious since frames {B} and {P} are always mutually parallel, i.e. $\theta_4 = \phi$. Considering this fact, the above joint coordinates vectors $\boldsymbol{\theta}_E$ and $\mathbf{L}_E$ for further considerations are reduced as:

$$
\boldsymbol{\theta} = [\theta_1 \quad \theta_2 \quad \theta_3]^T \tag{5}
$$

$$
\mathbf{L}_E = [l_{11} \quad l_{12} \quad l_{13}]^T \tag{6}
$$

where θ_i and l_{1i}, i=1,2,3 are scalar variables controlled by actuators.

Unit vectors:

- ${}^{B}\mathbf{a}_i = [c\gamma_i\cdot c\theta_i \quad s\gamma_i\cdot c\theta_i \quad s\theta_i]^T,\ i=1,2,3$ (7)

unit vectors ${}^{B}\mathbf{a}_i$ define vectors ${}^{B}\mathbf{l}_{1i}$ as ${}^{B}\mathbf{l}_{1i} = l_{1i}\cdot{}^{B}\mathbf{a}_i$.

For rotary joints, unit vectors ${}^{B}\mathbf{a}_i$ contain joint coordinates θ_i while $l_{1i} = l_1$, i=1,2,3 where l_1 is fixed arm length.

For translatory joints, unit vectors ${}^B\mathbf{a}_i$ define directions of translatory joints l_{1i} while $\theta_i = \theta$, i=1,2,3 is fixed inclination angle of translatory joints. In these cases ${}^B\mathbf{a}_i$ is expressed as

$$ {}^B\mathbf{a}_i = \begin{bmatrix} c\gamma_i \cdot c\theta & s\gamma_i \cdot c\theta & s\theta \end{bmatrix}^T,\ i = 1,2,3 \tag{8} $$

Other vectors and parameters are defined as shown in Fig. 3, where ${}^B\mathbf{w}_i$ and ${}^B\mathbf{z}_i$ are unit vectors while l_2 is fixed length of joint parallelograms.

Based on geometric relations shown in Figs. 3a and 3b the following generalized equations for both geometric models are derived:

$$ k_i \cdot {}^B\mathbf{w}_i = {}^B\mathbf{p}_{Op} + {}^{P=B}\mathbf{p}_i - {}^B\mathbf{b}_i \tag{9} $$

$$ k_i \cdot {}^B\mathbf{w}_i = l_{1i} \cdot {}^B\mathbf{a}_i + l_2 \cdot {}^B\mathbf{z}_i \tag{10} $$

Vectors $k_i \cdot {}^B\mathbf{w}_i$ in eq. (9) are common for all cases of DELTA robots and using eqs. (1), (3) and (4) can be obtained as

$$ k_i \cdot {}^B\mathbf{w}_i = \begin{bmatrix} k_{wxi} \\ k_{wyi} \\ k_{wzi} \end{bmatrix} = \begin{bmatrix} x_P + (r - R) \cdot c\gamma_i \\ x_P + (r - R) \cdot s\gamma_i \\ z_P \end{bmatrix} \tag{11} $$

By taking square of both sides in eq. (10) the following relation is derived

$$ l_2^2 = k_i^2 - 2 \cdot l_{1i} \cdot ({}^B\mathbf{a}_i \cdot k_i \cdot {}^B\mathbf{w}_i) + l_{1i}^2 \tag{12} $$

where $k_i^2 = kw_{xi}^2 + kw_{yi}^2 + kw_{zi}^2$.

From this equation inverse and direct kinematics for all cases of DELTA robots can be solved.

2.1 Inverse and direct kinematics for DELTA robot with rotary actuators

Taking into the account that for this case $l_{1i} = l_1$ and substituting eqs. (7) and (11), eq. (12) can be reduced to the well known type of trigonometric equation as

$$ c\theta_i \cdot (c\gamma_i \cdot kw_{xi} + s\gamma_i \cdot kw_{yi}) + s\theta_i \cdot kw_{zi} = \\ = \frac{l_1^2 - l_2^2 + k_i^2}{2 \cdot l_1} \tag{13} $$

from which joint coordinates θ_i, i=1,2,3 can be solved.

Equation (13) gives 2 solutions of inverse kinematics, Fig. 4a.

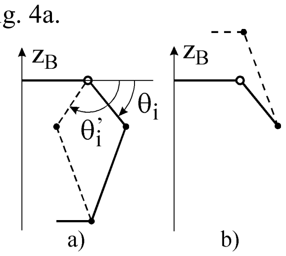

Fig. 4. Two solutions for inverse and direct kinematics

In order to avoid one of the DELTA structure singularities solution θ_i from Fig. 4a has to be chosen. It is important to mention that calculations of each chain in eq. (13) are independent and the algorithm may be implemented on a parallel architecture.

According to eq. (11), eq. (13) in case of direct kinemnatics represents the system of three equations from which world coordinates x_P, y_P and z_P can be obtained. Among two solutions of direct kinematics only one is physically possible, Fig. 4b.

2.2 Inverse and direct kinematics for DELTA robot with translatory actuators

In case of inverse kinematics, equation (12) is a second order polynomial in terms of l_{1i} and joint coordinates for all cases of DELTA robots with translatory actuated joints are obtained as

$$ l_{1i} = ({}^B\mathbf{a}_i \cdot k_i \cdot {}^B\mathbf{w}_i) + \sqrt{({}^B\mathbf{a}_i \cdot k_i \cdot {}^B\mathbf{w}_i)^2 + k_i^2 - l_2^2} \tag{14} $$

Equation (14) gives two solutions of inverse kinematics but only solution with positive square root can be chosen. It is also important to mention that calculations of each chain in eq. (14) are independent and the algorithm may be implemented in parallel architecture.

According to eq. (11), eq. (14) in case of direct kinematics, represents the system of three equations from which world coordinates x_P, y_P and z_P can be obtained. Among two solutions of direct kinematics only solution when $z_P < 0$, is physically possible.

3. DELTA ROBOT FIRST PROTOTYPE

On the basis of adopted concept and design parameters the first DELTA robot prototype with rotary actuators completely has been designed, built and tested in our laboratory, Fig. 5.

Fig. 5. DELTA robot first prototype

Parallel mechanism provides three degrees of freedom of end-effector positioning. At this stage, actuators are composed of step motors and timing bolts are located on the stationary base. The fourth degree of freedom provides orientation of end-effector and is also actuated by step motor located of the moving plate. End-effector is equipped with standard vacuum cup with pneumatically powered vacuum generator (venturi tube).

Figure 6 represents a simplified structure of control and programming system.

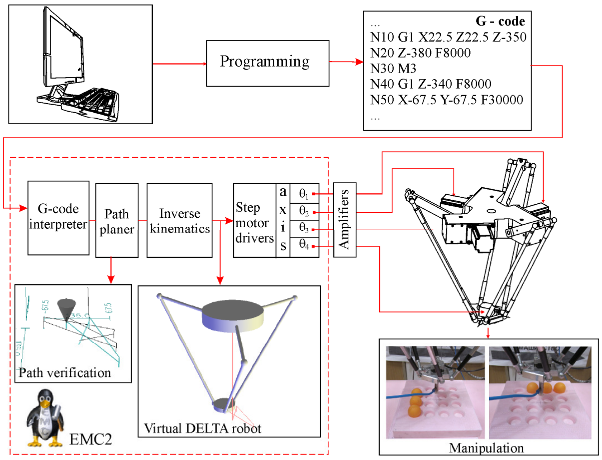

Fig. 6. The structure of control and programming system

Among several proposed OAC (Open Architecture Control), solution the first low-cost control system is based on PC real-time Linux platform with EMC2 software for computer control of machine tools, robots, parallel kinematics machine. EMC2 was initially created by the NIST and is free software released under the terms of the General Public License (http://linuxcnc.org). Based on equation (13) kinematics modul is programmed in C language and is integrate in EMC2 software system.

In this stage, the programming system is very conventional with the use of G code. During G-code loading, EMC2 software performs path verification. When the program starts running, G-code instructions are executed in real time and generated control signals are directed to real and/or virtual DELTA robot. The virtual DELTA robot makes possible simulation of the real DELTA robot for the user, i.e., verification of the program in robot workspace.

4. CONCLUSION

In order to develop reconfigurable DELTA robot with rotary or translatory actuated joints, the generalized modelling approach is discussed. The results of study on generalization of modelling approach without any use of non-actuated variables have been detailed reported in this paper. Based on analytically solved inverse and direct kinematics, control algorithms are implemented on low-cost hardware with EMC2 computer control software. The developed and investigated prototype indicates that proposed reconfigurable concept with generalized modelling approach will be superior to comparable approaches, which justify further research in this direction.

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