ROTATION TRANSFORMATION MATRIX OF THE JOINT COORDINATE SYSTEM WITH THE APPLICATION TO THE KNEE JOINT

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Abstract:

The human musculoskeletal system is comprised of various joints which are characterized by their unique morphology and kinematics. The joint coordinate system is tailored to comply with the observed mechanical properties of joints, making their research more intuitive and in accordance with standard clinical terminology. This coordinate system allows for spatial orientation to be described by three sequence independent Euler angles (the so-called "jaw-pitch-roll" angles), which mirrors the sequence independency of the final position of human limbs. The knee joint is the most complex, both anatomically and mechanically. It can be modelled as a four link open kinematic chain to which the joint coordinate system is applied. This coordinate system is usually described in Denavit-Hartenberg parameters, and the rotation transformation matrix is obtained using standard methods of vector algebra. In this paper another approach is used to derive the rotation transformation matrix, namely, the Rodrigues' rotation formula. The utility of this algorithm is evident: the Rodrigues' method provides an elegant solution which is obtained in a concise and efficient manner.

Key words: biomechanics, knee joint, joint coordinate system, rotation transformation matrix, Rodrigues' rotation formula

1. Introduction

Complex anatomical structures often require creative engineering solutions, adapted for the specific experiment or research. One of the most challenging in biomedical engineering is the human knee joint. Due to high dynamic loads and its susceptibility to injuries, there is a great need for replacement prosthetics, as well as individual surgical plans. In order for these requirements to be met, the problem has to be approached from various fields of engineering (kinematics, dynamics, tribology, FEM analysis, etc.) and medicine. Thus, it is important for the coordinate system, which is applied during research, to comply with mechanical and medical requirements and to bridge the gap between medical and engineering research groups in terms of terminology.

The joint coordinate system is created in relation to the bony landmarks of the joint [1], [2]. Also, it seamlessly fits the four link open kinematic chain, which conveniently describes the function of joints [1]. This coordinate system allows for three sequence independent Euler angles to be used [1], [3]. These angles (the so-called "jaw-pitch-roll" angles) are in accordance with clinical rotations of the joint (flexion-extension, abduction-adduction and external-internal rotation) [1], [2], [3]. Under certain conditions, translations along the axes of this coordinate system correspond to the clinical translations, as well [1]. These translations within the joints are extremely small and difficult to measure, due to the lack of precision of the measuring instruments and the reference points being obscured [3], [4]. The magnitude of the displacement vector greatly relies on the choice of reference points [1]. This gives an even higher importance to the choice and positioning of the coordinate system.

The manipulation of the four link open kinematic chain requires rotational transformation matrices to be derived. The usual mathematical approach involves careful examination of relative motions and vector algebra. Using the Rodrigues' rotation formula makes this process much more efficient, as one only needs to determine the unit vectors which specify the rotational axes.

1.1 The knee joint and its relevant anatomical and kinematical properties

The knee joint is the most complex joint in the human body. It is comprised of bones, ligaments and tendons, all of which are responsible for proper functioning and stability of this joint [4], [5], [6].

The bones which make the knee joint are femur, tibia and patella (see Fig. 2). The distal part of the femur is in contact with the proximal part of the tibia, and during various movements of the knee they perform a sliding-rolling motion. This motion occurs on the condylar surfaces (medial and lateral condyles of femur and tibia). These surfaces are protected by the articular cartilage and due to the presence of synovial fluid, the friction during motion of a healthy knee is often neglected [4], [6].

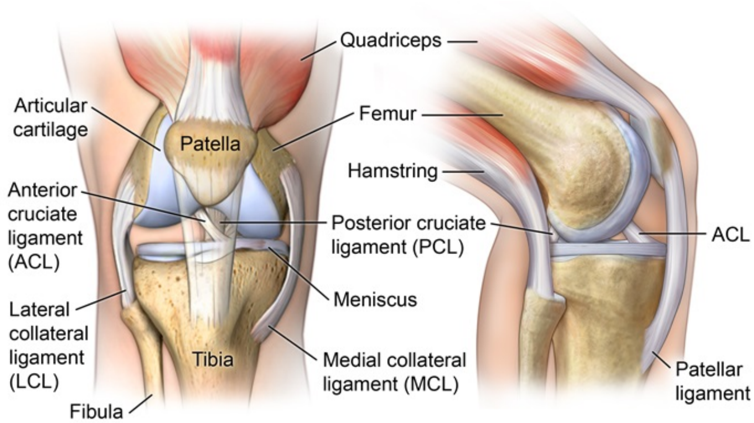

Fig. 1. On the left: frontal view of the right knee joint. On the right: side view of the right knee joint.[1]

The patella is a small bone embedded in the quadriceps tendon (see Fig. 1), which connects the quadriceps muscle group to the tibia. The function of the patella is to provide leverage, creating a much greater moment with the quadriceps force [4].

Apart from the quadriceps muscle group, there are other muscles which are connected to the knee joint, but they have been proven to act only sporadically, in certain circumstances, to provide stability [4]. These muscles are often neglected during engineering research. Also, medial and lateral ligaments are not taken into consideration for the same reasons.

During research, the coordinate system of the knee is often positioned in a way that provides the most convenient approximations. For example, when the total moment of forces acting in the knee is calculated, with a careful placement of the origin of the coordinate system, the influence

[1] Image taken from an open source: https://comportho.com/wp-content/uploads/2016/07/328031.jpg, accessed: 03.04.2021.

of cruciate ligaments can be neglected [4]. This is a widely accepted practice during evaluation of tibio-femoral and patello-femoral forces.

The reference points are located using pronounced markers on the bones (Fig. 2). The origin of the femoral coordinate system (XYZ, defined by unit vectors $\vec{I}, \vec{J}, \vec{K}$) is located in the middle of the line between medial and lateral condylar prominences [2]. The Z-axis points from the origin to the center of the femoral head. The Y-axis is normal to the frontal plane, which contains the Z-axis and the most posterior points of condyles equidistant to it [1].

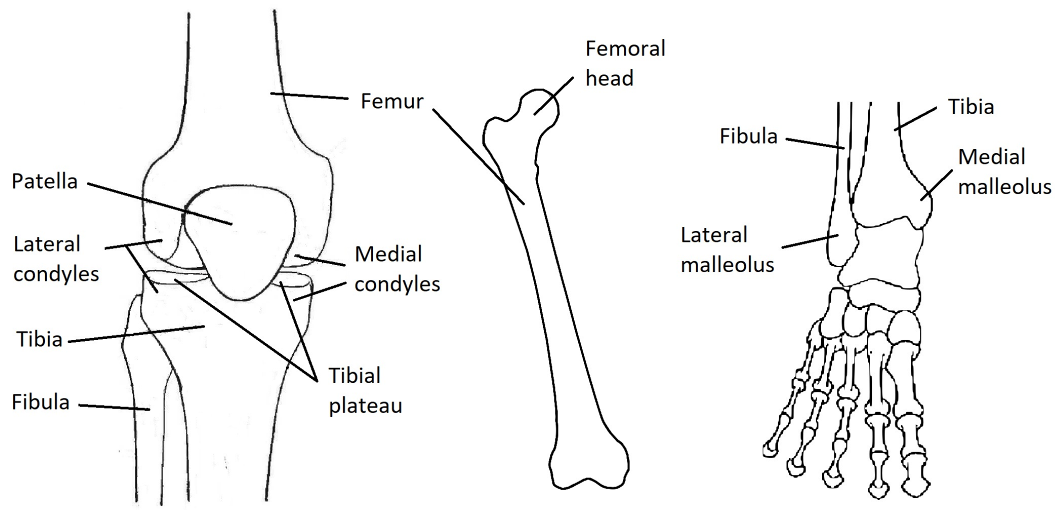

Fig. 2. On the left: anterior aspect of the knee joint. In the middle: femur. On the right: distal parts of tibia and fibula

The tibial coordinate system (xyz) is denoted by unit vectors $\vec{i}$, $\vec{j}$ and $\vec{k}$. The axis of tibial rotation (z-axis in Fig. 4) passes through the middle of the line connecting the medial and lateral condylar prominences, and through the middle of the line connecting the prominent points of medial and lateral malleolus. The y-axis is oriented anteriorly, passing through the middle of each tibial plateau [1].

The X-axis of the femoral coordinate system, as well as the x-axis of the tibial coordinate system, are defined by the right hand rule. These coordinate systems are shown in Fig. 3 and 4.

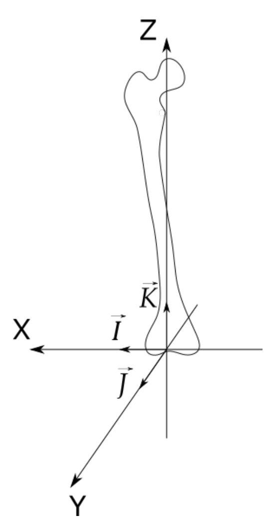

Fig. 3. Femur and its coordinate system (XYZ)

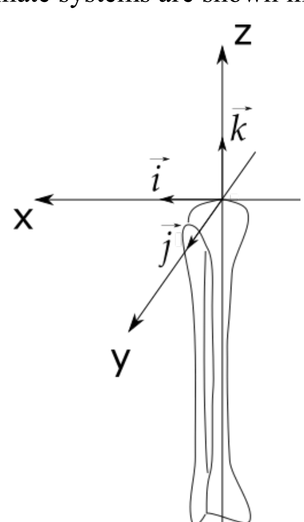

Fig. 4. Tibia and its coordinate system (xyz)

During movement, three rotations and three translations can be discerned. According to clinical nomenclature, the rotations are [1]:

1. Flexion-extension (around the X-axis of the femoral coordinate system),
2. External-Internal rotation of the tibia (around the z-axis) and
3. Abduction-adduction (around the common perpendicular to the X and z axes).

Clinical translations, which correspond to the translations along the axes of the joint coordinate system when the abduction-adduction angle is zero, are as follows:

4. Medial-lateral tibial thrust or shift (translation along the X-axis),
5. Anterior-poster. tibial drawer (along the common perpendicular to the X and z axes) and
6. Joint distraction-compression (translation along the z-axis).

In case when the abduction-adduction angle is not zero, coupling among these translations exists, and translations along the axes of the joint coordinate system do not correspond to clinical translations [1]. These translations are often neglected during research (and in this paper, as well). However, the knee joint is always considered to have six degrees of freedom.

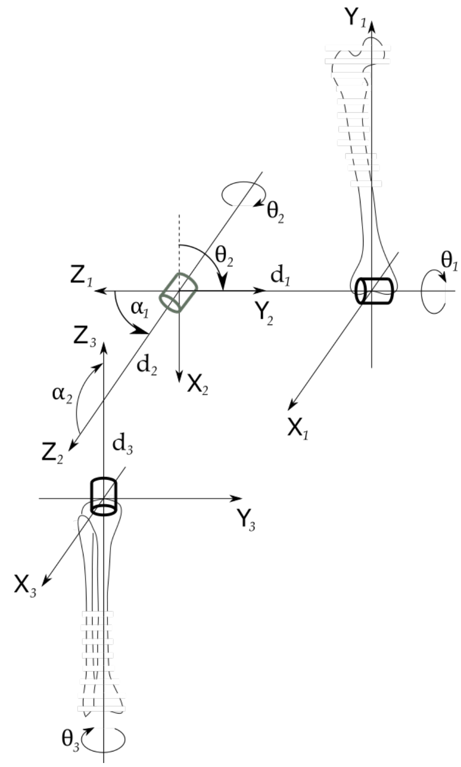

Fig. 5. Four link open kinematic chain, geometrically defined using Denavit-Hartenberg parameters

During research, the knee joint is often described as a four link open kinematic chain with three cylindrical joints. Two middle links in this mechanism are imaginary. This representation is convenient for mechanical analysis. Very often Denavit-Hartenberg parameters are applied (Fig. 5), as well, since they provide a very concise geometrical description of the system, in which the axes intersect ($l_1 = l_2 = l_3 = 0$), coordinate systems are rotated by 90° relative to each other ($\alpha_1 = \alpha_2 = 90^\circ$), θ_1, θ_2 and θ_3 denote the rotational angles, and d_1, d_2 and d_3 denote the translations. In reference position, when the abduction-adduction angle is zero, $\theta_2 = 90^\circ$.

2. The joint coordinate system

The joint coordinate system includes two fixed axes (Z and z, which belong to femoral and tibial coordinate systems), along with the so-called floating axis, which is a common perpendicular to the Z and z axes of these coordinate systems.

The direction of these axes is specified by non-orthogonal unit base vectors $\vec{e}_1$, $\vec{e}_2$ and $\vec{e}_3$.

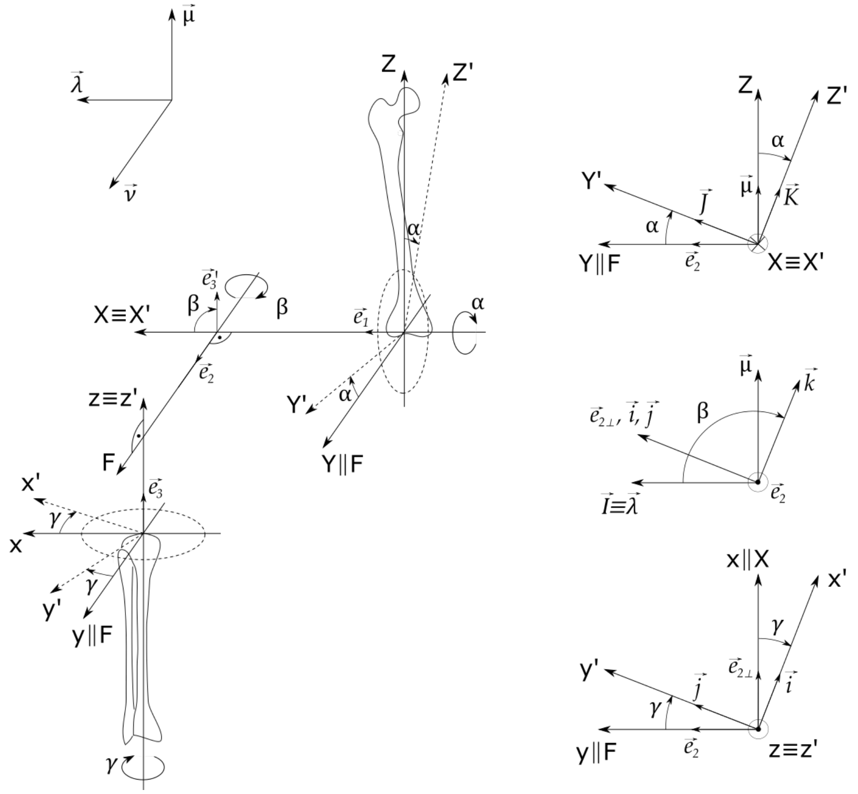

Fig. 6. On the left: a joint coordinate system applied to the knee joint. On the right: coordinate transformations shown in detail.

The femur is considered to rotate around the X-axis, while the tibia remains fixed. This rotation is denoted by the angle α and the X-axis is considered to be a fixed axis. The clinical positive direction of this rotation is the mathematically positive direction. Also, the tibia rotates around the z-axis, while the femur remains fixed. The angle of rotation is γ and the z-axis is also a fixed axis, but the clinically positive direction.is the mathematically negative direction.

Unlike these two rotations, the rotation which corresponds to abduction-adduction motion is considered to happen around the common perpendicular which is denoted as the floating axis (F). The angle of this rotation is β, and it is defined as the angle between X and z axes (between fixed axes). Mathematically, the sign of this clinical rotation is also negative. In reference position, when the abduction-adduction angle is zero, $\beta = 90^\circ$ (measured in reference to the vector $\vec{e}_3'$ which is parallel to $\vec{e}_3$).

These Euler angles correspond to the so-called jaw-pitch-roll angles (the gyroscopic system), which are often used in the field of aeronautics [3]. The final orientation of the leg should be independent of the sequence in which the clinical rotations are performed. The joint coordinate system reflects this characteristic, because it satisfies the two conditions defined by Roth, that the perpendicular distance and angle between adjacent screw axes are constant [1], [7].

If a Cartesian coordinate system is used, the final orientation of the body will depend upon the sequence in which the rotations around its axes are performed. When the sequence of rotations is changed, the axes of the joint coordinate system are changed as well. In this way, different sequence of rotations provides different motions in the joint coordinate system, but the order of rotations in the joint coordinate system itself is of no consequence [1]. The first rotation performed is the rotation around the fixed axis of the stationary body (femur in this case), the second rotation is the rotation around the floating axis, and the third rotation is performed around the second body fixed axis [1].

Thus, the final body orientation is independent of the sequence of rotations performed around the axes of the joint coordinate system.

3. Rotation transformation matrix

Often, during research, the femur is fixed, while the tibia is manipulated for purposes of measurements and evaluation. Thus, the need arises for a rotation transformation matrix, between femoral and tibial coordinate systems.

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \left[R \right] \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \tag{1}$$

To obtain the matrix $\left[R \right]$, dot products between unit base vectors of these coordinate systems have to be evaluated:

$$\left[R \right] = \begin{bmatrix} \vec{i} \cdot \vec{I} & \vec{j} \cdot \vec{I} & \vec{k} \cdot \vec{I} \\ \vec{i} \cdot \vec{J} & \vec{j} \cdot \vec{J} & \vec{k} \cdot \vec{J} \\ \vec{i} \cdot \vec{K} & \vec{j} \cdot \vec{K} & \vec{k} \cdot \vec{K} \end{bmatrix} \tag{2}$$

According to Fig. 6, and keeping in mind that the angle γ refers to the tibial rotation while the femur remains fixed, the angle α denotes the rotation of the femur around its fixed axis while the tibia remains fixed, as well as that the reference value of the angle β is 90°, the required dot products are as follows:

$$\vec{i} \cdot \vec{I} = \cos\left(\beta - 90^\circ\right)\cos\gamma = \sin\beta\cos\gamma \tag{3}$$

$$\vec{j} \cdot \vec{I} = \cos\left(\beta - 90^\circ\right)\sin\gamma = \sin\beta\sin\gamma \tag{4}$$

$$\vec{k} \cdot \vec{I} = \cos\beta \tag{5}$$

Here, an auxiliary inertial Cartesian coordinate system is used, denoted by unit vectors $\vec{\mu}$, $\vec{\lambda}$ and $\vec{\nu}$, which is parallel to the coordinate system XYZ in the reference configuration. Also, the unit vector perpendicular to $\vec{e}_2$, is denoted as $\vec{e}_{2\perp}$:

$$\begin{aligned} \vec{i} \cdot \vec{J} &= \left(-\sin\gamma\vec{e}_2 + \cos\gamma\vec{e}_{2\perp}\right) \cdot \vec{J} = -\sin\gamma\vec{e}_2 \cdot \vec{J} + \cos\gamma\vec{e}_{2\perp} \cdot \vec{J} \\ &= -\sin\gamma\cos\alpha + \cos\gamma\left(-\cos\beta\vec{\mu} + \sin\beta\vec{\lambda}\right) \cdot \left(\sin\alpha\vec{\mu} + \cos\alpha\vec{e}_2\right) \\ &= -\sin\gamma\cos\alpha - \cos\beta\cos\gamma\sin\alpha \end{aligned} \tag{6}$$

$$\begin{aligned}\vec{j}\cdot\vec{J} &= \left(\cos\gamma\vec{e}_2+\sin\gamma\vec{e}_{2\perp}\right)\cdot\vec{J}=\cos\gamma\vec{e}_2\cdot\vec{J}+\sin\gamma\vec{e}_{2\perp}\cdot\vec{J}\\ &=\cos\gamma\cos\alpha-\sin\gamma\sin\alpha\cos\beta\end{aligned} \tag{7}$$

$$\vec{k}\cdot\vec{J}=\left(\sin\beta\vec{\mu}+\cos\beta\vec{\lambda}\right)\cdot\left(\sin\alpha\vec{\mu}+\cos\alpha\vec{e}_2\right)=\sin\alpha\sin\beta \tag{8}$$

$$\begin{aligned}\vec{i}\cdot\vec{K} &= \left(-\sin\gamma\vec{e}_2+\cos\gamma\vec{e}_{2\perp}\right)\cdot\vec{K}=-\sin\gamma\vec{e}_2\cdot\vec{K}+\cos\gamma\vec{e}_{2\perp}\cdot\vec{K}\\ &=-\sin\gamma\vec{e}_2\cdot\left(\cos\alpha\vec{\mu}-\sin\alpha\vec{e}_2\right)+\cos\gamma\left(-\cos\beta\vec{\mu}+\sin\beta\vec{\lambda}\right)\cdot\left(\cos\alpha\vec{\mu}-\sin\alpha\vec{e}_2\right)\\ &=\sin\alpha\sin\gamma-\cos\beta\cos\gamma\cos\alpha\end{aligned} \tag{9}$$

$$\begin{aligned}\vec{j}\cdot\vec{K} &= \left(\cos\gamma\vec{e}_2+\sin\gamma\vec{e}_{2\perp}\right)\cdot\left(\cos\alpha\vec{\mu}-\sin\alpha\vec{e}_2\right)=-\sin\alpha\cos\gamma+\sin\gamma\cos\alpha\vec{e}_{2\perp}\cdot\vec{\mu}\\ &=-\sin\alpha\cos\gamma+\sin\gamma\cos\alpha\left(-\cos\beta\vec{\mu}+\sin\beta\vec{\lambda}\right)\cdot\vec{\mu}=-\sin\alpha\cos\gamma-\cos\alpha\cos\beta\sin\gamma\end{aligned} \tag{10}$$

$$\vec{k}\cdot\vec{K}=\left(\sin\beta\vec{\mu}+\cos\beta\vec{\lambda}\right)\cdot\left(\cos\alpha\vec{\mu}-\sin\alpha\vec{e}_2\right)=\cos\alpha\sin\beta \tag{11}$$

Finally, the rotation transformation matrix is:

$$\left[\mathrm{R}\right]=\begin{bmatrix}\sin\beta\cos\gamma & \sin\beta\sin\gamma & \cos\beta\\ -\sin\gamma\cos\alpha-\cos\beta\cos\gamma\sin\alpha & \cos\alpha\cos\gamma-\sin\alpha\cos\beta\sin\gamma & \sin\alpha\sin\beta\\ \sin\alpha\sin\gamma-\cos\beta\cos\gamma\cos\alpha & -\sin\alpha\cos\gamma-\cos\alpha\cos\beta\sin\gamma & \cos\alpha\sin\beta\end{bmatrix} \tag{12}$$

The transformation matrix can be derived using vector algebra only, without any need for visual representation. For example, the dot product $\vec{i}\cdot\vec{J}$ can be transformed using the properties of orthogonal unit base vectors, as is shown in (13), where [1]: $\left(\vec{i}\cdot\vec{e}_2\right)\cdot\vec{e}_2=0, \vec{e}_2\times\vec{k}=\vec{i}$ and $\left[\vec{i}\cdot\left(\vec{e}_2\times\vec{k}\right)\right]\left(\vec{e}_2\times\vec{k}\right)=\vec{i}$. Further on, the properties of the coordinate system are used to determine the values of dot products: $\vec{i}\cdot\vec{e}_2=-\sin\gamma$, $\vec{e}_2\cdot\vec{J}=\cos\alpha$, $\vec{e}_2\cdot\vec{j}=\cos\gamma$ and $\vec{k}\cdot\vec{I}=\cos\beta$. It is obvious that the final result, and the transformation matrix itself, does not depend upon the order in which the rotations are performed [1].

$$\begin{aligned}\vec{i}\cdot\vec{J} &= \left\{\left(\vec{i}\cdot\vec{e}_2\right)\cdot\vec{e}_2+\left[\vec{i}\cdot\left(\vec{e}_2\times\vec{k}\right)\right]\left(\vec{e}_2\times\vec{k}\right)\right\}\cdot\vec{J}\\ &=\left(\vec{i}\cdot\vec{e}_2\right)\left(\vec{e}_2\cdot\vec{J}\right)+\left[\vec{i}\cdot\left(\vec{e}_2\times\vec{k}\right)\right]\left[\left(\vec{e}_2\times\vec{k}\right)\cdot\vec{J}\right]\\ &=-\sin\gamma\cos\alpha+\left[\vec{e}_2\cdot\left(\vec{k}\times\vec{i}\right)\right]\left[\vec{k}\cdot\left(\vec{J}\times\vec{e}_2\right)\right]\\ &=-\sin\gamma\cos\alpha+\left(\vec{e}_2\cdot\vec{j}\right)\left[\vec{k}\cdot\left(-\vec{I}\sin\alpha\right)\right]\\ &=-\sin\gamma\cos\alpha-\cos\gamma\sin\alpha\vec{k}\cdot\vec{I}=-\sin\gamma\cos\alpha-\cos\beta\cos\gamma\sin\alpha\end{aligned} \tag{13}$$

4. Rodrigues method of obtaining the rotation transformation matrix

In this section the Rodrigues' rotational formula will be used to obtain the rotation transformation matrix in a more efficient and elegant way. The formula is:

$$\left[\mathrm{A}\right]=\left[\mathrm{I}\right]+\xi\left\{\left(1-\cos\theta\right)\left[e^d\right]^2+\sin\theta\left[e^d\right]\right\} \tag{14}$$

Where $\left[\mathrm{I}\right]$ is the identity matrix, θ is the angle of rotation and $\left[e^{d}\right]$ is the dual object of the unit vector of the rotational axis, defined as:

$$\vec{e}=\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} \Rightarrow \left[e^{d}\right]=\begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \tag{15}$$

The parameter $\bar{\xi}$ denotes the occurrence of rotation around the relevant axis. If $\bar{\xi}=1$, rotation occurs, and if $\bar{\xi}=0$, rotation does not occur (only translation is possible along this axis). Here, only the rotations around each axis of the joint coordinate system are considered. Thus, the final formula that will be used is:

$$\left[\mathrm{A}\right]=\left[\mathrm{I}\right]+\left(1-\cos\theta\right)\left[e^{d}\right]^{2}+\sin\theta\left[e^{d}\right] \tag{16}$$

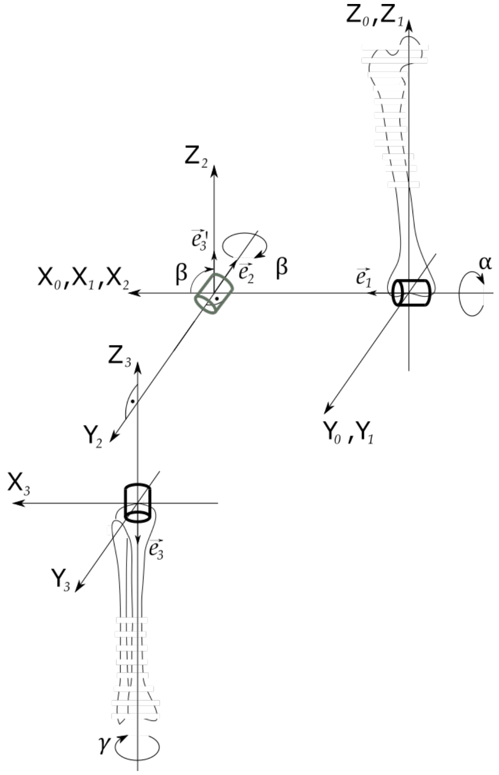

Fig. 6. The knee joint represented as a four link open kinematic chain.

The knee joint is represented as a four link open kinematic chain with three cylindrical joints (Fig. 6). Each cylindrical joint in the system has its own Cartesian coordinate system ($X_1Y_1Z_1$, $X_2Y_2Z_2$, $X_3Y_3Z_3$). The Cartesian coordinate system $X_0Y_0Z_0$ represents the inertial frame of reference. In reference position, all of these Cartesian coordinate systems are parallel to each other. The unit vectors which correspond to the axes of rotation are denoted in the same way in which it has been done previously (see Fig. 6). These are the unit vectors $\vec{e}_1$, $\vec{e}_2$ and $\vec{e}_3$. However, they are also in accordance with the positive mathematical direction of rotation. As it has been stated above, the direction of rotation is the clinical positive direction, which is not always the mathematical positive, relative to the relevant axis.

For the purposes of using the Rodrigues' formula, these vectors have to be defined in the coordinate system of the axis they refer to. For example: $\vec{e}_1^{(1)}$ denotes that the unit vector $\vec{e}_1$ in the coordinate system $X_1Y_1Z_1$. Thus, they are defined as follows:

$$\vec{e}_1^{(1)} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}, \vec{e}_2^{(2)} = \begin{Bmatrix} 0 \\ -1 \\ 0 \end{Bmatrix}, \vec{e}_3^{(3)} = \begin{Bmatrix} 0 \\ 0 \\ -1 \end{Bmatrix} \tag{17}$$

Their dual objects are then:

$$\left[e_1^{d(1)}\right] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \left[e_2^{d(2)}\right] = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \left[e_3^{d(3)}\right] = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{18}$$

The Rodrigues' formula is now applied for each rotation:

$$\left[A_{0-1}\right] = [I] + (1-\cos\alpha)\left[e_1^{d(1)}\right]^2 + \sin\alpha\left[e_1^{d(1)}\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \tag{19}$$

$$\left[A_{0-2}\right] = [I] + (1-\cos\beta)\left[e_2^{d(2)}\right]^2 + \sin\beta\left[e_2^{d(2)}\right] = \begin{bmatrix} \sin\beta & 0 & \cos\beta \\ 0 & 1 & 0 \\ -\cos\beta & 0 & \sin\beta \end{bmatrix} \tag{20}$$

$$\left[A_{2-3}\right] = [I] + (1-\cos\gamma)\left[e_3^{d(3)}\right]^2 + \sin\gamma\left[e_3^{d(3)}\right] = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{21}$$

As femur and tibia rotate independently, the final rotation transformation matrix referring from the tibial coordinate system $X_3Y_3Z_3$ to the femoral coordinate system $X_1Y_1Z_1$ is:

$$\begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} = \left[A_{0-1}\right]\begin{Bmatrix} X_1 \\ Y_1 \\ Z_1 \end{Bmatrix} \Rightarrow \begin{Bmatrix} X_1 \\ Y_1 \\ Z_1 \end{Bmatrix} = \left[A_{0-1}\right]^{-1}\begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} \tag{22}$$

$$\begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} = \left[A_{0-2}\right]\begin{Bmatrix} X_2 \\ Y_2 \\ Z_2 \end{Bmatrix}, \begin{Bmatrix} X_2 \\ Y_2 \\ Z_2 \end{Bmatrix} = \left[A_{2-3}\right]\begin{Bmatrix} X_3 \\ Y_3 \\ Z_3 \end{Bmatrix} \Rightarrow \begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} = \left[A_{0-2}\right]\left[A_{2-3}\right]\begin{Bmatrix} X_3 \\ Y_3 \\ Z_3 \end{Bmatrix} \tag{23}$$

$$\begin{Bmatrix} X_1 \\ Y_1 \\ Z_1 \end{Bmatrix} = \left[A_{0-1}\right]^{-1}\left[A_{0-2}\right]\left[A_{2-3}\right]\begin{Bmatrix} X_3 \\ Y_3 \\ Z_3 \end{Bmatrix} = \left[A_{1-3}\right]\begin{Bmatrix} X_3 \\ Y_3 \\ Z_3 \end{Bmatrix} \tag{24}$$

$$\left[A_{1-3}\right] = \begin{bmatrix} \sin\beta\cos\gamma & \sin\beta\sin\gamma & \cos\beta \\ -\sin\gamma\cos\alpha - \cos\beta\cos\gamma\sin\alpha & \cos\alpha\cos\gamma - \sin\alpha\cos\beta\sin\gamma & \sin\alpha\sin\beta \\ \sin\alpha\sin\gamma - \cos\beta\cos\gamma\cos\alpha & -\sin\alpha\cos\gamma - \cos\alpha\cos\beta\sin\gamma & \cos\alpha\sin\beta \end{bmatrix} \tag{25}$$

which is the same rotation transformation matrix as shown in (12).

5. Conclusions

Due to the described anatomical and mechanical complexity of the knee joint, an adequate coordinate system and the choice of reference points are of crucial importance.

The joint coordinate system allows for the knee joint to be analyzed in terms of standard clinical translations and rotations. This makes the research of this joint more intuitive and eases the communication between clinicians and engineers.

The Euler angles, which correspond to the clinical rotations around the axes of the joint coordinate system, are the jaw-pitch-roll angles commonly used in aeronautics. It has been shown in existing literature that the final body orientation is independent of the sequence of these rotations around the axes of the joint coordinate system.

It is often convenient to represent the knee joint as a four link open kinematic chain, with two imaginary links. This system is easily manipulated, when provided with the rotation transformation matrix which refers the tibial coordinate system to the femoral coordinate system. While the four link mechanism is usually geometrically described by Denavit-Hartenberg parameters, it is more convenient to tie a Cartesian coordinate system to each cylindrical joint, all parallel to each other in reference position. Then, a straightforward way to obtain the rotation transformation matrix has been shown, using the Rodrigues' transformation formula. Obtaining the rotation transformation matrix is crucial, as kinematics and dynamics of the knee joint can not be studied without it. Furthermore, this method is especially useful for implementation in programing codes, due to the simplicity of the algorithm.

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