International Congress Motor Vehicles & Motors 2012

Kragujevac, October 3rd-5th, 2012

MVM2012-024

Predrag Mrđa[1]
Nenad Miljić[2]
Marko Kitanović[3]
Slobodan Popović[4]
Miroljub Tomić[5]

MODEL BASED APPROACH IN YAMAHA R6 FORMULA STUDENT ENGINE CONTROL PARAMETERS OPTIMISATION

ABSTRACT: An involvement of the Internal Combustion Engines Department in the support activities to the University of Belgrade Formula Student team provided an opportunity to conduct some tests and research in the field of the engine control parameters optimization on a specific racing class engine –Yamaha YZF-R6s. The main goal of this research was getting optimal look-up tables for ignition timing and fuelling in order to obtain maximal performance in terms of effective torque and power of the air-flow restricted version of the engine, used in Formula Student competition vehicle. In order to improve accuracy and provide more detailed calibration data a model based approach is suggested with a detailed 1D thermodynamic model as a basis for additional testing point simulation. The method developed, provides a mean for model calibration to be based solely on data available from the limited number of measured engine effective parameters. This approach shows that a lack of test point data, due to test bench equipment limitations, can be successfully overcomed in engine optimisation process by using model based approach. Furthermore, the method used, demonstrated a capability of identifying combustion process parameters without in-cylinder pressure measurements and analysis.

KEYWORDS: internal combustion engine, optimisation, spark advance map, combustion parameters identification

INTRODUCTION

As part of the support activities for the newly formed Formula Student (FS) team, of the University of Belgrade, extensive engine testing was carried out at the Internal Combustion Engine Department testing facilities. The main goal of the project was to obtain as good as possible fuelling and advance look-up tables based on real-time readings from test-bench acquisition systems for the forthcoming competition season. All measured data was sorted for later analysis in an effort to match the engine simulation model and the tested engine on particular operating regimes. These data were used for validation of a mathematical model of the engine created within Ricardo WAVE engine simulation software. Reliable mathematical model allows numerous and low cost investigation on relations between input and output parameters of the model. Because of complexity and high dynamics of processes taking place in the IC engine, particular attention needs to be devoted to correlations between simulation input parameters. This paper a methodology for deriving correlations between Wiebe heat

[1] *Predrag Mrđa, Research Assistant, Internal Combustion Engines Department, Faculty of Mechanical Engineering, University of Belgrade, Kraljice Marije 16, 11120 Belgrade 35, Serbia, pmrdja@mas.bg.ac.rs*
[2] *Nenad Miljić, Research and Teaching Assistant, Internal Combustion Engines Department, nmiljic@mas.bg.ac.rs*
[3] *Marko Kitanović, Research Assistant, Internal Combustion Engines Department, mkitanovic@mas.bg.ac.rs*
[4] *Slobodan Popović, Research and Teaching Assistant, Internal Combustion Engines Department, spopovic@mas.bg.ac.rs*
[5] *Miroljub Tomić, Professor, Internal Combustion Engines Department, mtomic@mas.bg.ac.rs*

release form factor and angular duration of combustion for full load and engine speed of 9000 RPM based on measurements of engine effective parameters.

ENGINE TESTING

The testing was performed on a stock 599 cc Yamaha R6, naturally aspirated engine with a 20mm diameter restrictor intake pipe section which is in accordance with the FS competition rules. Due to the rotational speed limit of the hydraulic dynamometer used, the engine was coupled to the dynamometer through the transmission gear which is the part of the engine assembly. That fact introduced some uncertainty level in determining the engine brake effective efficiency, since the mechanical efficiency of the transmission gear was unknown. Moreover, direct coupling of the engine and dynamometer is fairly limited because of specific power take off design of the engine, which is located not at the crankshaft end, but in the middle of it.

Table 1 *Engine and ECU specifications*

Engine	YAMAHA R6, four stroke, spark ignition, inline four cylinder, 20mm restrictor
Ignition order	1-2-4-3
Bore [mm]	67
Stroke [mm]	42.5
Compression ratio [-]	12.8
Valves per cylinder	4
Intake Valve Opening	39° CAD BTDC
Intake Valve Closing	65 CAD ABDC
Exhaust Valve Opening	64 CAD BBDC
Exhaust Valve Closing	24 CAD ATDC
Engine Control Unit (ECU)	DTA S60 PRO
ECU interface software	DTASwin 63.1
Injection	Sequential
Ignition	Sequential
Lambda sensor	Wide band
Communication	Serial, CAN

Engine testing points and guidance in engine mapping

Engine operating regimes are represented as a combination of input parameters which have an influence on the engine working process and consequently on the output parameters. There are numerous input parameters being the control parameters at the same time: air/fuel ratio, spark advance, intake and exhaust valve timing, EGR valve position, variable length of the intake runners etc. They have to be defined for different operating regimes specified by, mainly, the throttle position and the engine speed. All these input parameters are mutually independent, and all of them have a direct influence on the actual engine torque, fuel consumption, exhaust gas temperature, emissions of pollutants and other parameters. Certain combinations of input parameters are not realistically feasible during engine testing at particular operating regimes and operation on such regimes could lead to engine damage.

Engine mapping represents the process of getting data for injector opening time and spark advance look-up tables that give predefined value of air to fuel equivalence ratio (AFR) and maximal output power for the steady state operating regime, which is defined by the percentage of throttle opening and engine speed. For this purpose, an engine control unit with calibration interface, which enables rapid manual change of control parameters, is required.

Setting and maintaining the desired engine operating point requires skilled engine test-bench operator especially if non-automatic controlled hydraulic brake is used. Each modification of spark advance for a given throttle position, engine speed, and AFR results in a change of engine output torque which further leads to changes of angular speed for constant brake torque. In order to obtain maximum engine torque, engine speed unsteadiness needs to be compensated for, manually. When the steady state engine operating point is reached, look-up tables can be updated. It is recommended that moving through engine operating points goes from minimal to maximal throttle openings, and for every throttle position from minimal engine speed to its maximum. There is always a risk of creating conditions for engine knock on high loads and low engine speed because of low intake mixture velocity and low level of turbulence in the combustion chamber, for relatively high compression ratio values. Generally, for this type of engine, knock can be experienced at almost each operating point if the spark advance is too high which results in a noticeable loss of the engine torque [1], [2].

Initial fuelling and spark advance maps can be designed by means of virtual testing through the thermodynamically based engine working process simulation. With the initial guess of model input parameters, the simulation results can provide maps which can be successfully used for starting the engine and even operation on the most of the engine regimes. It is desirable to create an initial spark advance map with intentionally retarded values (about 10° to 15° CA when compared to the optimal ones) which provides delayed combustion. The described preparation of the spark advance maps provide the simple and direct way toward the optimal spark advance during engine calibration process. Since the initial spark advance is retarded, this control parameter sweep is done only in the advancement direction until the maximum brake torque (MBT) or knock conditions are reached.

Combinations of throttle position and engine speed on which injector timing and spark advance optimization was performed are shown in Figure 1. Beside, examined operating regimes, two zones of unreachable operating regimes are noticeable. Zone 1 represents high throttle position values and low engine speed. Knock cannot be avoided even with reduction of spark advance angle. Small throttle position values and low load regimes are positioned on the border of the zone 2, indicating the area of almost zero value effective mean pressure.

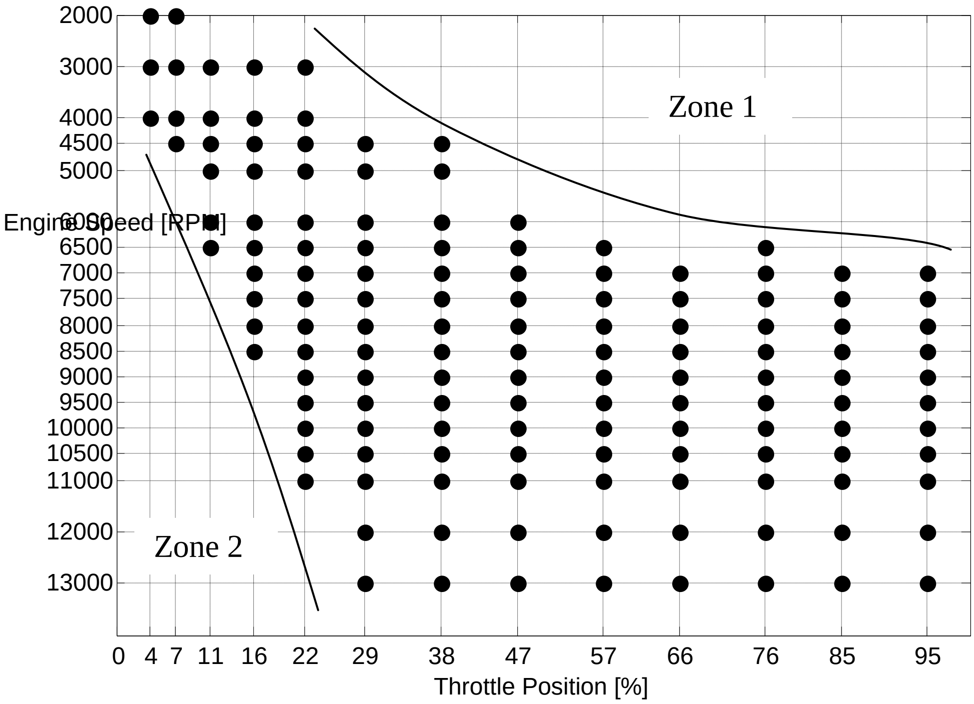

Figure 1 *In-laboratory optimized operation regimes. Grid steps and figure orientation are copied from the ECU calibration software [3]*

Measurement setup and data acquisition

The data acquisition, during engine testing, was accomplished through several data channels. Serial communication between the ECU and PC allows real-time adjustment of control parameters (spark timing, injector timing and others) and monitoring of stock engine sensor readings (engine speed, throttle position, lambda sensor reading, intake air, coolant and oil temperature, intake air and oil pressure, manifold absolute pressure, battery voltage and others). At the same time, established high speed CAN communication enabled the parallel dataflow of the all available ECU information to a dedicated acquisition computer based on the National Instruments (NI) PXI system.

The engine test bench was equipped with additional sensors providing monitoring of the parameters, like effective brake torque, transmission output shaft angular speed, exhaust gas temperature, intake air, coolant and oil pressure and temperatures and A/F ratio. The graphical monitoring application, in-house developed in the NI Labview environment, provided additional real-time engine test bench monitoring and data logging. Along with the data acquisition, multifunctional acquisition devices of the PXI system were simultaneously used for controlling and supervising of engine test bench subsystems, like the intake air conditioning system, the engine throttle positioning, oil cooling and fuel supply system. The concept of the engine test bench and instrumentation used is shown in Figure 2.

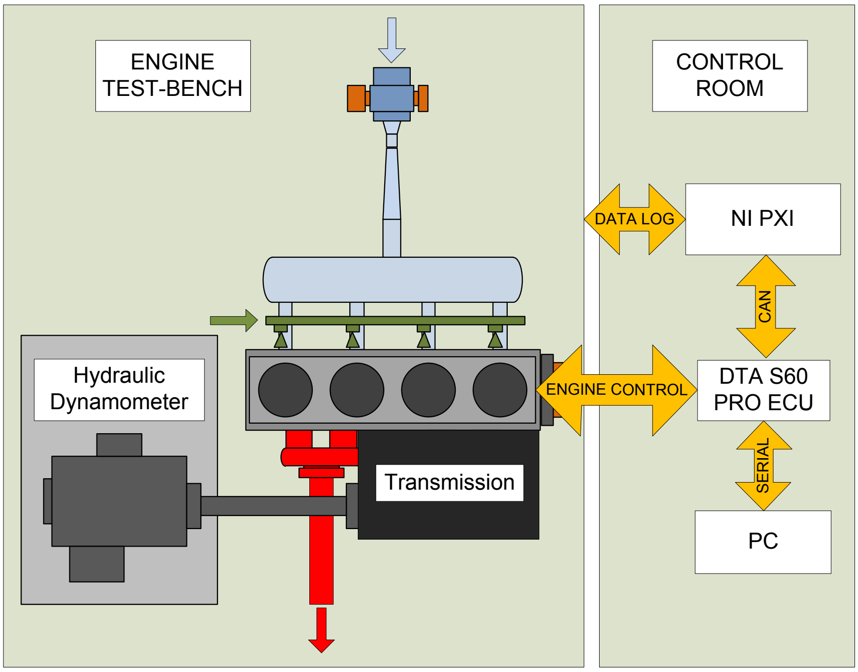

***Figure 2** The engine test bench and instrumentation used during calibration process*

Obtained look-up tables

Look-up tables shown in Figure 3 and Figure 4 have been obtained directly by engine mapping on the test bench. This is an expensive and time consuming process which can be even impossible to accomplish when the number of engine control variables becomes high, which is often the case with modern engines with high degree of freedom in controlling parameters. The model based calibration process is, therefore, the fundamental approach in solving this problem. Although the focus of the calibration process, of this particular engine, was on only a few control parameters, the model based approach is not of less importance in this case. The limitation of the manualy controlled dynamometer and its open-loop nature of engine speed control, makes the map fine tuning a time consuming process which can be conducted on a limited number of engine operating points. Therefore, the model based approach can provide significant and valuable support in the calibration of this particular engine.

Presented results are valid for steady state regimes and additional correction for transients needs to be added relying on experience and specific methodologies.

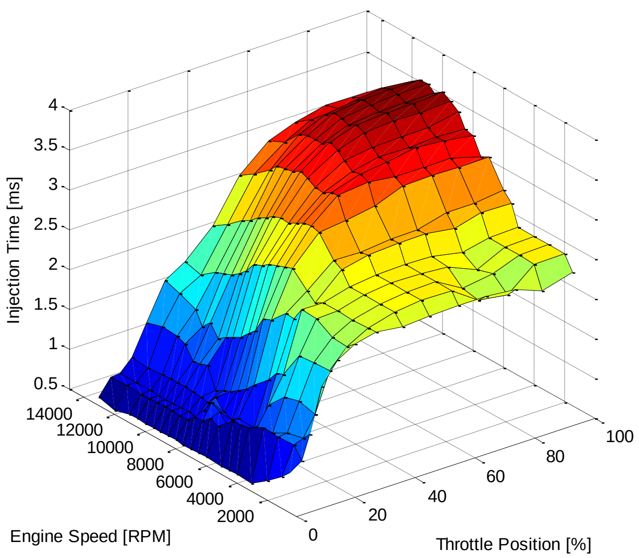

***Figure 3** Injection time look-up table obtained by engine test-bench optimization*

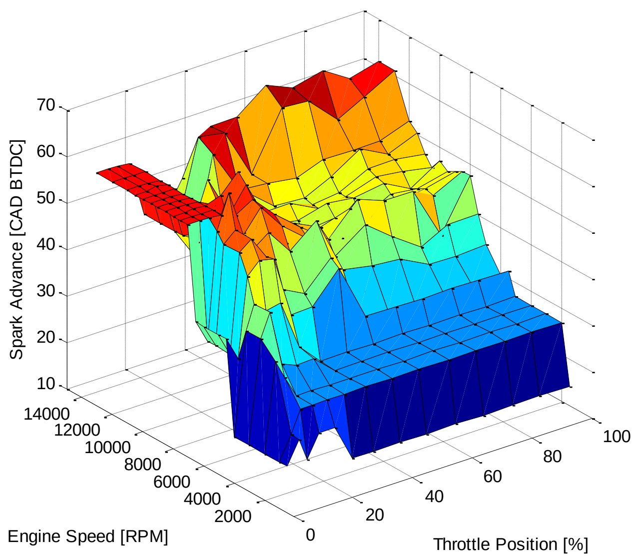

***Figure 4** Spark advance timing look-up table obtained by engine test-bench optimization*

SIMULATION MODEL

Simulation of the internal combustion engine working cycle was performed using Ricardo WAVE simulation software package. The quality and the usability of the simulation results are vastly dependent on how well the thermodynamic model of the engine is defined. A well-defined model can give results which are able to shed light on some important relations between the engine model parameters and control and effective parameters relationships. An absolute accuracy of the, estimated engine output parameters is always questionable when the model definition is not a result of extensive experimental engine research.

In order to provide results, that can be used as an estimation of engine output parameters, the model should be fed with the initial values of numerous parameters. These parameters define combustion process nature, amount of air and fuel taken, air to fuel ratio, pneumatic losses in the intake and exhaust engine sections, heat transfer process, heat and mass losses, mechanical friction loses and many others.

The most reliable method of model input parameter identification is based on experimental data analysis which should lead to modelled and measured engine parameters matching. The identification process can be accomplished with trial and error method, or by more sophisticated optimisation based techniques. The crucial set of model input parameters can be easily identified by measured in-cylinder pressure analysis [4]. Unfortunately, the tested engine was not equipped with the in-cylinder pressure indicating system, and lack of this data significantly narrowed the path of model parameter identification to only available data – measured effective parameters. Having in mind the difficulties associated with model parameter estimation, with no in-cylinder data available, the main idea was to establish some method that can be helpful in combustion model parameter estimation and plausible even with this reduced experimental data availability.

The simulation model used within the WAVE simulation software is a 1D model which includes the sub-models of various phenomena occurring in real engine working cycle [5]. The model used, represents the flow through the engine as a flow network of quasi-one dimensional compressible flow equations. The flow network is meshed into the individual volumes which are connected via energy, momentum and mass conservation equations and boundary conditions. The combustion model used is based on the well-known Wiebe heat release function within single-zone combustion chamber. The heat transfer sub-model used is based on the Woschni model [6] with simplified approach assuming a uniform heat flow coefficient on all cylinder wall, cylinder head and piston surfaces. The same heat transfer model setup is applied to all cylinders.

Estimation of the mechanical friction losses is achieved by means of a modified Chen-Flynn model [7] which determines the overall friction losses by calculation of terms dependent on peak in-cylinder pressure, mean piston velocity and term which takes into account the driving torque of the auxiliaries. All input parameters of the model were set in accordance with the recommendations for this particular type of the engine. The overall capabilities of the WAVE simulation software provided a detailed simulation of the engine filling / emptying and combustion processes. The Yamaha R6 base model, represented in WAVE simulation environment, is shown in Figure 5.

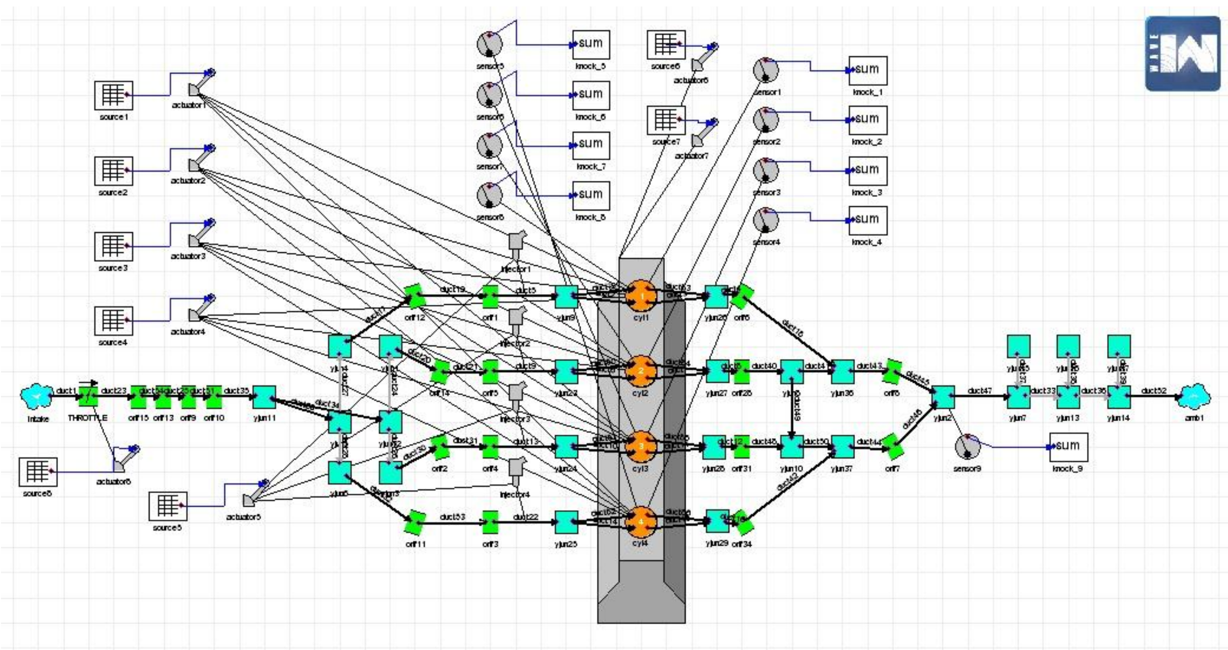

Figure 5 *Ricardo WAVE simulation model*

MODEL CALIBRATION AND VALIDATION

Mass airflow matching

The simulation model was made taking into account all available data of the intake and exhaust manifold geometry, combustion chamber dimensions and crank slide and valve-train kinematics. The engine mass airflow is affected by various factors influencing the volumetric efficiency like the intake runner length, the intake manifold plenum volume, in-pipe friction losses and pipe wall temperatures. By varying the model parameters defining the effective flow area and friction losses of a particular intake section, it is possible to identify the model parameters which can lead to the model being capable of matching the real engine flow within acceptable accuracy. A relative difference of simulated and measured airflow is used as an evaluation criteria in the intake system parameter identification process. It can be calculated as:

$$G_{a_{REL}} = \frac{G_{a_{SIM}} - G_{a_{MEAS}}}{G_{a_{SIM}}} \cdot 100 \qquad (1)$$

where $G_{a_{MEAS}}$ represents the measured mass airflow and $G_{a_{SIM}}$ a simulated mass airflow for different input parameters datasets. Figure 6 shows how a variation of the engine model intake section parameters influence the difference between the simulated and measured air flow (defined by equation (1)) on the wide open throttle (WOT) regimes and a 8000 to 11000 RPM engine speed range. As an acceptable overall $G_{a_{REL}}$ limit, a value of 1% is chosen (solid line on Figure 6).

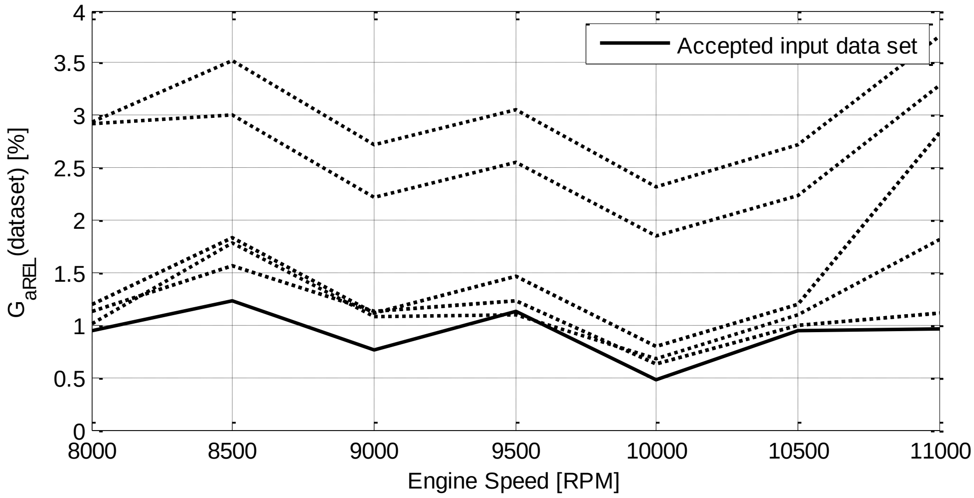

Figure 6 *Relative differences between simulated and measured mass airflow (WOT, model parameter identification)*

According to Formula Student demands and race track specification, the engine operating range chosen, is the most critical one from the aspect of overall vehicle performance. Therefore the research focus is put on engine operating points covering the higher speed range and loads.

The engine experimental results for the operating points described shown in Figure 6, are shown in Table 2.

Table 2 *Experimental results for particular regimes*

Engine Speed	Throttle Position	Spark Advance	Excess air ratio	Brake Torque	Brake Power	Mass Airflow
n [RPM]	φ [%]	SOC [CAD BTDC]	λ [-]	M_b [Nm]	P_b [kW]	G_a [kg/h]
8000	100	49,4	0,9	42,0	35,2	144,8
8450	100	48,5	0,9	43,5	38,5	151,7
9000	100	50,0	0,9	44,0	41,5	163,7
9500	100	47,6	0,9	44,0	43,8	170,0
10000	100	49,5	0,9	43,5	45,6	182,2
10500	100	53,2	0,9	42,6	46,8	189,7
11000	100	55,0	0,9	40,6	46,8	195,0

Combustion model

The Wiebe function is widely used, in thermodynamic calculations, to describe the cumulative rate of the in-cylinder fuel mass burned. The cumulative mass fraction burned x_b as a function of crank angle θ is given by the following equation:

$$x_b(\theta) = 1 - exp\left[-a \cdot \left(\frac{\theta - SOC}{BDUR}\right)^{m+1}\right] \tag{2}$$

where the SOC is the start of combustion angle, $BDUR$ is the total combustion duration and parameter m is called the combustion mode parameter and defines the shape of the combustion profile. The value of constant a in the equation **(2**) represents the chosen definition of the end of combustion completeness level. In the case of $x_{b,EOC} = 99.9$ % MFB level at the end of combustion, a has the value:

$$a = -\ln\left(1 - x_{b_{EOC}}\right) = -\ln(0.001) = 6.9 \tag{3}$$

The parameters of the Wiebe function, mainly $BDUR$ and m, are usually determined by means of measured in-cylinder pressure analysis through common methods like Rassweiler & Withrow [8] or Hohenberg based [9] (or even more sophisticated methods like [10]). On the other hand, by sweeping these unknown parameters within a simulation model, it is possible to get a large number of simulated testing points. If the simulated and measured testing points are close enough, this could indicate a matching between the actual and estimated combustion parameters. Hopefully, this idea would lead to combustion parameter estimation within the whole area of engine operating points.

The simulation input parameters of a WAVE model, defining combustion profile by Wiebe function, is possible to set in two different ways:

- by setting the angular position of 50% mass fraction burned $CA50$, combustion duration $BDUR$ (from 10% to 90% fuel mass burned) and Wiebe parameter m
- or by setting the combustion start angle SOC, combustion duration $BDUR_{10-90}$ (from 10% to 90% fuel mass burned) and Wiebe parameter m.

The only available parameter, defining combustion, in this experiment, was the spark advance angle. In fact, this parameter coincides with the SOC, timing of combustion start that is defined so as to indicate exactly the 0% of heat released due to combustion. This further leads to a conclusion that the second set of parameters (SOC, $BDUR_{10-90}$ and m) should be used in engine simulation, since one of these parameters is explicitly available from the test bench experiment.

The combustion duration between 10% and 90% of heat released, is correlated with the overall (99.9%) combustion duration angle $BDUR$ via equations (2) and (3). Combination of these equations leads to $BDUR_{10-90}$ estimation formula:

$$BDUR_{10-90} = BDUR \cdot \left(\left(\frac{\ln(1-0.9)}{a}\right)^{\frac{1}{m+1}} - \left(\frac{\ln(1-0.1)}{a}\right)^{\frac{1}{m+1}}\right) \tag{4}$$

Relying on "known" SOC information, assumed total combustion duration $BDUR$ and Wiebe parameter m, estimation of the angular location of 50% mass fraction burned (ATDC) can be done by the following equation:

$$CA50 = SOC + BDUR \cdot \left(\frac{\ln(1-0.5)}{a}\right)^{\frac{1}{m+1}} \tag{5}$$

Simulation initialization and results analysis

In order to test the suggested concept, a test point at WOT and 9000 RPM is chosen (see Table 2). Full-factorial sweep of $BDUR_{10-90}$ (20º…50º CA) and m (2…8) parameters was applied on mass airflow matched simulation model. A variation of the combustion duration $BDUR_{10-90}$ and Wiebe function shape parameter m has an influence on in-cylinder heat release process, and therefore, on simulation output results such as engine effective power. Summed results of combustion parameter sweep are presented in Figure 7, which shows how the brake indicated power depends on it. An analysis of the Figure 7 leads to a conclusion that the power isolines can be approximated by a second order polynomial.

Taking into account the transmission gear efficiency level, η_{trans} the brake effective power isolines can be created. The measured brake power (WOT @ 9000 RPM), was $Pe_{meas} = 41.5\,kW$ and depending on the assumed η_{trans} efficiency different simulated brake effective isolines can be created which are also shown on the Figure 7.

It is noticeable that a 3D surface, presented in Figure 7, has a ridge which gathers the points of maximum power and these downhill gradients, in orthogonal direction, are almost the same. The same power isolines can be found from either side of the "ridge", but only one set of them is thermodynamically correct and can be achievable. Small values of $BDUR_{10-90}$ and m combustion input parameters lead to high in-cylinder maximum pressures (Figure 8) which are far from the actual and real ones which imply that the correct simulated test point has to be in the region of higher m. Furthermore, relatively high values of spark advance, for this particular engine, indicate indicate an asymetric heat release w.r.t. to TDC, and this can be represented with higher m values. Besides this reasoning, a more correct conclusion can be deduced from the combustion indicators analysis of the simulated testing points.

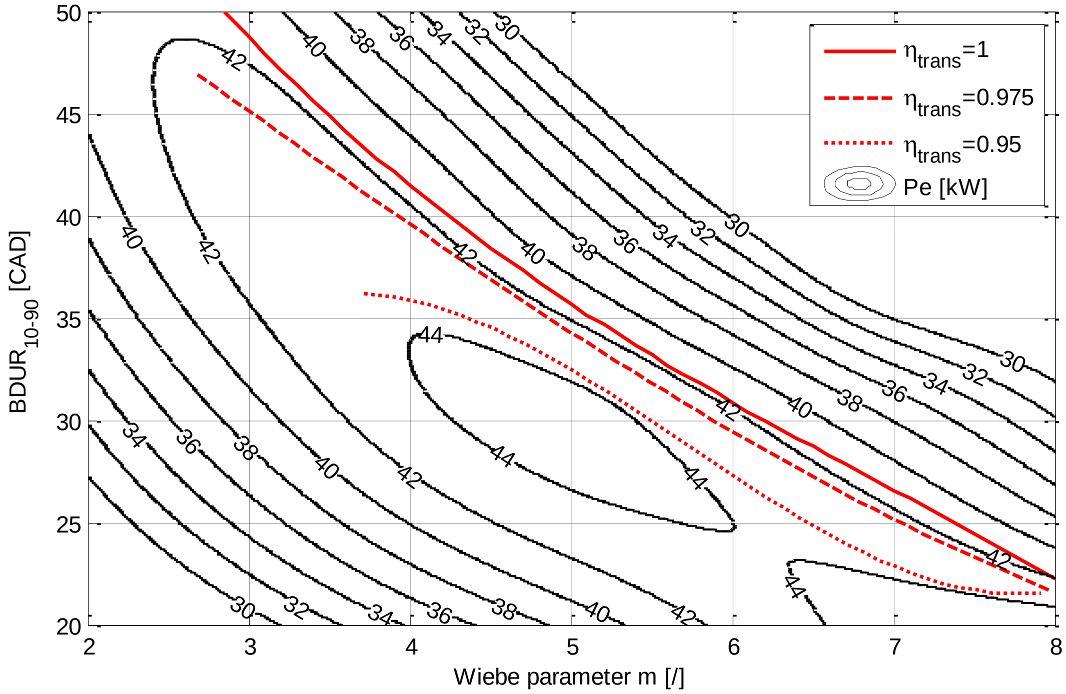

Figure 7 *Simulated engine effective power as function of $BDUR_{10-90}$ and Wiebe shape parameter m*

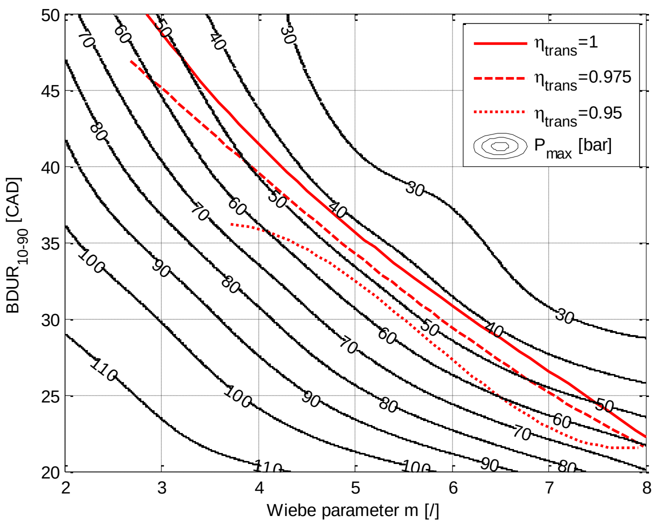

Figure 8 *Maximum cycle pressure as a function of $BDUR_{10-90}$ and Wiebe parameter m*

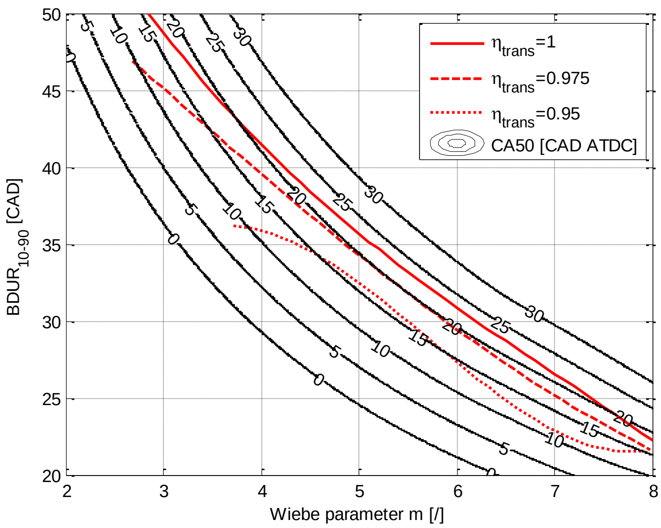

Figure 9 *CA50 ATDC position as a function of $BDUR_{10-90}$ and Wiebe parameter m*

It is important to mention, that all recorded measurements points are gathered as optimal ones, i.e. with optimised value of the spark advance. For example, this particular testing point had a spark advance of 50º CA BTDC. It is well known that the MBT and the spark advance are firmly correlated. Bargende [11] showed that the MBT can be achieved by positioning $CA50$ point within the range of 8º-10º ATDC and this can be easily accomplished by setting the correct spark advance. This "rule" has a sound thermodynamic foundation which is mainly based on the minimal entropy change during combustion process [12]. This optimal spark advance can be accomplished in some parts of the engine operating area, only, since the spark advance toward MBT can lead to knock conditions which limit the further torque rise. If an engine knocking is not experienced, on the particular testing point, then it can be assumed that the MBT optimal spark advance is achieved in accordance with optimal CA50 position.

With an assumption that the overall transmission gear efficiency is $\eta_{trans} = 0.95$, the actual engine brake effective power is estimated as $Pe_{eng_{meas}} = 41.5/0.95 = 43.7kW$. By identifying the second order polynomial coefficients it is possible to obtain the functional relationship between the combustion duration and m shape parameter at WOT testing points as:

$$BDUR_{10-90} = -93.37 + 164 \cdot \eta_{trans} + 7.012 \cdot m - 15.39 \cdot m \cdot \eta_{trans} + 0.2945 \cdot m^2 \tag{6}$$

i.e with $\eta_{trans} = 0.95$ the equation (6) can be transformed and $BDUR_{10-90}$ takes a simpler form:

$$BDUR_{10-90} = -17.63 + 35.05 \cdot m - 7.068 \cdot m^2 + 0.4125 \cdot m^3 \tag{7}$$

The relations, formulated in equations (6) and (7) are in a good accordance with the ones derived by Lindström [13].

The region of interest is located in the area in which the Wiebe shape parameter m is in the range $6 \ldots 7$ and $BDUR_{10-90} = 20 \ldots 25$. By superpositioning the simulation results with the measured brake effective power (± 1%) exhaust temperature (± 5° K) (Figure 10) and brake specific fuel consumption (Figure 11), in the area of achieved $\mathrm{CA50}$=8°…10° ATDC (Figure 9), it is possible to distinguish the most probable Wiebe combustion parameters which are representative of the measured (WOT @ 9000 RPM) testing point:

1. $BDUR_{10-90} = 24°\,\mathrm{CA}$ @ $m = 6.4$, or
2. $BDUR_{10-90} = 25°\,CA$ @ $m = 6.1$

Although a lot of assumptions are introduced in this analysis, a good agreement between the simulated and measured effective parameters can lead to a conclusion that this approach is able to deliver some important information on what is happening in the combustion chamber even without in-cylinder pressure measurement. Having in mind that optimal spark advance corresponds to the optimal position of the $CA50$ combustion indicator, it is possible to narrow the range of the most probable Wiebe combustion parameters which , for the test point discussed, are $BDUR_{10-90} = 24°..25°\,\mathrm{CA}$ @ $m = 6.1…6.4$.

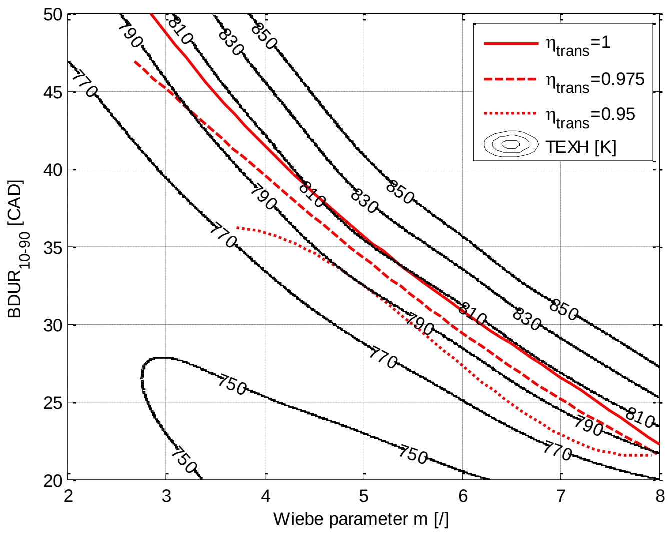

***Figure 10** Exhaust gas temperature as a function of Wiebe parameter m and $BDUR_{10-90}$*

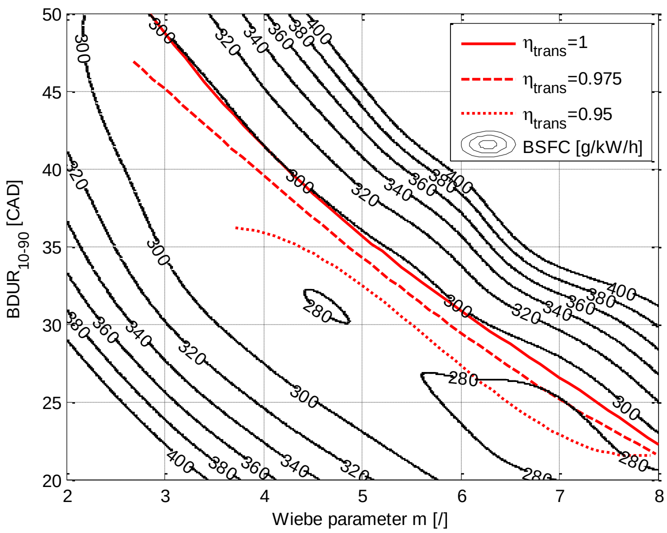

***Figure 11** Brake specific fuel consumption as a function of Wiebe parameter m and $BDUR_{10-90}$*

By advancing and retarding the optimal spark advance and conducting an analyzing these operating points, it is possible to derive the relationship between the SOC and the combustion parameters also. Further spread of this analysis on partial load engine regimes can provide the overall perspective of relationships between the load levels, engine speed, spark advance and combustion parameters.

By using Design of Experiment (DOE) approach, testing plan can be developed which includes only limited number of testing points thus minimising the effort and costs of in detailed test bench engine optimisation. The suggested analysis, relying on one-dimensional thermodynamic simulation of the engine, provides parameters which can be incorporated in more detailed model based calibration process.

CONCLUSIONS

Calibration of the engine maps is a time consuming and a costly process which requires experimental, test bench based research with high quality equipment involved. Due to various limitations, engine testing can be sometimes accomplished on only a limited number of testing points which further diminish the accuracy of estimated spark advance map. A method, suggested in this paper, provides means for establishing the model based calibration process which can significantly improve the quality of the spark advance map. The method is based on the comparison of one dimensional thermodynamic model simulation results and measured effective engine parameters. Relating the on-test-bench accomplished, optimal spark advance, with the thermodynamics and combustion process background enables the estimation of the most probable combustion parameters representing the tested engine point within acceptable accuracy limits. Retarding and advancing the optimal spark advance angle can further give some insight into the relation between the angular position of the combustion start and combustion parameters.

By applying the suggested method on a set of testing points, predefined by a DOE testing plan, enough information can be extracted to form an overall model which describes the relations between the engine load, speed and spark advance on one side, and combustion parameters on the other in the wide area of the engine map.

Commonly used model based calibration methods rely solely on experimentally acquired test bench data, gathered with high quality closed loop controlled test bench equipment. The method suggested here gets the most out of the limited number of measured points and compensates the lack of test data by acceptable accurate thermodynamic simulation.

Although the method is demonstrated on the WOT testing points only, its encouraging results give a clue that the results on partial load regimes can be equally good.

REFERENCES

[1] R. D. Atkins, *An Introduction to Engine Testing and Development*. SAE International, 2009.

[2] A. J. Martyr and M. A. Plint, *Engine Testing: The Design, Building, Modification and Use of Powertrain Test Facilities*. Elsevier, 2012.

[3] “S Series User Manual.” DTA Competition Engine Management Systems, 2012.

[4] J. Fredrick, “In-Cylinder Combustion Analysis of a Yamaha R6 FSAE Engine: Achieving Improved Engine Performance and Efficiency through Burn Rate Combustion Diagnostics,” Master Thesis, Auburn University, 2011.

[5] “Ricardo WAVE 8.4.1, User Manual.” Ricardo, 2012.

[6] G. Woschni, “A Universally Applicable Equation for the Instantaneous Heat Transfer Coefficient in the Internal Combustion Engine,” SAE International, Warrendale, PA, 670931, Feb. 1967.

[7] S. K. Chen and P. F. Flynn, “Development of a Single Cylinder Compression Ignition Research Engine,” SAE International, Warrendale, PA, 650733, Feb. 1965.

[8] G. M. Rassweiler and L. Withrow, “Motion Pictures of Engine Flames Correlated with Pressure Cards,” SAE International, Warrendale, PA, 380139, Jan. 1938.

[9] G. K. Hohenberg, “Basic findings obtained from measurement of the combustion process,” Melbourne, 1982.

[10] M. Tomic, S. Popovic, N. Miljic, S. Petrovic, M. Cvetic, D. Knezevic, and Z. Jovanovic, “A quick, simplified approach to the evaluation of combustion rate from an internal combustion engine indicator diagram,” *Thermal Science*, vol. 12, no. 1, pp. 85–102, 2008.

[11] M. Bargende, “Schwerpunkt-Kriterium und automatische Klingenerkennung Bausteine zur automatischen Kennfeldoptimierung bei Ottomotoren,” *MTZ*, vol. 56, no. 10, pp. 632–638, 1995.

[12] A. Beccari, S. Beccari, and E. Pipitone, “An Analytical Approach for the Evaluation of the Optimal Combustion Phase in Spark Ignition Engines,” *J. Eng. Gas Turbines Power*, vol. 132, no. 3, pp. 032802–11, Mar. 2010.

[13] F. Lindström, H.-E. Ångström, G. Kalghatgi, and C. E. Möller, “An Empirical SI Combustion Model Using Laminar Burning Velocity Correlations,” SAE International, Warrendale, PA, 2005-01-2106, May 2005.