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Identification of a Coupled-Tank Plant and Takagi-Sugeno Model Optimization Using a Whale Optimizer

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***Abstract*— The process industries have continually combated the problem concerning liquid level control. Effective control of a system depends largely on the accuracy of the mathematical model that predicts its dynamic behavior. In this paper the Takagi-Sugeno fuzzy model for the coupled-tank system was acquired based on empirical technique. Furthermore, a metaheuristic algorithm was used as an optimizer on the coupled-tank model. Then, a juxtaposition was made when comparing models which were identified and optimized, leading to satisfactory results. Experimental results obtained on the coupled-tank system are provided.**

***Keywords*—coupled-tank system; Takagi-Sugeno; metaheuristic optimization algorithm; identification; discrete-time systems.**

I. Introduction

The liquid level control is used in the process industries such as petro-chemical, biochemical, spray coating, waste water treatment and purification, beverages and pharmaceutical industries, and within the scope of them presents with an extensive number of applications.

In [1] authors have expressed and emphasized the issue of performance analysis of three control schemes for couple tank system, PI (based on pole placement, Ziegler Nichols and Ciancone correlation tuning methods), PI-plus-feedforward and model predictive control. Moreover, this paper is mostly based of off the research in our previous paper [2], where we dabbled on the similar complexities considering only one tank. In addition, the article [3] combats the problem of the fuzzy-PID controller applied to the nonlinear dynamic model of the liquid level of the coupled tank system, not neglecting the effects of noise. Described by fuzzy IF-THEN rules, the fuzzy model proposed by Takagi and Sugeno [4] depicts local linear input-output relations of a nonlinear system. Furthermore, when it comes to control purposes fuzzy logic has many forms that can be implemented. A procedure used to make two-variable fuzzy logic controllers (FLCs) set for the levels in a laboratory coupled-tank system is submitted in the paper [5].

The fuzzy design can be considered as an optimization problem, where the structure, antecedent, and consequent parameters of fuzzy rules are prerequisites that need to be identified. Metaheuristic methods can deal with non-convex, nonlinear, and multimodal problems subjected to linear or nonlinear constraints with continuous or discrete decision variables as global optimization algorithms. In the literature [6] appealing points of view on this grouping are discussed. In recent years, a dozen metaheuristic methods have been proposed. Some of them include the genetic algorithm (GA) [7], particle swarm optimization (PSO) [8], gray wolf optimization (GWO) [9], whale optimization algorithm (WOA) [10] and ant colony optimization algorithm (ACO) [11]. The WOA has proven to be outstanding at resolving a variety of models, multimodal and problems that are not linear. The foremost supremacies of this algorithm, and all of the metaheuristic algorithms in general, are that it avoids getting stuck in the local minimum because of random distributions.

In this paper, the structure and consequent parameters are known (number of rules, shapes of input membership functions and linear models in the consequent part of the rules), while the antecedent parameters are concluded using the whale optimizing algorithm.

II. Mathematical Modeling

The Coupled Tanks plant is a “Two-Tank” module made up of a pump with a water basin and two tanks as is shown on Fig. 1.

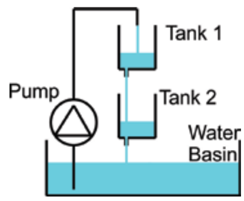

Fig. 1 The Coupled Tanks plant

A. Analytical model

The input into the process is the voltage to the pump V_p and its output is the water level in tank, H_2. The volumetric inflow rate to tank 1, Q_{i1}, is intended to be directly proportional to the applied pump voltage, $Q_{i1} = KV_p$. When applying Bernoulli’s equation for small orifices, the outflow velocity from tank 1, V_{o1}, can be expressed by a succeeding relationship,

$$V_{o1} = \sqrt{2gH_1}, \quad Q_{o1} = A_o V_{o1}, \tag{1}$$

where A_o is an area of the outlet orifice of tank 1 and tank 2, while Q_{o1} is the outflow rate from tank 1. In attaining the tank’s equation of motion the mass balance principle can be applied to the water level in tank, i.e.

$$A_t\dot{H}_1 = Q_{i1} - Q_{o1} = K\,V_p - A_o V_{o1} =$$
$$= K\,V_p - A_o\sqrt{2gH_1}\,, \quad (2)$$

where A_t is the area of tank 1 and tank 2. The nonlinear differential equation that describes the change in level in tank 1 is

$$\dot{H}_1 = \frac{K}{A_t}V_p - \frac{A_o}{A_t}\sqrt{2gH_1}. \quad (3)$$

The water level equation of motion in tank 2 still needs to be obtained. The input to the tank 2 process is the water level, H_1, in tank 1 (generating the outflow feeding tank 2) and its output variable is the water level, H_2, in tank 2 (i.e. bottom tank). The obtained equation of motion should be a function of the system's input and output, as defined beforehand,

$$A_t\dot{H}_2 = Q_{i2} - Q_{o2}\,, \quad (4)$$

$$Q_{i2} = Q_{o1}\,, \quad V_{o2} = \sqrt{2gH_2}. \quad (5)$$

The nonlinear differential equation that describes the change in level in tank 2 is described as

$$A_t\dot{H}_2 = A_o\sqrt{2gH_1} - A_o\sqrt{2gH_2}\,. \quad (6)$$

B. Takagi-Sugeno fuzzy model and identification

The main idea of the TS fuzzy modeling method is to partition the nonlinear system dynamics into several locally linearized subsystems, so that the overall nonlinear behavior of the system could be captured by fuzzy blending of such subsystems. Thus, a fuzzy model and identification of a liquid level system will be implemented in accordance with the TS model containing three rules. The fuzzy rule associated with the i-th linear subsystem, can then be defined as i-th rule:

IF $x_2(k)$ is M_i THEN

$$\boldsymbol{x}(k+1) = A_i\boldsymbol{x}(k) + B_i u(k), \quad i = 1,2,3, \quad (7)$$
$$y(t) = C_i\boldsymbol{x}(t), \quad i = 1,2,3.$$

Here M_i is the fuzzy set, $\boldsymbol{x}(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}$ is the input, $y(k) \in \mathbb{R}$ is the output variable, $A_i \in \mathbb{R}^{2x2}$, $B_i \in \mathbb{R}^{2x1}$, $C_i \in \mathbb{R}^{1x2}$. In our case, the selected state space variable is equal to the output variable $x_2(k) = y(k) = H_2(k)$.

The overall output, using the fuzzy blend of the linear subsystems, will then be as follows:

$$\boldsymbol{x}(k+1) = \frac{\sum_{i=1}^{3} w_i(x_2(k))\{A_i\,\boldsymbol{x}(k) + B_i\,u(k)\}}{\sum_{i=1}^{3} w_i(x_2(k))}, \quad (8)$$

$$h_i(x_2(k)) = \frac{w_i(x_2(k))}{\sum_{i=1}^{3} w_i(x_2(k))}\,, \quad (9)$$

$$\boldsymbol{x}(k+1) = \sum_{i=1}^{3} h_i(x_2(k))\{A_i\,\boldsymbol{x}(k) + B_i\,u(t)\}, \quad (10)$$

$$y(k) = \frac{\sum_{i=1}^{3} w_i(x_2(k))C_i\,x(k)}{\sum_{i=1}^{3} w_i(x_2(k))} = \sum_{i=1}^{3} h_i(x_2(k))C_i\,\boldsymbol{x}(k)\,, \quad (11)$$

where $w_i(x_2(k)) = M_i(x_2(k))$ is the grade of membership of $x_2(k)$ in M_i and $h_i(x_2(k))$ is normalized weight. The linear models in the consequent rules (7) can be obtained by utilizing an analytical linearization of a non-linear equation. Besides that, another approach is to apply the methods of identification in accordance with the measured input output data. The identification methods were used based on the step response. Since models obtained by identification experimentally turned out to be more of an adequate approximation, in comparison with the analytically obtained linearized models, they were used. Linear models can be represented by transfer functions as the relationships of outputs and inputs,

$$G_{1i}(z) = \frac{H_{1i}(z)}{V_{pi}(z)}\,, G_{2i}(z) = \frac{H_{2i}(z)}{H_{1i}(z)}, i = 1,2,3. \quad (12)$$

Nominal levels in the tanks H_{1Ni}, H_{2Ni}, nominal voltages V_{pNi} and corresponding identified Z-transfer functions, for the sampling time $T = 0.01$ second, are given in Table 1. Matrices for the state space plant model A_i and B_i are given in Table 2.

TABLE I
NOMINAL VALUES AND LINEAR MODELS

i	1	2	3
$V_{pNi}\ [V]$	4.4	6	7.1
$H_{1Ni}\ [m]$	0.077	0.1665	0.2415
$H_{2Ni}\ [m]$	0.075	0.1645	0.233
$G_{1i}(z)$	$\frac{2.3124\cdot10^{-5}}{z-0.99952}$	$\frac{2.626\cdot10^{-5}}{z-0.9996}$	$\frac{2.642\cdot10^{-5}}{z-0.9997}$
$G_{2i}\ (z)$	$\frac{6.6976\cdot10^{-4}}{z-0.99928}$	$\frac{5.269\cdot10^{-4}}{z-0.9994}$	$\frac{4.216\cdot10^{-4}}{z-0.9995}$

TABLE II
MATRICES FOR THE STATE SPACE SYSTEM MODEL

i	A_i	B_i
1	$\begin{bmatrix}0.99952 & 0\\ 6.6976\cdot10^{-4} & 0.99928\end{bmatrix}$	$\begin{bmatrix}2.3124\cdot10^{-5}\\ 0\end{bmatrix}$
2	$\begin{bmatrix}0.99958 & 0\\ 5.2685\cdot10^{-4} & 0.99944\end{bmatrix}$	$\begin{bmatrix}2.6264\cdot10^{-5}\\ 0\end{bmatrix}$
3	$\begin{bmatrix}0.99965 & 0\\ 4.216\cdot10^{-4} & 0.99954\end{bmatrix}$	$\begin{bmatrix}2.6415\cdot10^{-5}\\ 0\end{bmatrix}$

In this article a nonlinear TS fuzzy model is obtained by combining three linear models around 0.08 m, 0.16 m and 0.24 m. The membership functions are depicted in Fig. 2.

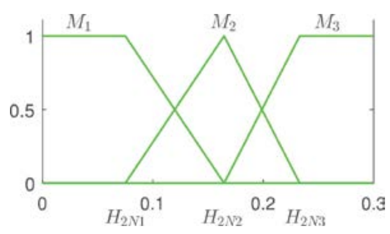

Fig. 2 Membership functions.

III. THE WHALE OPTIMIZER

Whale Optimization Algorithm has demonstrated to be remarkable at solving a variety of nonlinear and multimodal problems. The advantages of this method, and all metaheuristic algorithms in the main, are the random distribution. This distribution allows avoiding getting stuck in the local minimum. WOA has been first proposed by Seyedali Mirjalili and Andrew Lewis in [10]. The paper was inspired by the idea of the attack of dozen whales. The flock consists of a number of whales that hunt, on the principle of surrounding prey, immersion whales to a greater depth, then gradually spiraling to the surface with the release of bubbles which form a "wall" and thus prevent prey from leaving the formed area. The hunt contains three phases. The leader whale has a job to find the fish. The rest follow information. Each takes exactly the same position in every lunch. The first phase is encircling the prey by defining the best search agent and updating the position of others. The mathematical model of this phase is proposed using the distance vector $\boldsymbol{D}$ and vector $\boldsymbol{X}$ which is used to update the position:

$$\boldsymbol{D} = |\boldsymbol{C}\boldsymbol{X}'(t) - \boldsymbol{X}(t)| \,, \tag{13}$$

$$\boldsymbol{X}(t+1) = \boldsymbol{X}'(t) - \boldsymbol{A}\boldsymbol{D} \,, \tag{14}$$

$$\boldsymbol{A} = 2\boldsymbol{a}\boldsymbol{r} - \boldsymbol{a}, \quad \boldsymbol{C} = 2\boldsymbol{r}, \tag{15}$$

where t is the current iteration, $\boldsymbol{A}$ and $\boldsymbol{C}$ are coefficient vectors. Coefficient $\boldsymbol{a}$ is linearly decreased from 2 to 0 and $\boldsymbol{r}$ is a random vector in [0,1]. $\boldsymbol{X'}$ is the position vector of the best solution obtained so far and $\boldsymbol{X}$ is the position vector. The shrinking encircling mechanism (defining the new position of the searching agent using $\boldsymbol{A}$) and the spiral-shaped path (first calculation distance between whale and prey using helix-based movement) are the basic mathematical models that mimic the hunt of the second phase. The new position of the agent is located between the current best agent and the original position. The function for this approach is:

$$\boldsymbol{X}(t+1) = \begin{cases} \boldsymbol{X}'(t) - \boldsymbol{A}\boldsymbol{D} \text{ if } p < 0.5 \\ \boldsymbol{D}' - e^{bl} \cdot cos(2\pi l) + \boldsymbol{X}'(t) \text{ if } p \geq 0.5 \end{cases} \tag{16}$$

where p is a random number in [0,1], b is a constant for defining the shape of the logarithmic spiral, l is a random number in [-1,1] and $\boldsymbol{D}'$ indicates the distance of the i-th whale from the prey [10]. The third phase is based on adoptive variation that depends of the value search vector $\boldsymbol{A}$, which provides good correspondence between first two phases.

IV. Takagi-Sugeno Model Optimization

In the Fig. 2 we observe the beforehand mentioned TS model which was obtained based on the symmetric shape of the membership functions. The configuration of the functions is triangular and the centers of the membership functions are located in the selected nominal points in which the linear models are identified. However, in order to achieve a better approximation of the non-linear characteristics and overall behavior of the plant, a more adequate approximation of the non-linear model is presented by adjusting the parameters of the membership functions. We can view the parameters as the width of the membership functions. So in conclusion, in this case we only optimized the parameters that were located in the rule premise. Moreover, the mentioned TS parameters are all coded into one whale, per say one agent, that is presented with a vector which contains the premise parameters, in our case it has four parameters. In the proposed WOA algorithm the population is set to 40, while the total number of iterations is set to 20. The population size and the number of iterations, viewed as a criteria of stopping, are determined based on a series of experiments with different values, all the while taking in account the specificity of our problem which is that the dimensionality of the problem is small (only 4 unknown parameters). Furthermore, in this optimization method, one agent represents one potential optimal fuzzy model. The mean square error (MSE) is taken as an objective function and it can be calculated as

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y(i) - y_m(i))^2, \tag{17}$$

where n is the number of data points, $y(i)$ is the measured output of the plant, $y_m(i)$ is the output of the model.

A dataset for the learning process of the WOA algorithm, in other words for the optimization of the TS model, is obtained from the plant operation in 1600 seconds. All of the parameter values that were used in the implementation of the WOA were taken from the original paper [10]. In the aim of identification we bring the input voltage which has a shape as depicted in Fig. 3.

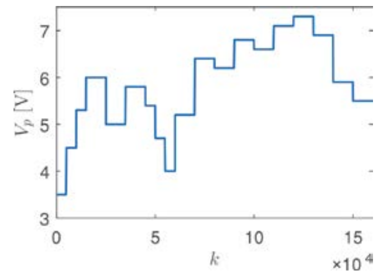

Fig. 3 Voltages used for model optimization

There it should be observed that the values are between the nominal voltages, this is done in order to cover the range of interest. Optimized membership functions are shown on Fig. 4.

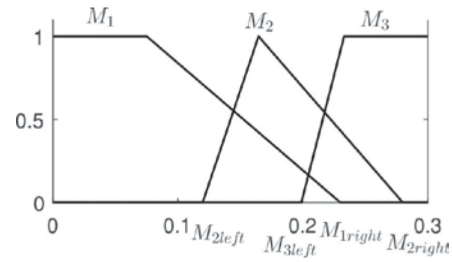

Fig. 4 Optimized membership functions

where $M_{2left} = 0.11975$, $M_{3left} = 0.19875$, $M_{1right} = 0.23$, $M_{2right} = 0.28$.

The same input output data is utilized, with 160001 points that where used for the previous identification, based on the Fig. 3. On the other hand, we now have the second way of identification. This means that we will also acquire unspecified parameters in the rule premise. This approach gives smaller dimensionality of unknown parameters. Fuzzy rules of this system have the same shape as in the eq. (7), with the difference being the selection of the Gaussian membership functions. Three Gaussian membership functions, each with one parameter - slope s_i, for $i = 1,2,3$, were used. Utilizing the WOA for the estimation of the unknown parameters we can conclude that each agent consists of three parameters. Their optimized numerical values are $s_1 = 0.05, s_2 = 0.0134, s_3 = 0.05$. Moreover, the membership functions are shown on Fig. 5.

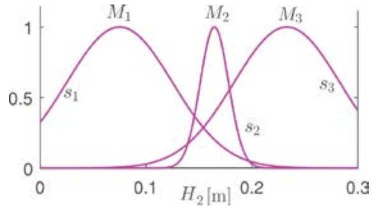

Fig. 5 Optimized Gaussian membership functions

V. Simulation and Experimental Results

Comparison of the nonlinear analytical model (analytical, red color), TS model based on initial membership functions (TS identified, green color), TS model based on optimized membership functions (TS optimized widths and TS optimized slopes, black and magenta colors) with experimentally obtained results (experiment, blue color) is showed on Fig. 6.

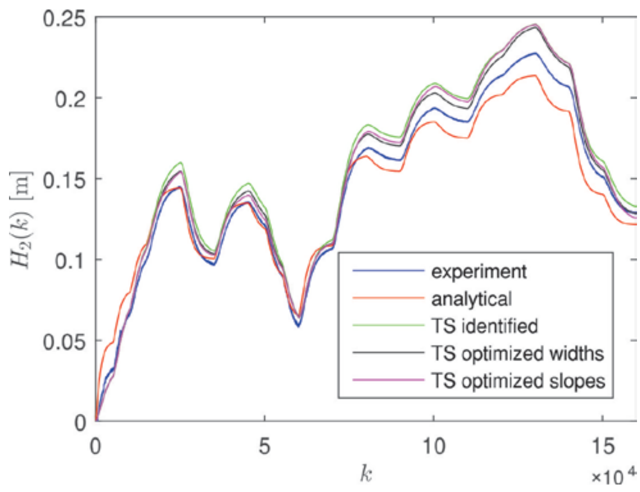

Fig. 6 Comparison of different models with experiment

To explicitly see the improvement caused by the optimization, the numerical values of the MSE, calculated using eq. (17), are given. Mentioned values for four different models (analytical, TS identified, TS optimized widths, TS optimized slopes) are respectively: $1.0054 \cdot 10^{-4}$, $1.4614 \cdot 10^{-4}$, $7.122 \cdot 10^{-5}$, $9.6 \cdot 10^{-5}$. These results show us that the analytical nonlinear model is the least accurate. The initial TS model is better than analytical nonlinear and with optimization we get an even greater improvement in the accuracy of the TS models.

VI. Conclusions

At first, mathematical model of the liquid level system was analytically and experimentally acquired. Further, TS fuzzy model was obtained built on three identified local linear models. In order to improve the model, an optimization, using WOA metaheuristic, was performed using triangular and Gaussian membership functions. Numerical values of MSE are given in order to show the achieved better approximation using optimized membership functions in relation to the analytical and identified model. The initial TS model is better than analytical nonlinear model while optimization further increases the accuracy of the TS fuzzy models. In addition, it is simple and consists only of three fuzzy rules. Future research will focus on other metaheuristic algorithms, using more fuzzy rules and bringing neuro-fuzzy controller in to the set.

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