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Modelling and Speed Control in a Series Direct Current (DC) Machines Using Feedback Linearization Approach

Mitra VESOVIĆ, Radiša JOVANOVIĆ, Vladmir ZARIĆ

Faculty of Mechanical Engineering, University of Belgrade, Kraljice Marije 16, 11120 Belgrade, Serbia
mvesovic@mas.bg.ac.rs, rjovanovic@mas.bg.ac.rs, vzaric@mas.bg.ac.rs, llaban@mas.bg.ac.rs

***Abstract*— In this paper the nonlinear feedback control system is presented for the speed control in direct current - DC motor. Nonlinear functions of dead zone, Coulomb and viscous friction were investigated and used for obtaining the mathematical model. The effectiveness and the comparison between linear and nonlinear control signal have been confirmed using Matlab/Simulink software. From the conclusions, based on the experimental results, it is easy to see that nonlinear control system is more acceptable and has a better performance for speed control. The validity of using feedback linearization in DC motors has been proven.**

***Keywords*— feedback linearization; nonlinear systems; nonlinear control; identification**

I. INTRODUCTION

Speed control in a direct current motor (DC) has been challenging, widely studied task. Many researches have been done to model electrical machines. For example, serial DC motor has often been modelled as linear object. On the other hand, models in which motor current or flux are found as essential parameters are considered to be nonlinear [1]. This paper presents the design and implementation concerning both, linear and nonlinear models for the system and it represents a continuation of the research done by the authors on the similar topic [2]. Disparate controllers have been proposed to lead the speed of DC machines into the desired value. For example Proportional-Integral-Derivative (PID) controller is a popular controller in industries due to simple structure, low cost and easy to implementation. It provides reliable performance for the system if PID parameter is identified properly. But it suffers due to lack of robustness [1]. The linear approximation, of the nonlinear state space representation of the series DC motor, around the equilibrium point and PI controller design the tracking performance is deteriorated in the periods in which the speed is reduced. This is due to the fact that the input signal is limited to a minimum of 0 [V]. That is, in this condition the motor is actually operating in open loop [3].

Besides linear, there are plenty of nonlinear controllers: the fuzzy logic and genetic – based new fuzzy models [4], artificial neural networks [5], adaptive control technique [6], and others.

It is important to make this comparison to find out under what conditions a technique presents a superior performance over the other one and thus have the certainty when it is useful to implement nonlinear controllers, which have greater complexity [7].

The aim of this study is the development and later implementation of a nonlinear control system, by the feedback linearization method, for a laboratory installed DC motor, SRV02 Rotary Servo Base Unit, which has been considered as a single-input-single-output (SISO) system.

Feedback linearization is an approach to nonlinear control design which has attracted a great deal of research interest in recent years. By a combination of a nonlinear transformation and state feedback (feedback linearization), the nonlinear control design is reduced to designing a linear control law [8]. The central idea of the approach is to algebraically transform a nonlinear system dynamics into a (fully or partly) linear one, so that linear control techniques can be applied. This differs entirely from conventional linearization in that feedback linearization is achieved by exact state transformations and feedback, rather than by linear approximations of the dynamics [9]. This technique has been successfully implemented in many applications of control, such as industrial robots, high performance aircraft, helicopters and biomedical dispositifs, more tasks used the methodology are being now well advanced in industry [10].

II. LINEAR MODEL OF SYSTEM DYNAMICS

One of the first steps in the synthesis of a control system is constructing an accurate model, because it saves time and it brings the cost-effectiveness. An appropriately developed system model is essential for reliability of the designed control. A DC series motor is an example of a simple, controlled process that can serve as a vehicle for the evaluation of the performance of the various controllers [4].

A schematic diagram of the DC motor is given in Fig. 1.

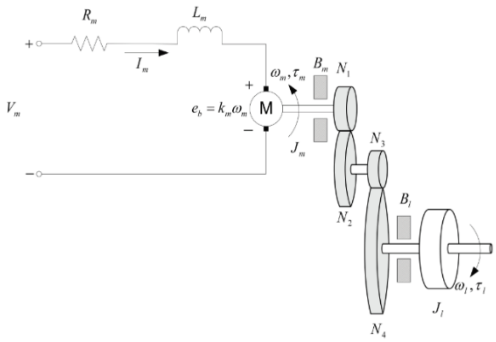

Fig. 1 SRV02 DC motor armature circuit and gain train [11]

The equations that describe the motor electrical components are as follows:

$$V_m(t) = R_m I_m(t) + L_m \frac{dI_m(t)}{dt} + e_b(t) \qquad (1)$$

$$e_b(t) = k_m \omega_m(t) \qquad (2)$$

where V_m , e_b , k_m and ω_m are motor voltage back electromotive voltage, back electromotive voltage constant and speed of the motor shaft, respectively. Since the motor inductance L_m is much less than its resistance R_m , it can be ignored [11]. Solving the system of equations for motor current I_m , we get an electrical equation of DC motor:

$$I_m(t) = \frac{V_m(t) - k_m \omega_m(t)}{R_m}.$$

The linear model can be obtained using the Second Newton's Law of Motion and connection between moment of inertia of the load J_l and of the motor shaft J_m, speed of the load shaft ω_l, viscous friction acting on the motor shaft B_m and on the load shaft B_l, total torque applied on the load τ_l and on the motor τ_m, with resulting torque acting on the motor shaft from the load torque denoted as τ_{ml}:

The linear model can be obtained using the Second Newton's Law of Motion and connection between moment of inertia, viscous friction constants, and torque of load and motor:

$$J_l \frac{d\omega_l(t)}{dt} + B_l \omega_l(t) = \tau_l(t) \qquad (4)$$

$$J_m \frac{d\omega_m(t)}{dt} + B_m \omega_m(t) + \tau_{ml}(t) = \tau_m(t) \qquad (5)$$

so the mechanical equation is:

$$J_{eq} \frac{d\omega_l(t)}{dt} + B_{eq} \omega_l(t) = \eta_g K_g \tau_m(t) \qquad (6)$$

where J_{eq} and B_{eq} are total moment of inertia and damping term. η_g and K_g are, respectively, the gearbox efficiency and the total gear ratio. Combining electrical and mechanical equations, assuming that motor torque is proportional to the voltage, the final equation becomes:

$$\left(\frac{d}{dt}\omega_l(t)\right) J_{eq} + B_{eq,v} \omega_l(t) = A_m V_m(t) \qquad (7)$$

where the equivalent damping term is given by:

$$B_{eq,v} = \frac{\eta_g K_g^2 \eta_m k_t k_m + B_{eq} R_m}{R_m} \qquad (8)$$

and the actuator gain equals:

$$A_m = \frac{\eta_g K_g \eta_m k_t}{R_m} \qquad (9)$$

Linear mathematical model is:

$$J_{eq} \dot{\omega}_l(t) + B_{eq,v} \omega_l(t) = A_m V_m(t). \qquad (10)$$

Choosing $y = \omega_l$ as output variable and $u = V_m$ as input signal, state equation of the system is obtained as follows:

$$J_{eq} \dot{y}(t) + B_{eq,v} y(t) = A_m u(t). \qquad (11)$$

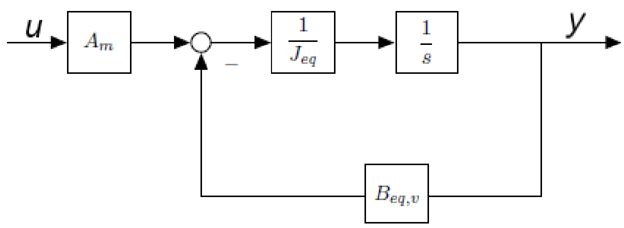

Fig. 2. Block diagram of a linear system

III. EXPERIMENTAL VERIFICATION OF THE OBTAINED LINEAR MATHEMATICAL MODEL

Responses of the system represented with the block diagram in the Fig. 2 are shown in the Fig. 3 and Fig. 4. After recording the responses of the object, comparisons were made with the responses obtained by simulations of the linear model, for step and sinusoidal inputs [2].

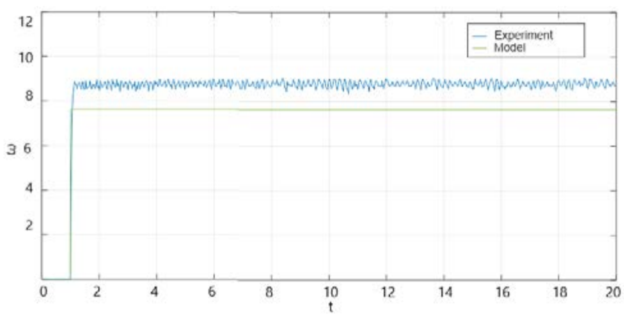

Fig. 3 Experimental results: comparison between real and model data for step input

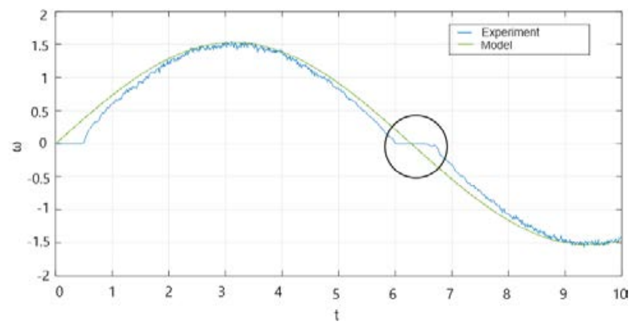

Fig. 4 Experimental results: comparison between real and model data for sinusoidal input

IV. FEEDBACK LINEARIZATION

Feedback linearization approach differs from the classical linearization (about the desired equilibrium point) in that no approximation is used; it is exact. Exactness, however, assumes perfect knowledge of the state equation and uses that knowledge to cancel the nonlinearities of the system. Since perfect knowledge of the state equation and exact mathematical cancellation of terms are almost impossible, the implementation of this approach will almost always result in a close-loop system, which is a perturbation of a nominal system whose origin is exponential stable. The validity of the method draws upon Lyapunov theory for perturbed systems [12] (that can be further studied in Chapter 9 of literature [12]).

Consider the single – input – single – output nonlinear SISO system [12]:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + g(\boldsymbol{x})u \qquad y = h(\boldsymbol{x}) \tag{12}$$

where $\boldsymbol{f}(\boldsymbol{x})$, $g(\boldsymbol{x})$ and $h(\boldsymbol{x})$ are sufficiently smooth in a domain $D \subset R^n$ (the mapping $f : D \longrightarrow R^n$, $g : D \longrightarrow R^n$ are vector fields on D) and $\dot{\boldsymbol{x}} = [x_1\, x_2 \dots x_n]^T$ is a state vector. It is necessary to find a state feedback control u , that transforms the nonlinear system into an equivalent linear system. Clearly, generalization of this idea is not possible in every nonlinear system: there must be a certain structural property that allows performing in such a manner of cancellation.

Using feedback to cancel nonlinearities requires the nonlinear state equation to have a structure:

Definition [12]:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + B\gamma(\boldsymbol{x})[u - \alpha(\boldsymbol{x})] \tag{13}$$

where $\boldsymbol{A}$ is $n \times n$ and B is $n \times p$ matrix, the functions $\alpha : R^n \longrightarrow R^p$, $\gamma : R^n \longrightarrow R^{p \times p}$ are defined on domain $D \subset R^n$ that contains the origin. Furthermore, two conditions must be satisfied. The first one is that the pair (A, B) must be controllable. The second one is that $\gamma(x)$ must be nonsingular for all $x \in D$. This is consequence of the control law form: $u = \alpha(x) + \frac{1}{\gamma(x)}v$ that provides a new control signal v. Even if the state equation does not have the structure (13), sometimes it is possible to execute feedback linearization for another choice of variables. Therefore, a more comprehensive definition is given [12].

A nonlinear system:

$$\dot{x} = f(x) + G(x)\,u \tag{14}$$

where $f : D \longrightarrow R^n$ and $G : D \longrightarrow R^{n \times p}$ are sufficiently smooth on a domain $D \subset R^n$, is said to be feedback linearizable (or input – state linearizable) if there exist a diffeomorphism $T : D \longrightarrow R^n$ such that $D_z = T(D)$ contains the origin and the change of variables $z = T(x)$ transforms the system (12) into the form:

$$\dot{z} = Az + B\gamma(x)[u - \alpha(x)] \tag{15}$$

with (A,B) controllable and $\gamma(x)$ nonsingular for all $x \in D$.

V. DETERMINATION OF THE RELATIVE DEGREE

The relative degree of a linear system is defined as the difference between the poles (degree of the transfer function's denominator polynomial number) and zeros (degree of its numerator polynomial). To extend this concept to nonlinear systems more mathematical treatment will be needed. The following definition is given and repeated here for completeness: Definition [13]: The system, outlined in (12), is said to have relative degree r at a point x_0 if:

i) $L_g\, L_f^k\, h(x) = 0$ for all x in a neighborhood of x_0 and all $k < r - 1$

ii) $L_g\, L_f^{r-1}\, h(x) \neq 0$

The terms L_g and L_f^k represent the Lie derivative of $h(x)$ taken along $g(x)$ and $k - \textit{times}$ along (x) , respectively.

NONLINEAR MATHEMATICAL MODEL

The nonlinear mathematical model of the DC motor was obtained considering the speed dependent friction nonlinearity. Reference [14] shows that in this case the nonlinear mathematical model of DC motor can be adopted as follows:

$$J_{eq}\dot{\omega}_l + T_{st}(\omega_l) + B_{eq,n}\,\omega_l = A_m V_m \tag{16}$$

The part of the obtained friction curve $T_{st}(\omega_l)$, for low angular velocity values, where the Stribeck effect is dominant, is shown in Fig. 5. It is assumed that friction characteristics are symmetrical, for negative and positive values of angular velocity.

(13)

TABLE I THE NUMERICAL VALUES OF THE PLANT PARAMETERS

Parameters	Values and units
Jeq	0.0021 kgm^2
Rm	2.6 Ω
kt	0.0077 Nm/A
ηm	0.69
ηg	0.9
Kg	70

It is assumed that friction characteristics are symmetrical, for negative and positive values of angular velocity. Applying standard optimization techniques with Matlab, the friction parameters were obtained (16).

In order to overcome the jump discontinuity of the proposed friction model, at $\omega_l = 0$, that jump is replaced by a line of finite slope, up to a very small threshold ε, whose boundaries are given with red dash line, as is shown in Fig. 5 [14].

$$T_{st} = 0.0174 sgn(\omega_l) + 0.0087 e^{-\frac{\omega_l}{0.064}} sgn(\omega_l), \; B_{eq,v} = 0.0721 \tag{17}$$

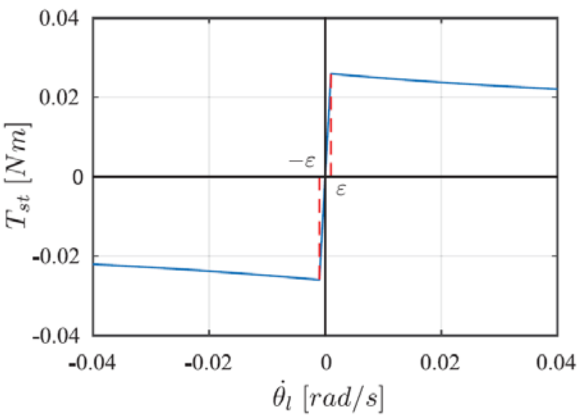

Fig. 5. Friction characteristics of DC motor [14]

This the line of finite slope will be used only for comparison with the hyperbolic tangent function (Fig. 6), because method of feedback linearization requires differentiable functions (as can be seen from the given definitions in the previous section). In this way only Coulomb and viscous friction is modelled and static friction is neglected [2].

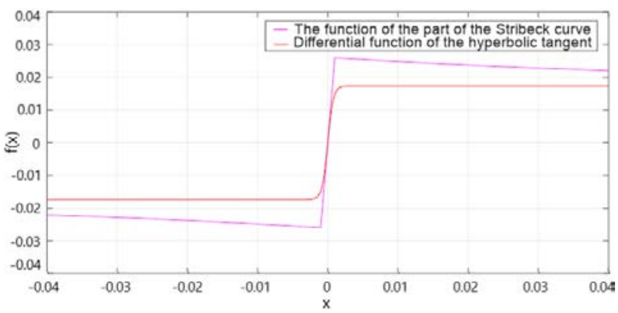

Fig. 6. Differential function of the hyperbolic tangent

Choosing $x = \omega_l$ as state variable, $y = \omega_l$ as measured variable and $u = V_m$ as control variable and denoting nonlinearity by $f(x)$, state equation of the system was obtained as follows:

$$\dot{x} = -\frac{B_{eq,n}}{J_{eq}} x - f(x) + \frac{A_m}{J_{eq}} u \tag{18}$$

$$y = x \tag{19}$$

To ensure that this model is an equivalent representation of the original system, an experiment was performed, with the results shown below on Fig. 7 for step and Fig. 8 for sinusoidal response.

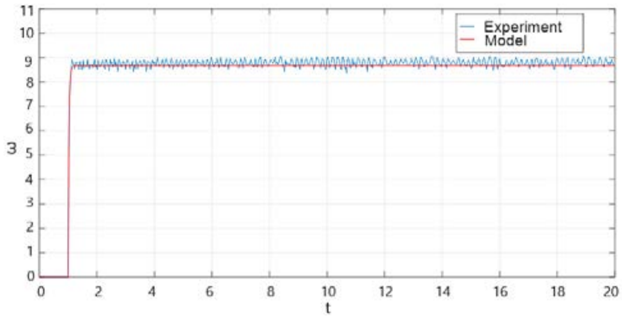

Fig. 7. Experimental results: comparison between real and model data for step input

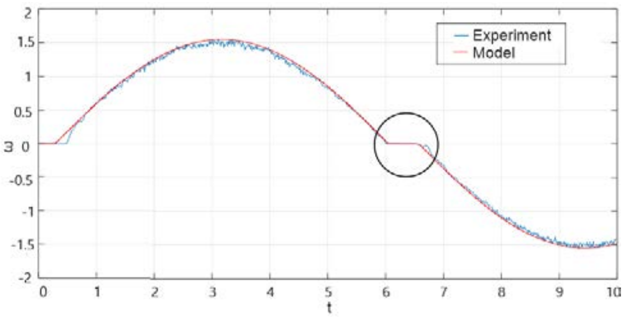

Fig. 8. Experimental results: comparison between real and model data for sinusoidal input

VI. EXPERIMENTAL RESULTS

Applying Definition [12] to the system (18) – (19) yields:

$$A = -\frac{B_{eq,n}}{J_{eq}} \tag{20}$$

$$B = \frac{A_m}{J_{eq}} \tag{21}$$

$$\alpha(x) = \frac{J_{eq}}{A_m} f(x) \tag{22}$$

$$\gamma(x) = 1. \tag{23}$$

First condition is met:

$$U = B. \tag{24}$$

Order of system is n = 1 and, because rank U =n, the pair (A, B) is controllable:

$$U = B = \frac{A_m}{J_{eq}}. \tag{25}$$

System transformation is not required and all functions are smooth and differentiable. γ(x) is not equal to zero, so the second condition is also met. With both conditions met feedback linearization is permitted.

The first derivative of the system (18) – (19) output depends on the control signal, which means that the relative degree of the system is 1:

$$y = x. \tag{26}$$

$$\dot{y} = \dot{x} = L_f h(x) + L_g h(x) u \tag{27}$$

$$\dot{x} = -\frac{B_{eq,n}}{J_{eq}}x - f(x) + \frac{A_m}{J_{eq}}\,u \tag{28}$$

$$L_f\,h(x) = -\frac{B_{eq,n}}{J_{eq}}x - f(x) \tag{29}$$

$$L_g h(x) = \frac{A_m}{J_{eq}} \tag{30}$$

Conclusion is that relative degree of this system is equal to the system order $r = 1$. The desired time – domain specifications for controlling the position of the load shaft are overshoot: $PO < 5\%$ and peak time: $t_p \leqslant 0.05\ s$. Choosing the control signal in the following form:

$$u = \frac{J_{eq}}{A_m}[f(\boldsymbol{x}) + v] \tag{31}$$

with $v = K_p\varepsilon + K_i\int_0^t \varepsilon d\tau$, where $K_p = 1.34$, $K_i = 124.9$, are obtained by calculating the minimum damping ratio and natural frequency and x_{ref} is desired output or reference, system is linearized. Linear control is obtained in the same way, with the same coefficients, but without canceling the nonlinearity:

$$u_l = K_p\varepsilon + K_p\int_0^t \varepsilon d\tau \tag{32}$$

The experiments were performed with Quanser rotary servo motor, SRV02. This model is equipped with the optical encoder and tachometer, for motor position and speed measuring, respectively [14].

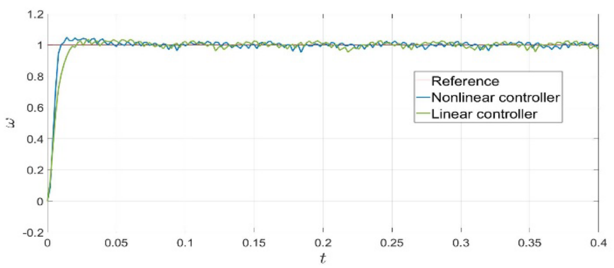

Fig. 9. Experimental results: speed tracking of step signal for the linear and nonlinear controller

The advantages of a nonlinear controller, for a step input, are seen in the shorter peak time and a little bit faster system response, although the output controlled with linear controller has a slightly smaller overshoot. Greater superiority of the nonlinear controller can be observed from the sinusoidal inputs (or any other inputs that consist change of the direction in the rotation of the load shaft in the motor because of the friction effect, which is the most noticeable in those cases).

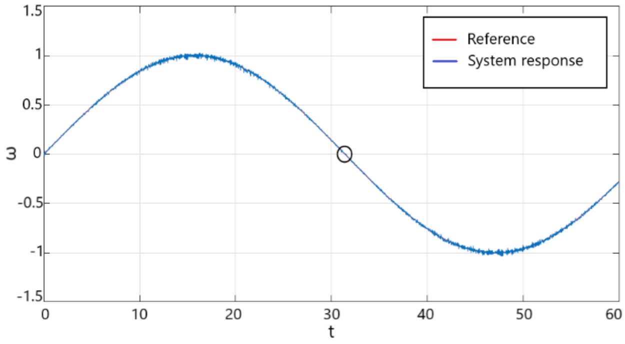

Fig. 10. Experimental results: speed tracking of sine signal for the nonlinear and nonlinear controller

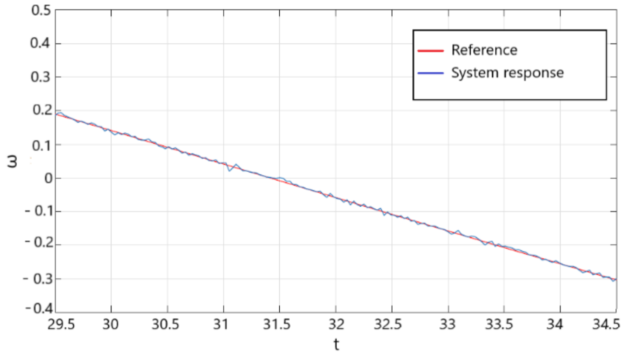

Fig. 11. Detail from Fig. 10.

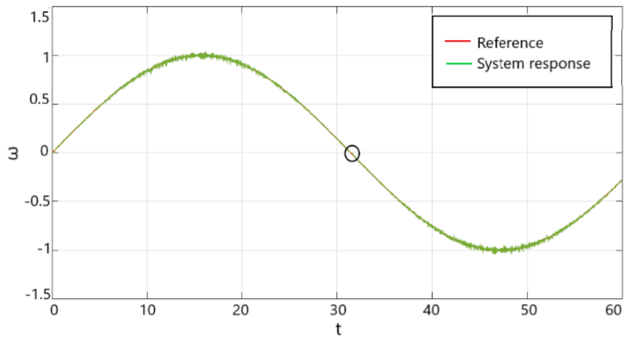

Fig. 12. Experimental results: position tracking of sine signal for the linear and nonlinear controller

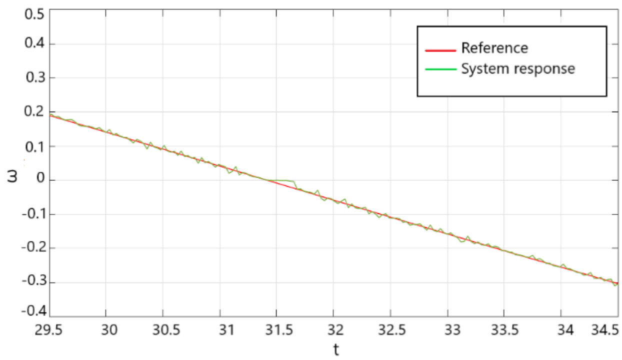

Fig. 13. Detail from Fig. 12.

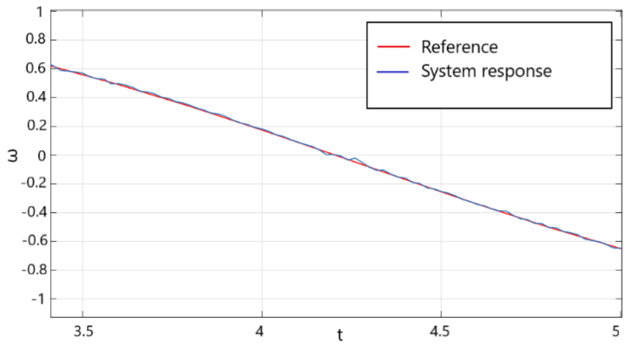

Fig. 14. Detail from experimental results: position tracking of chirp signal for the nonlinear controller

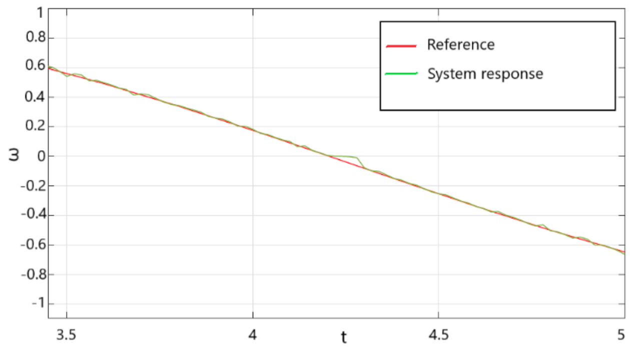

Fig. 15. Detail from experimental results: position tracking of chirp signal for the linear controller.

It can be observed, from the Fig. 9 - 15, that the specific requirements are met. The overshoot and the peak time are in the domain of desired values. Furthermore, it is observed that the nonlinear controller is more convenient and has better achievements for speed management.

VII. CONCLUSION

The feedback linearization method was proposed for controlling the DC motor. The goal was to confirm this method for controlling speed of the load shaft. After it has been shown that linear equations do not track the behavior of the object well enough, the nonlinear model was proposed by including Stribeck model of the friction. The conditions for fulfilling feedback linearization approach were studied. In order to satisfy those conditions an approximation of the function, which represent nonlinearity, was found as hyperbolic tangent.

It could be noted, through the experiment and analysis results, that the desired response was followed by the plant response. The comparison of the linear and nonlinear controller is given. The results show that both of the controllers are able to satisfy requirements, but that nonlinear controller gives better results.

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